Aggressive Design in Turbomachinery

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CFD Methods, DSE

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Introduction

Uncertainty Quantification (UQ)

- UQ is not just about an error bar

- It is a rapidly developing field encompassing
  - CFD prediction
  - Meshing and geometry generation and processing
  - Algorithms for efficient sensitivity analysis
  - Computationally tractable frameworks for robust design
  - Statistical analysis on sparse data

- Must be factored when designing with models for engine
  - Uncertainties (variability) exist in both models & engine

- Goal of UQ research in CFD methods is to
  - **Increase engine efficiency** given variability
  - **Maintain engine efficiency** given variability

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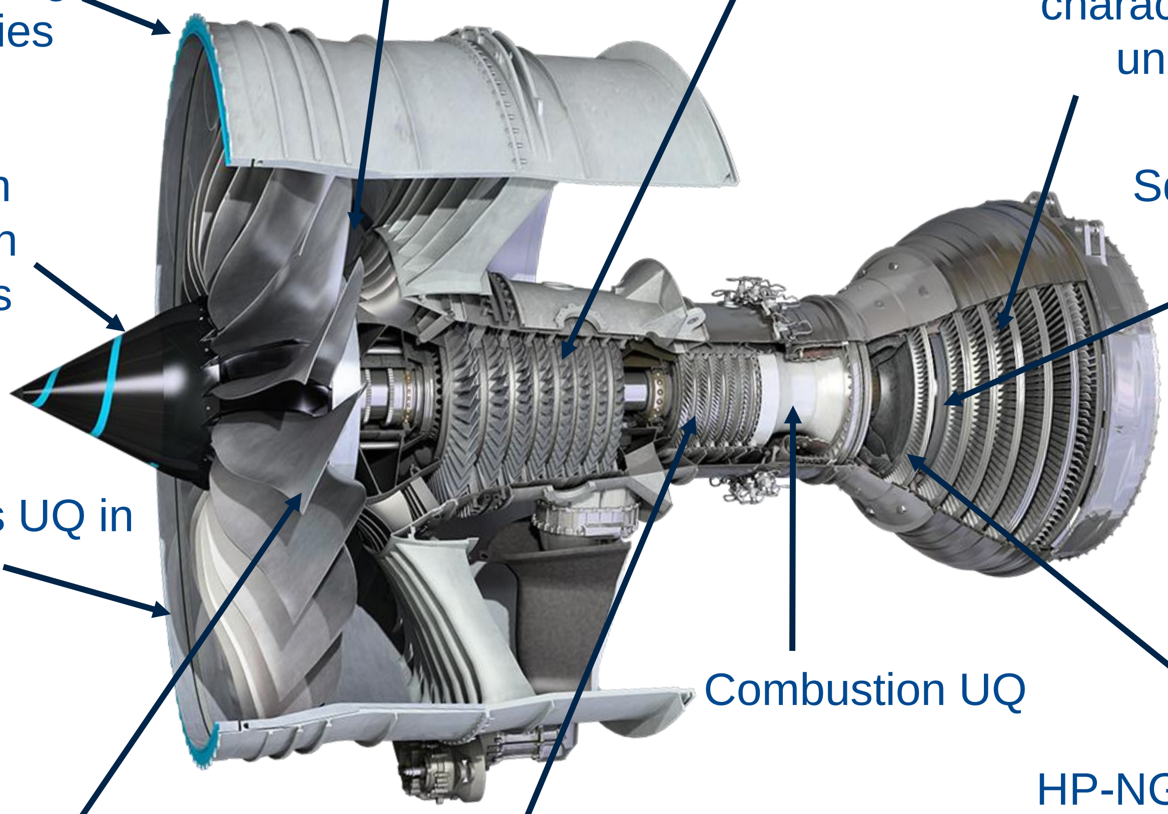
Engine Aleatory Uncertainties



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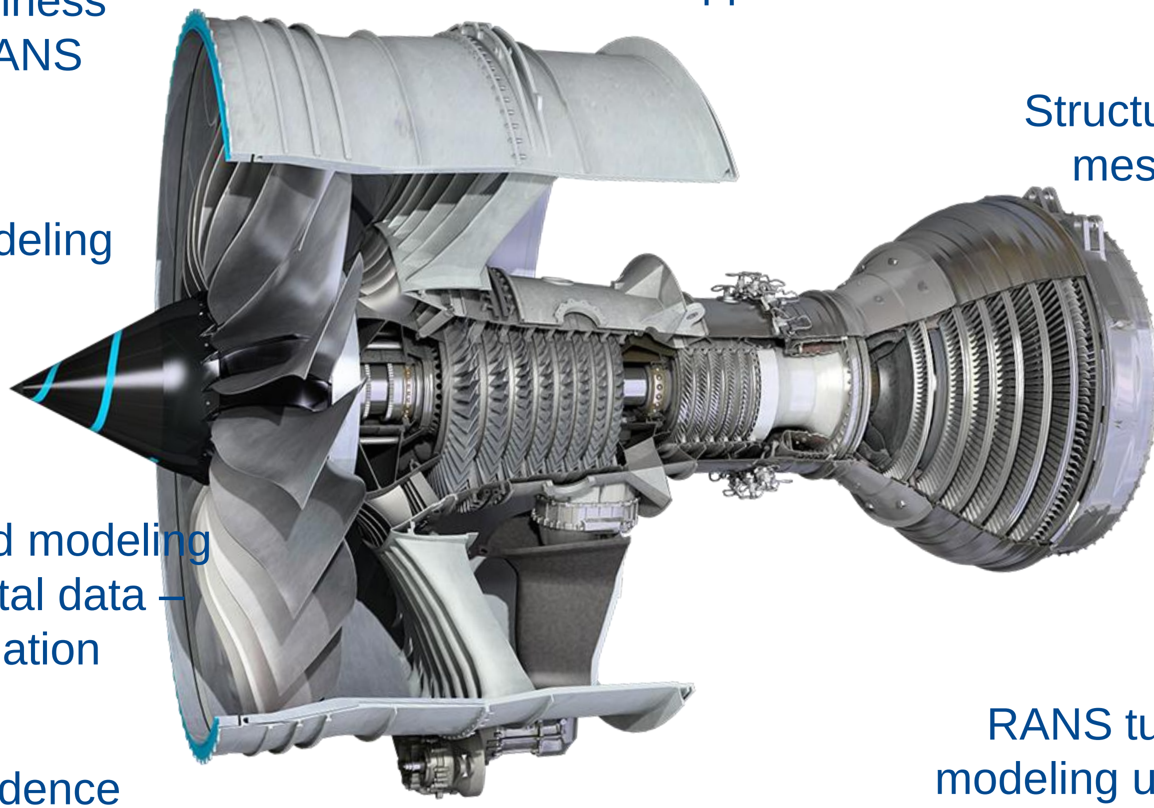
Engine Epistemic Uncertainties

