Variance-based and Probability-based Sequential Robust Design Optimization of Fluid-Structure Interaction Processes.

Dirk Roos[1*] & Johannes Einzinger[2]

[1]Institute of Modelling and High-Performance Computing
Niederrhein University of Applied Sciences
[2]ANSYS Germany GmbH

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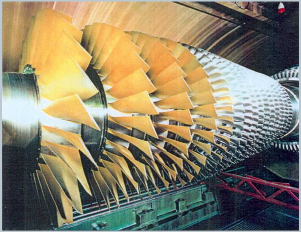
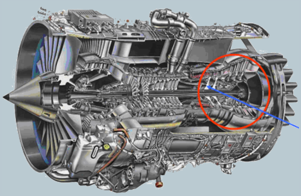
Robust Design Optimization

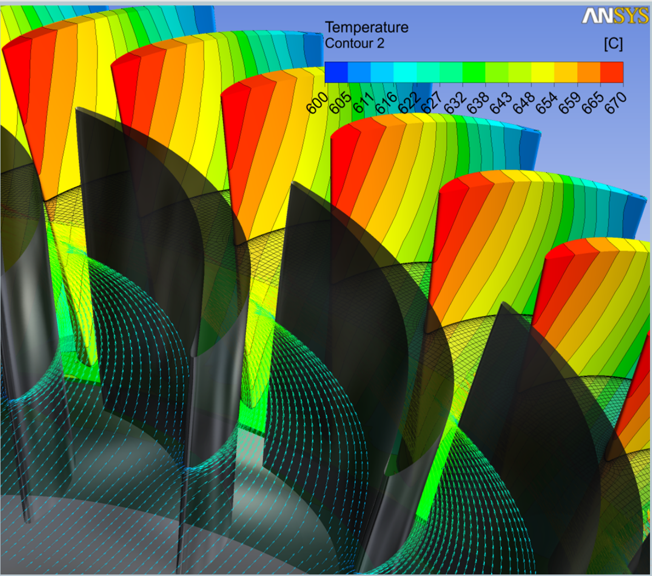


- Power plant 1000 MW, η ca. 50%
- Increasing of **4%** results in additional +80 MW power
- Per person, P = 1/6 kW
- Electricity for 80 MW / 1/6 kW = **480.000 inhabitants**
- Minimal mass, maximal life time
- **Six Sigma Design**
  $P(\mathscr{F}) \leq 3.4 \cdot 10^{-6}$



including material, manufacturing uncertainties and process parameters

RDO of an axial turbine blade



- Structural, thermal and fluid analysis
- CAD- and CAE-based parameterization
- Automatic process integration and meshing using **ANSYS Workbench**

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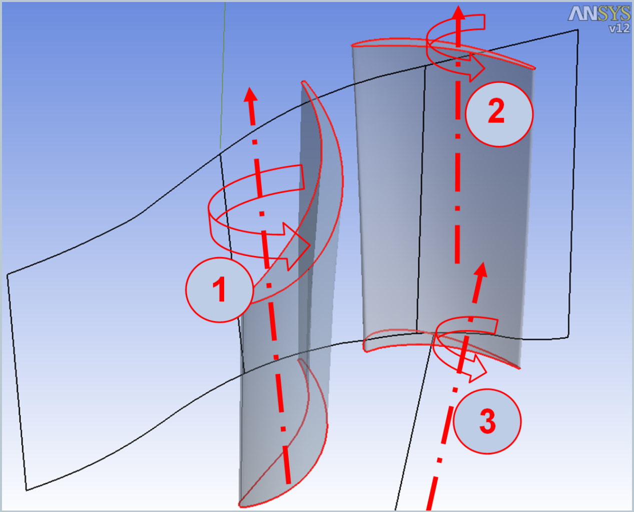
Design parameters



- $n_d = 4 + 3 = 7$ geometry parameters of the shroud profile and guide vane
- Rotational velocity of the rotor
- Temperature and pressure ratio
- Gaussian distributions

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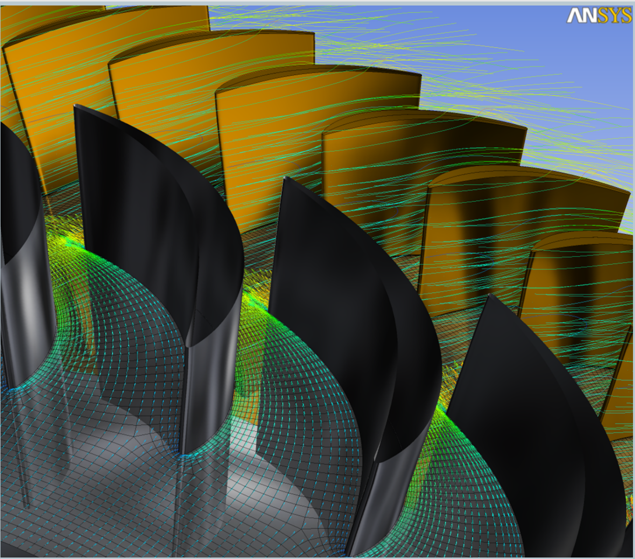
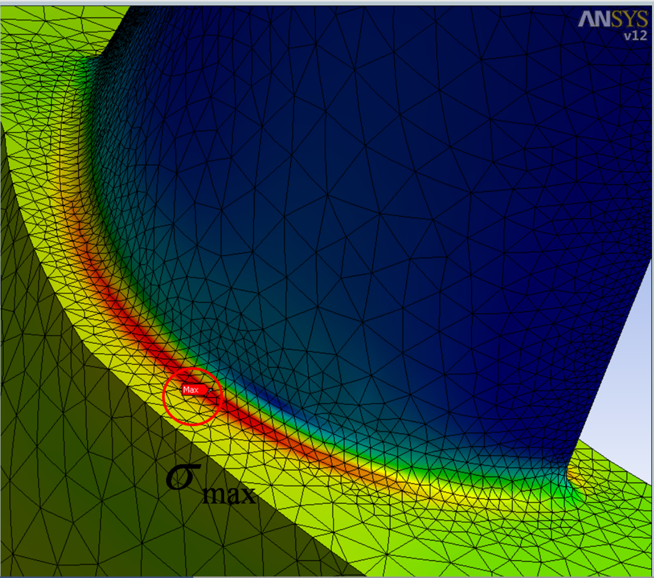
Stochastic objective function



- Random material parameters: (thermal conductivity of air and steel; mass flow rate, E, ν, ρ: steel)
- **Six Sigma Design** criteria
- Maximize
  - isentropic **efficiency**
  - **power** at rotor and
- Minimization of **maximal v. Mises stress**

Stochastic responses



- Location and mesh independency of the v. Mises stress
- Rotor blend radius

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