Hybrid RANS-LES approaches

Surface roughness models in RANS

Structured / unstructured meshing techniques

Transition modeling



Bayesian hybrid modeling for experimental data – CFD validation

RANS turbulence modeling uncertainties

Mesh independence during optimization

Conjugate heat transfer modeling uncertainties

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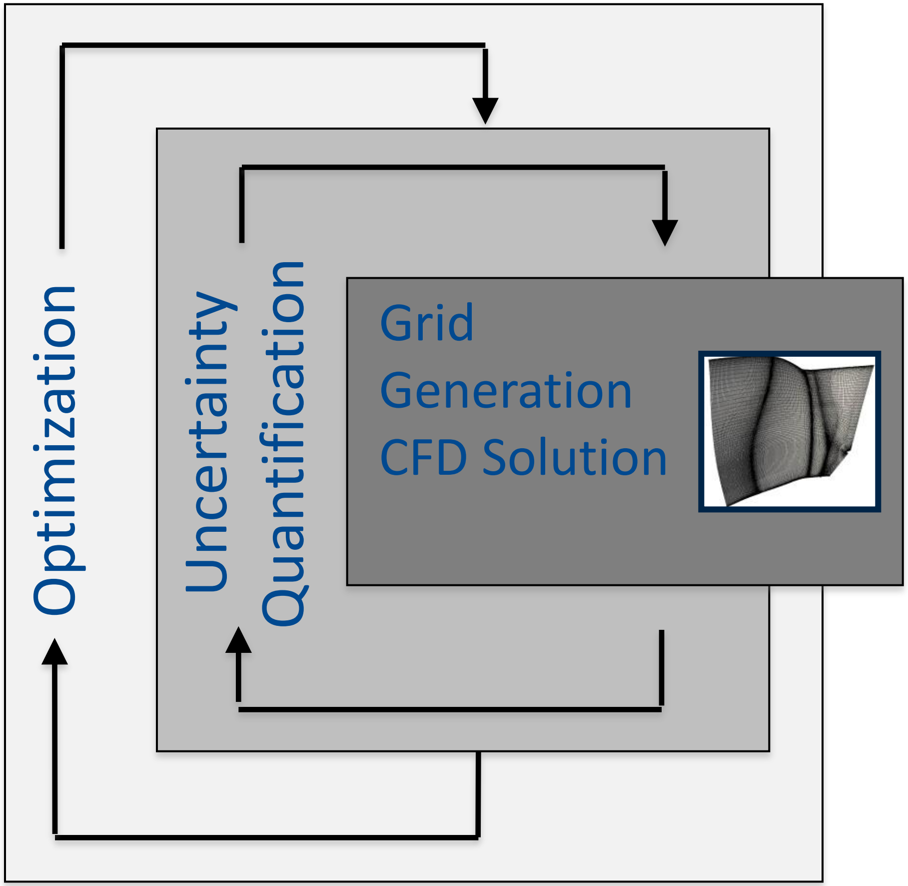
Design Methodology with Uncertainty



Rolls-Royce CFD Methods
3D Designs

Optimization: SOFT
Uncertainty Quantification: SOFT+UQ
Grid & Geometry Generation: PADRAM
CFD Solution: HYDRA

Mathematical Formulation

- Optimization under uncertainty

$$\underbrace{f}_{\text{physical model output}} = f(\underbrace{s}_{\text{design variables}}, \underbrace{\omega}_{\text{uncertain variables}})$$

$$\underset{s}{\text{minimize}} \; f(s, \omega)$$

Mathematical Formulation

- Optimization under uncertainty

$$f = f(s, \omega)$$

physical model output

design variables

uncertain variables

$$\underset{s}{\text{minimize}} \; f(s, \omega)$$

- Methods for optimization under uncertainty
  - Robust design, reliability based design optimization, first order reliability method, second order reliability method, most probable point,…
  - Some methods optimize moments, others optimize tails.

Mathematical Formulation

$$\underset{s}{\text{minimize}} \; f(s, \omega)$$

optimization goal is a random function

$$\underset{s}{\text{minimize}} \quad \mathbb{E}\{f(\omega, s)\}$$

Take the expectation

Mathematical Formulation

$$\underset{s}{\text{minimize}} \; f(s, \omega)$$

optimization goal is a random function

$$\underset{s}{\text{minimize}} \quad \mathbb{E}\{f(\omega, s)\}$$

Take the expectation

Scalarization

$$\underset{s}{\text{minimize}} \quad |\alpha \mathbb{E}\{f(\omega, s)\} + (1-\alpha)\sigma\{f(\omega, s)\}|$$

Multi-objective optimization

$$\underset{s}{\text{minimize}} \quad \begin{array}{l} \mathbb{E}\{f(\omega, s)\} \\ \sigma^2\{f(\omega, s)\} \end{array}$$

optimization goals are deterministic functions

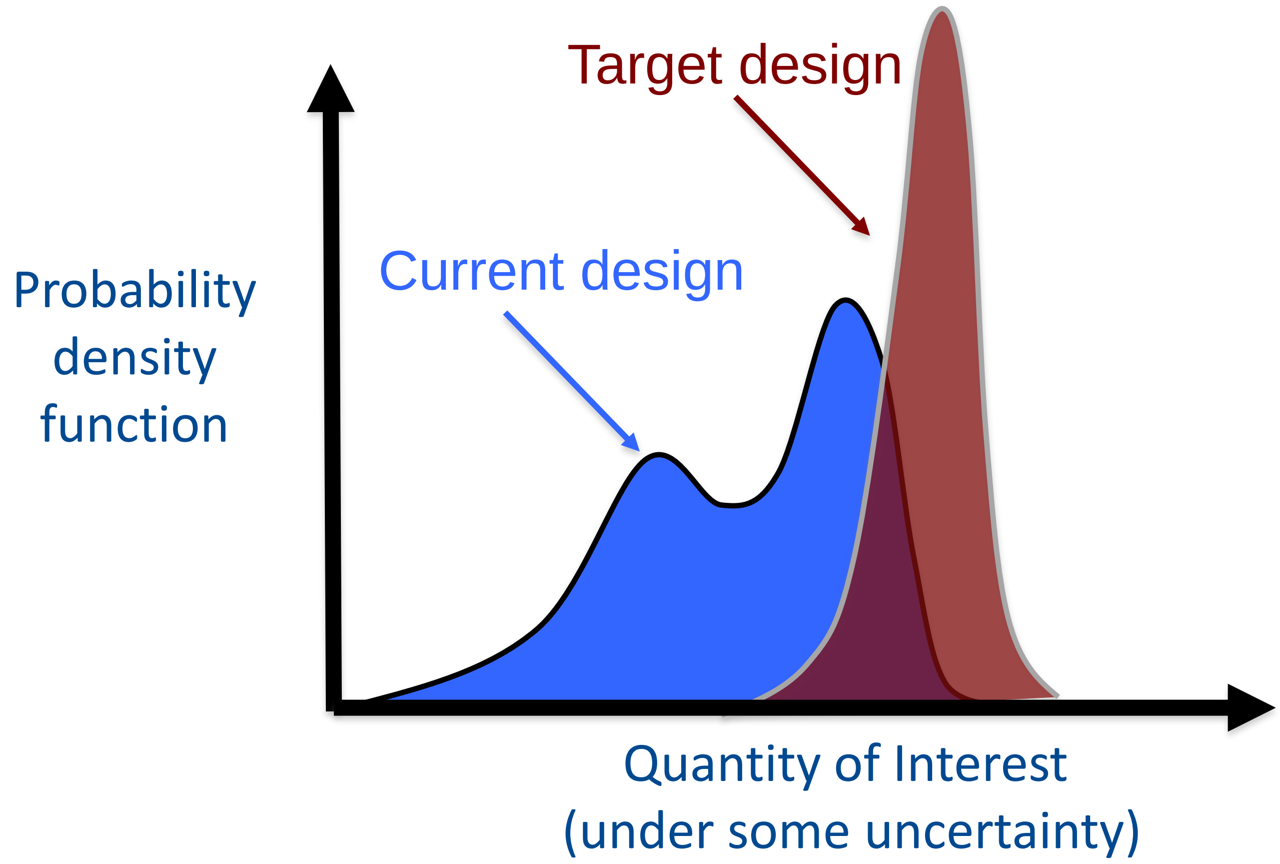
Some Issues

- Scalarization requires apriori knowledge
- Cost of Multi-Objective Optimization
  - Requires many objective function evaluations
  - Order of magnitude more expensive than single-objective problem
- What if a large variance is permissible (the PDF has a favorable skew)?
  - Skewness is not factored in robust design
  - Mean and variance do not uniquely define a PDF
- What if mean and variance are correlated?
- Challenging to optimize for a certain tail probability

These issues motivate the present work

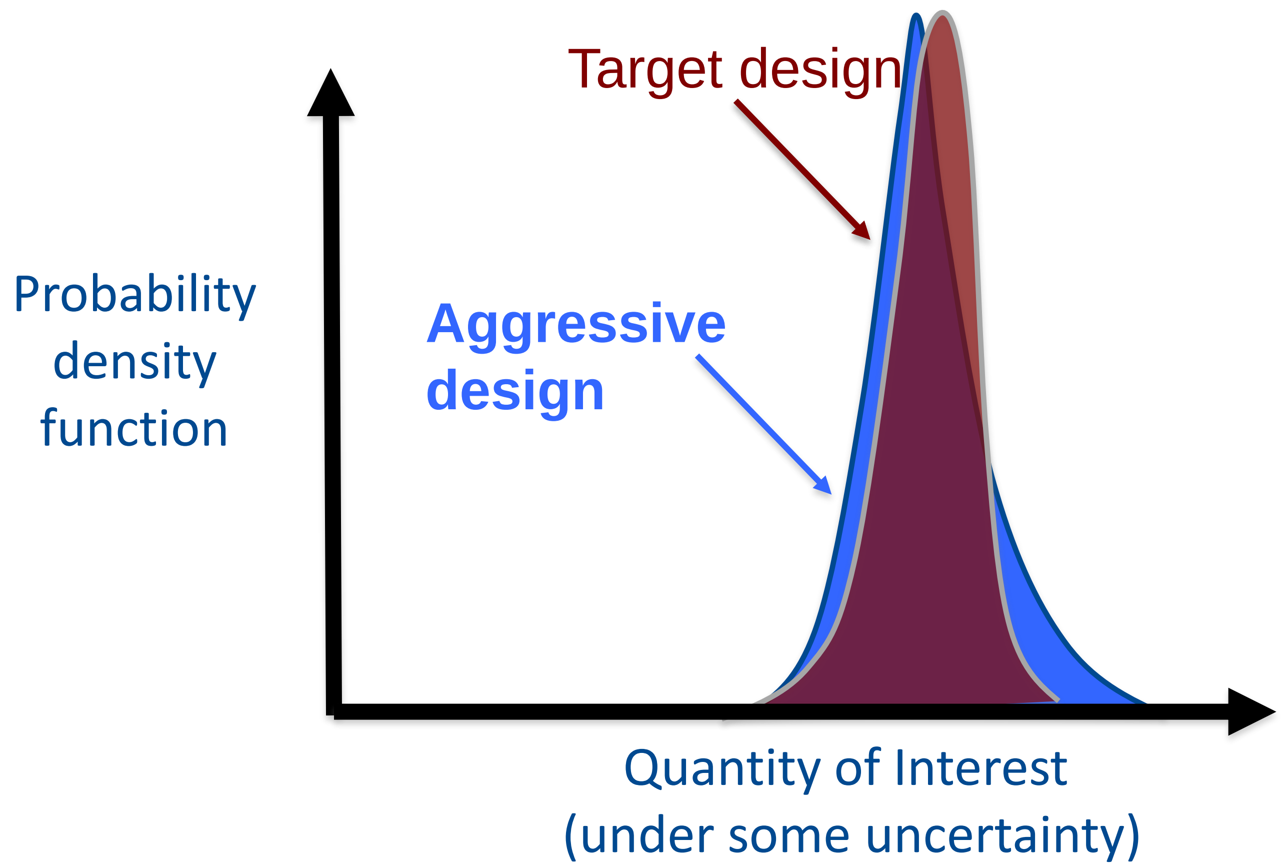
Mathematics of Aggressive Design

Aggressive design



Aggressive design seeks to minimize the “distance” between the current design PDF and the target design PDF

Aggressive design



Aggressive design seeks to minimize the "distance" between the current design PDF and the target design PDF

Mathematical Formulation

$$f = f(s, \omega)$$

physical model output

design variables

uncertain variables

Designer specifies target pdf of output

$$t = t(f) \geq 0, \quad \int t(f)\, df = 1$$

For a fixed design s, uncertainty produces pdf of f

$$u_s = u_s(f) \geq 0, \quad \int u_s(f)\, df = 1$$

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Mathematical Formulation

Goal is to find the design so that model pdf is as close as possible to the designer's target.

$$\underset{s}{\text{minimize}} \quad \delta(t, u_s)$$

where delta is a distance metric. We choose

$$\delta(t, u_s) \;=\; \int (t - u_s)^2 \, df$$

because it is differentiable.

Discretize the 'distance'

Choose an integration rule in the range of the model output

$$\delta \approx \tilde{\delta} = \sum_{i=1}^{M} \left(t(f_i) - u_s(f_i)\right)^2 w_i$$

$$= (\mathbf{t} - \mathbf{u}_s)^T \boldsymbol{W} (\mathbf{t} - \mathbf{u}_s)$$

where W is a diagonal matrix of integration weights, t is a vector of target pdf evaluations, and u_s is a vector of model pdf evaluations.

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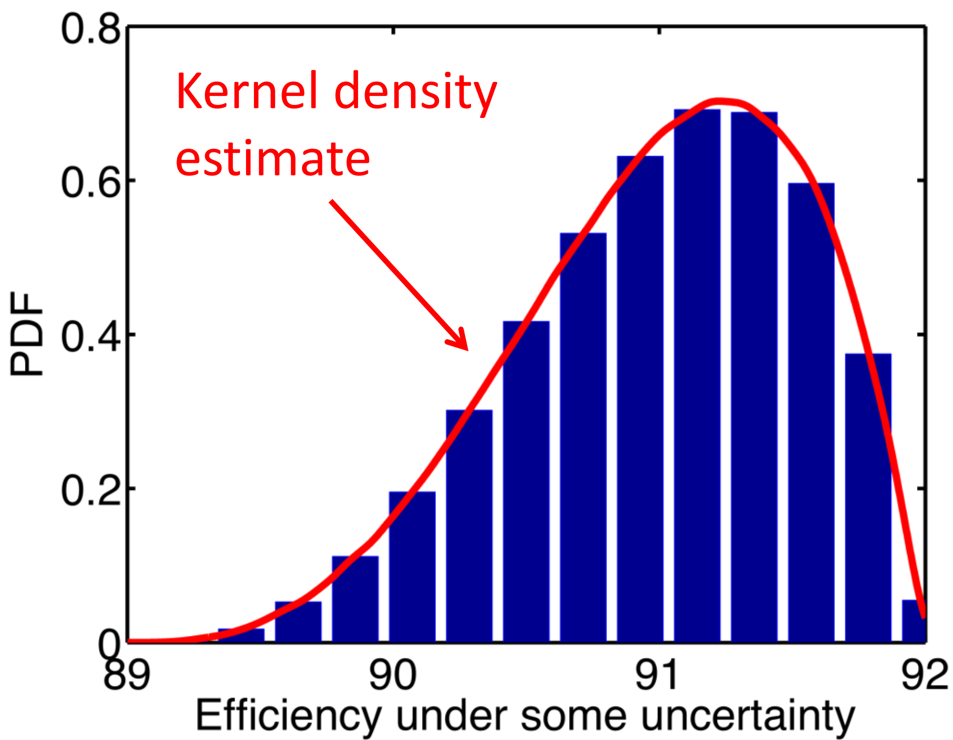
Quick detour: Kernel Density Estimation (KDEs)

Kernel density estimation is a statistically well-known alternative to histograms

Idea is to replace discrete bins with a unimodal kernel function to obtain an analytic definition for a PDF



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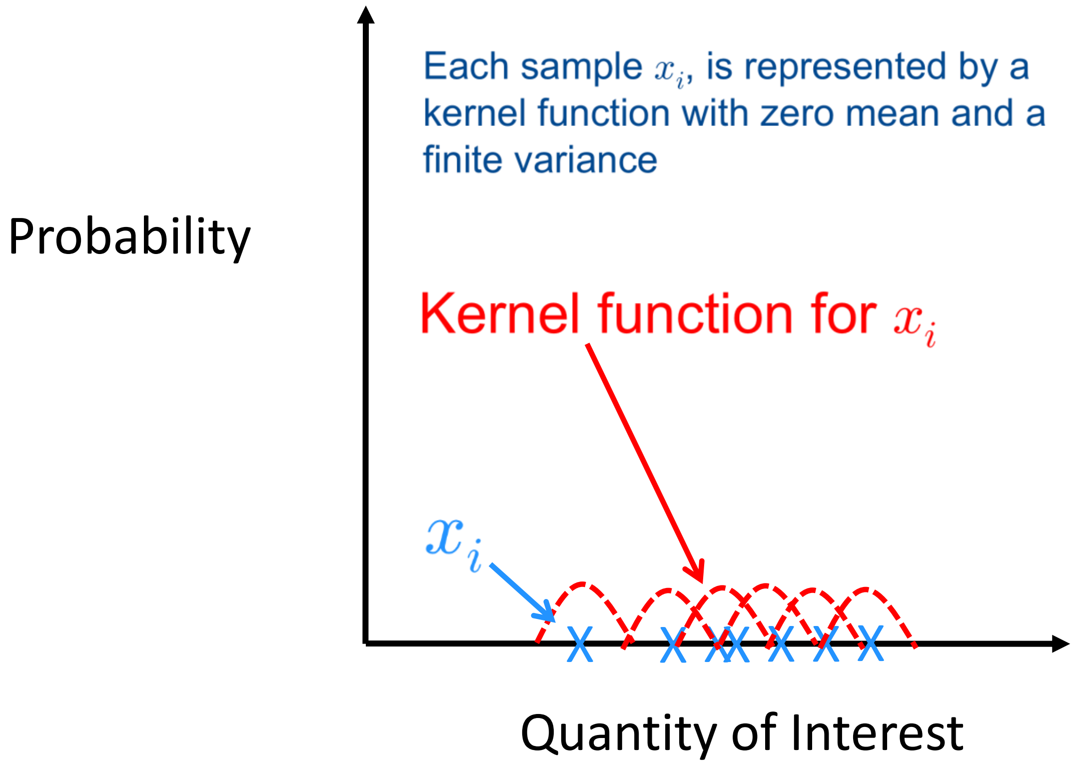
Quick detour: Kernel Density Estimation (KDEs)

$$f\left(x\right)=\frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{x-x_i}{h}\right)$$

xi – random samples
h – bandwidth
K – kernel function



Quick detour: Kernel Density Estimation (KDEs)

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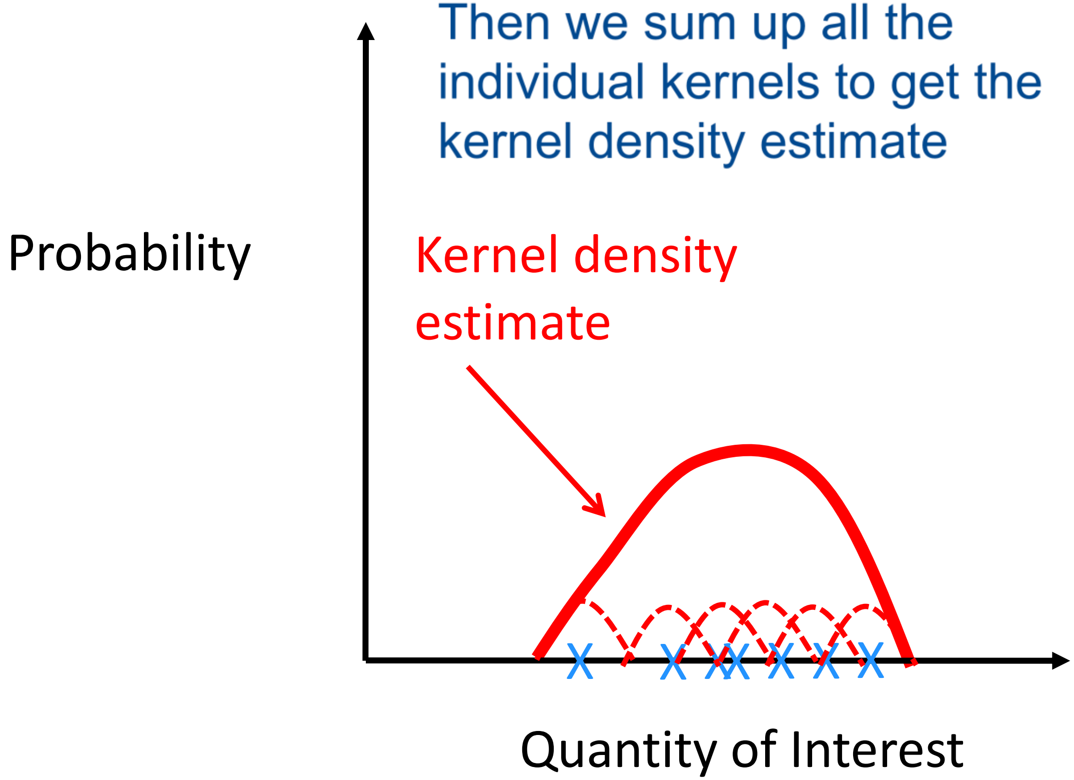

Kernel density estimation of model PDF

Choose a discretization in the random space (e.g., Monte Carlo samples)

$$f_j(s) \;=\; f(s, \omega_j)$$

Use a kernel density estimate of the model pdf with kernel $K=K_h$ with bandwidth parameter h

$$u_s(f_i) \;\approx\; \frac{1}{N}\sum_{j=1}^{N} K\big(f_j(s) - f_i\big)$$

model eval'd at point in uncertain space

numerical integration points for objective

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Kernel density estimation of model PDF

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model eval'd at point in uncertain space

numerical integration points for objective

In vector notation with 'e' a vector of ones,

$$\mathbf{u}_s \approx \boldsymbol{K}_s \mathbf{e}, \qquad \boldsymbol{K}_s(i,j) = \frac{1}{N} K\big(f_j(s) - f_i\big)$$

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Discrete optimization

$$\underset{s}{\text{minimize}} \quad (\mathbf{t} - \boldsymbol{K}_s\mathbf{e})^T \boldsymbol{W} (\mathbf{t} - \underbrace{\boldsymbol{K}_s\mathbf{e}}^{\text{kernel density estimate}})$$

target pdf

diagonal matrix of integration weights

Accelerating the Process

$$\tilde{\delta}(s) \;=\; (\mathbf{t} - \boldsymbol{K}_s\mathbf{e})^T \boldsymbol{W} (\mathbf{t} - \boldsymbol{K}_s\mathbf{e})$$

Can we use gradient information?

YES!

Gradient of the Objective

$$\tilde{\delta}(s) \;=\; (\mathbf{t} - \boldsymbol{K}_s\mathbf{e})^T \boldsymbol{W} (\mathbf{t} - \boldsymbol{K}_s\mathbf{e})$$

Use the gradient of f with respect to design variables s to compute the gradient of the objective with respect to s.

$$\nabla_s \tilde{\delta}(s) \;=\; 2(\mathbf{t} - \boldsymbol{K}_s\mathbf{e})^T \boldsymbol{W} \boldsymbol{K}'_s \boldsymbol{F}'$$

Derivative of kernel

$$\boldsymbol{K}'_s(i,j) = \frac{1}{N} K'\big(f_j(s) - f_i\big)$$

Partials of model with respect to design variables evaluated at points in uncertain space

$$\boldsymbol{F}' = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_M}{\partial s_1} & \cdots & \frac{\partial f_M}{\partial s_m} \end{bmatrix}$$

These are obtained as part of the CFD solution using adjoints

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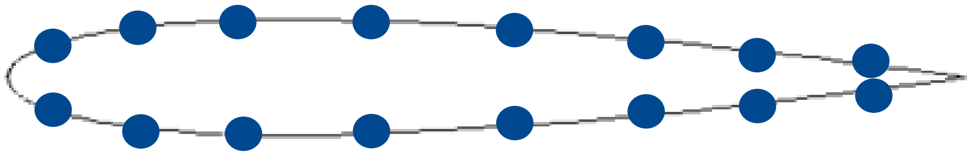
Computational Airfoil Design Using Aggressive Design

Computational airfoil design

$$M_\infty = 0.6752$$



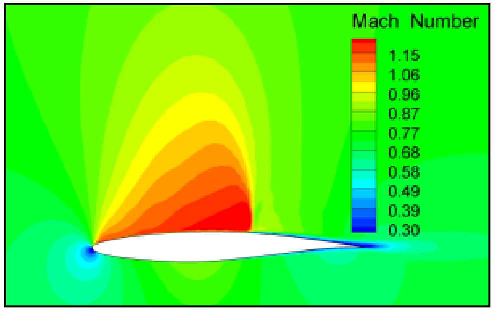
Inlet mach number for airfoil

Airfoil with Hicks-Henne bump functions

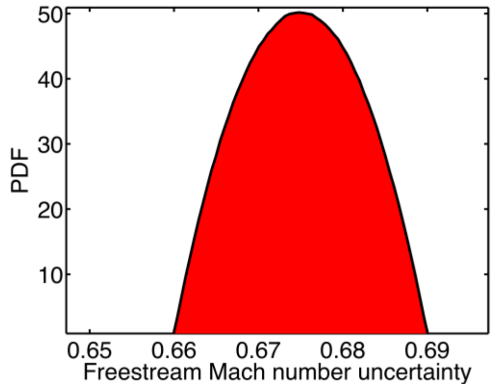
w

s

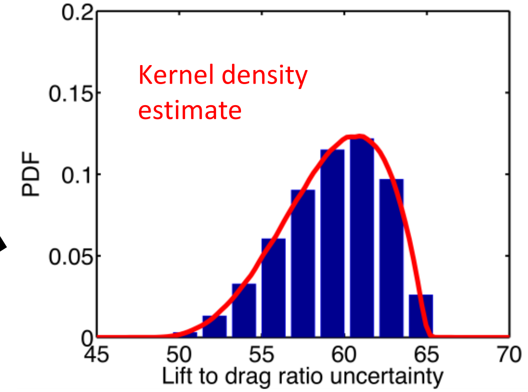
$$f(w, s)$$ = L/D of airfoil



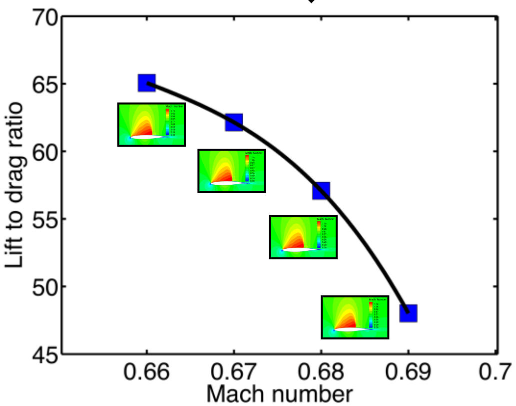
Computational airfoil design under uncertainty



Mach number PDF

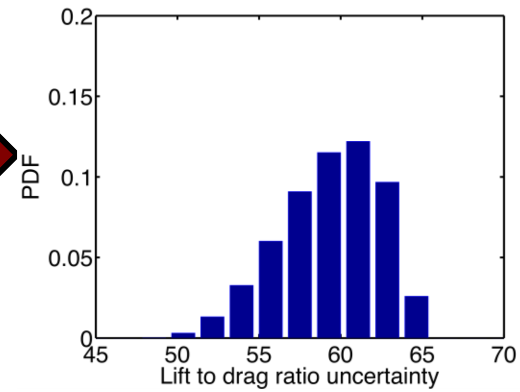


$$\mu\left(\frac{Lift}{Drag}\right), \ \sigma^2\left(\frac{Lift}{Drag}\right), \ \mathbf{K_se}$$



Response surface
(4 CFD runs)

Sample response surface with Mach number distribution (Stochastic Collocation)



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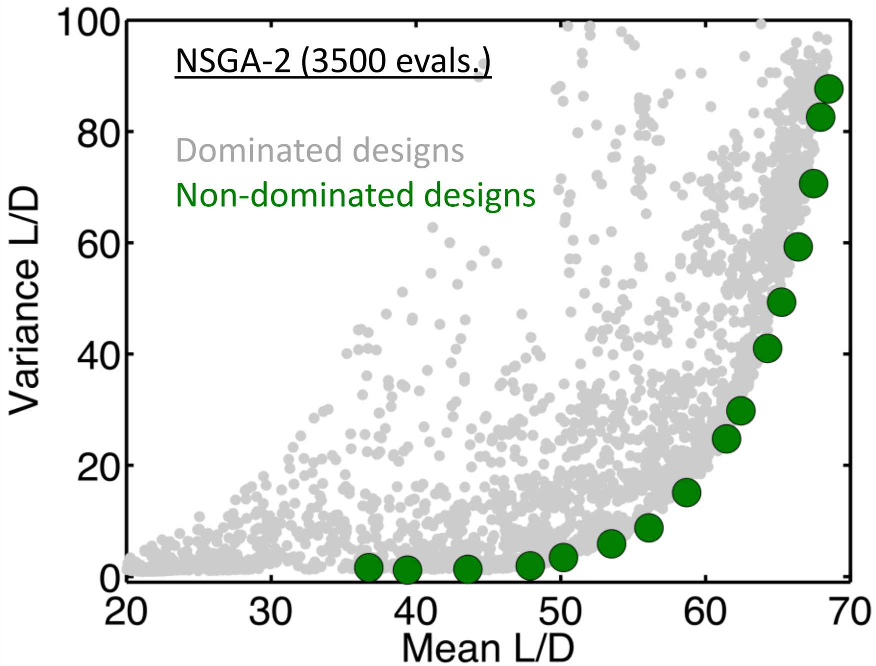
Robust Design Approach

Values of interest: $\mu\left(\frac{Lift}{Drag}\right), \ \sigma^2\left(\frac{Lift}{Drag}\right)$

Optimization problem: $$\underset{s}{\text{minimize}} \quad \mu\left(\frac{Lift}{Drag}\right)^{-1}, \quad \sigma^2\left(\frac{Lift}{Drag}\right)$$

Result:



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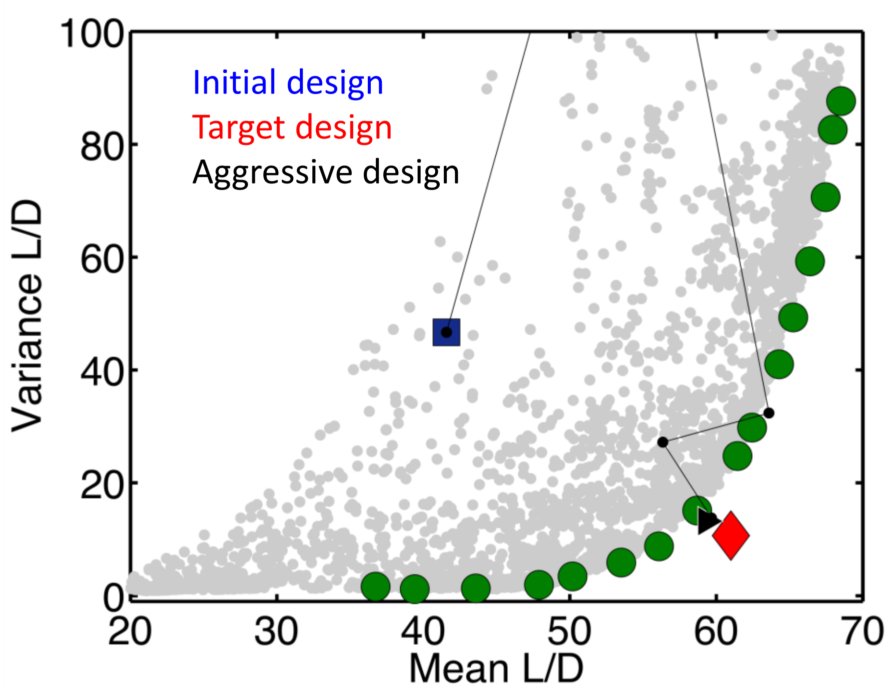
Aggressive Design Approach

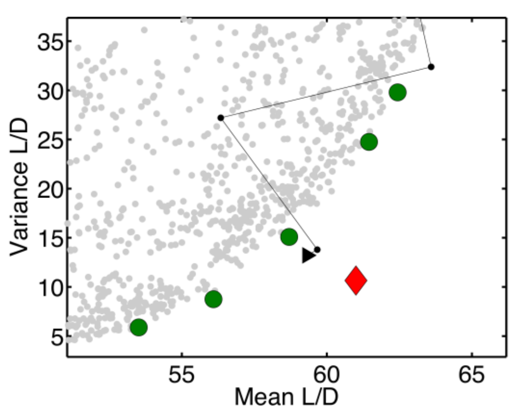
Values of interest: $\mathbf{K_s e}, \quad \mathbf{t}$

Optimization problem: $$\underset{s}{\text{minimize}} \quad (\mathbf{t} - \mathbf{K_s e})^{\mathbf{T}} \mathbf{W} (\mathbf{t} - \mathbf{K_s e})$$

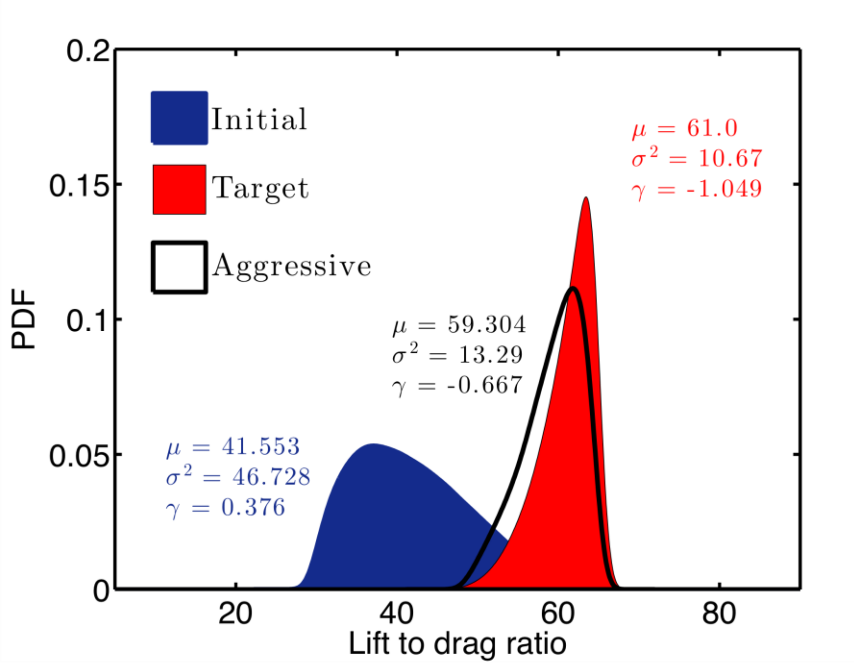
Result:

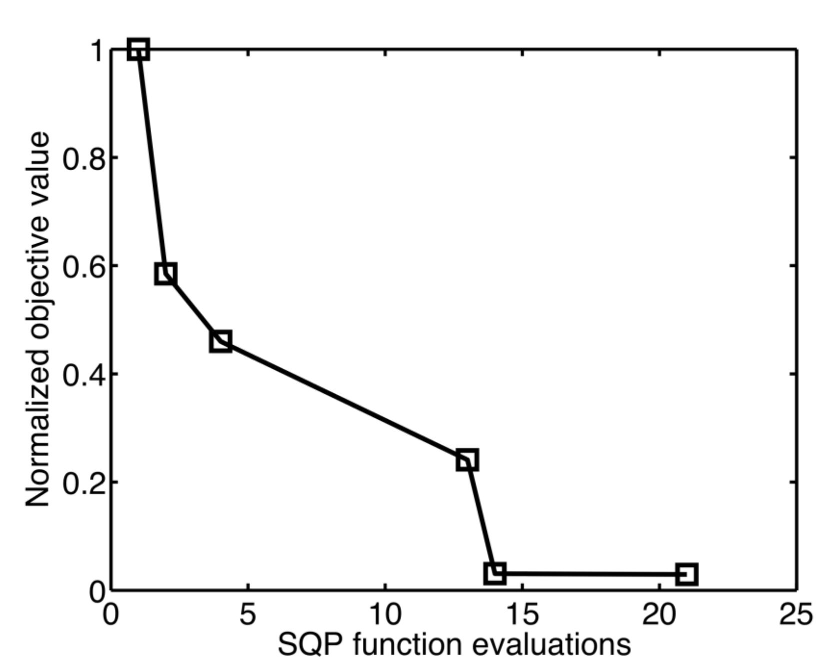


Close-up



Aggressive Design Approach





Metrics	Aggressive design	Robust design
Optimization Iterations	8	35 (generations)
Adjoint CFD	21 x 8 (4 lift, 4 drag)	-
Euler CFD	21 x 4	3500 x 4
Wall-clock time	37.7 minutes	~1 day

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Aggressive Design Highlights

We are not making a direct comparison between aggressive design and robust design – one is a single objective problem the other is a multi-objective one

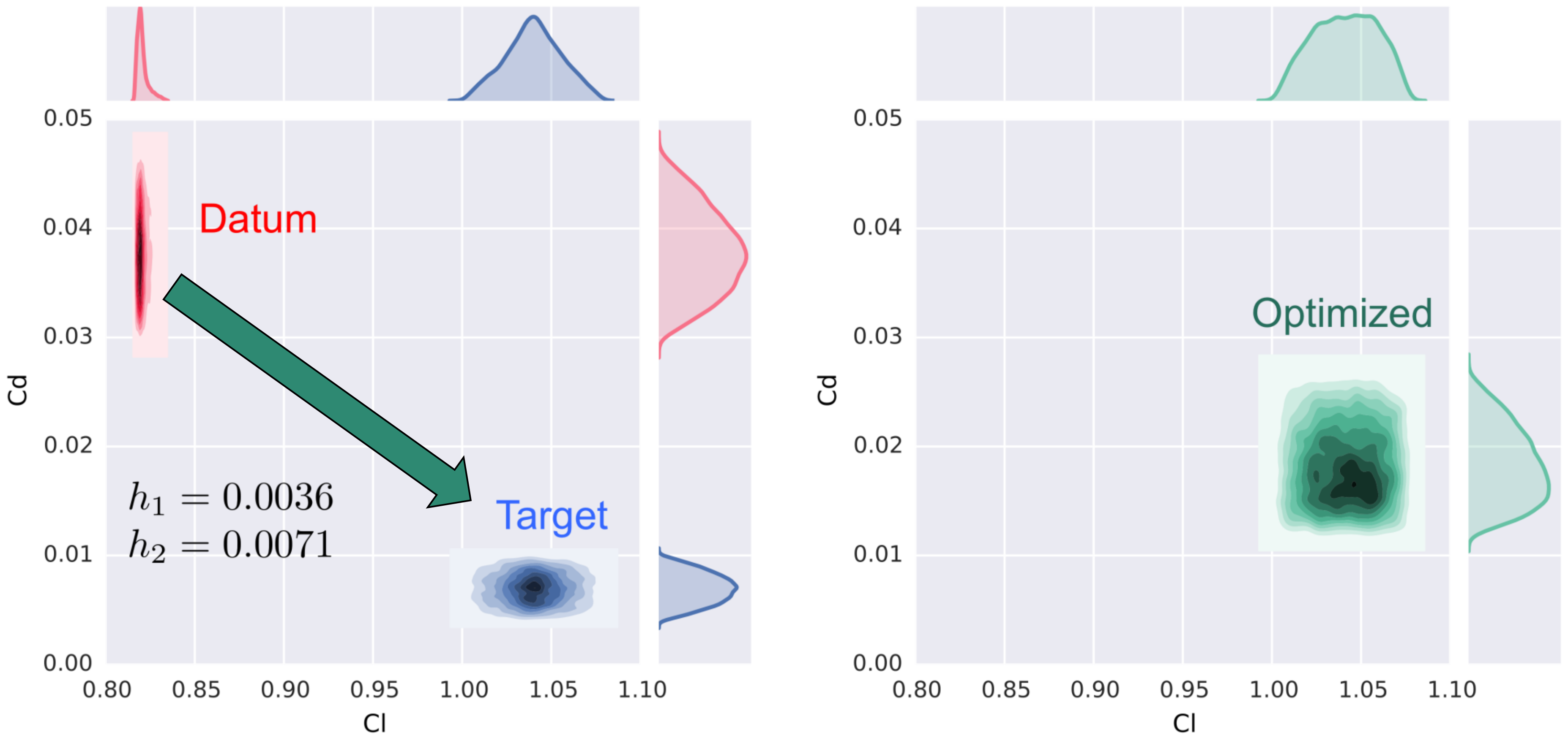
Like comparing apples with oranges

What we are presenting is a new approach for design under uncertainty – based on a target design that the designer has selected

Aggressive design is a simple, single-objective method with a smooth objective function

It leverages gradient information when present

Future Work: Multivariate aggressive design



The present framework extends nicely to multiple quantities of interest

Acknowledgements

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- Special thanks to Gianluca Iaccarino & Paul Constantine

Aggressive Design Literature

Seshadri, P., Parks, G.T., Shahpar, S., *Aggressive Design and Active Subspaces For Robust Redesign in Turbomachinery,* (Abstract accepted) ASME Turbo Expo 2015, Montreal, Canada

Seshadri, P., Constantine, P.G., Iaccarino, G., Parks, G.T., *Aggressive Design: A Density Matching Approach for Optimization Under Uncertainty*, Submitted. CMAME, 2014
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Iaccarino, G., Seshadri, P., Constantine, P.G., *Beyond Variance-based Robust Optimization: Aggressive Design*, SIAM Conference on Optimization 2014
(slides: web.stanford.edu/~jops/tmp/iaccarino_siam_opt14.pptx)

Seshadri, P., Constantine, P. G., Iaccarino, G., *Aggressive Design Under Uncertainty*, AIAA SciTech 2014, January 13-17, National Harbor, Maryland, 2014
(paper: http://arc.aiaa.org/doi/abs/10.2514/6.2014-1007)

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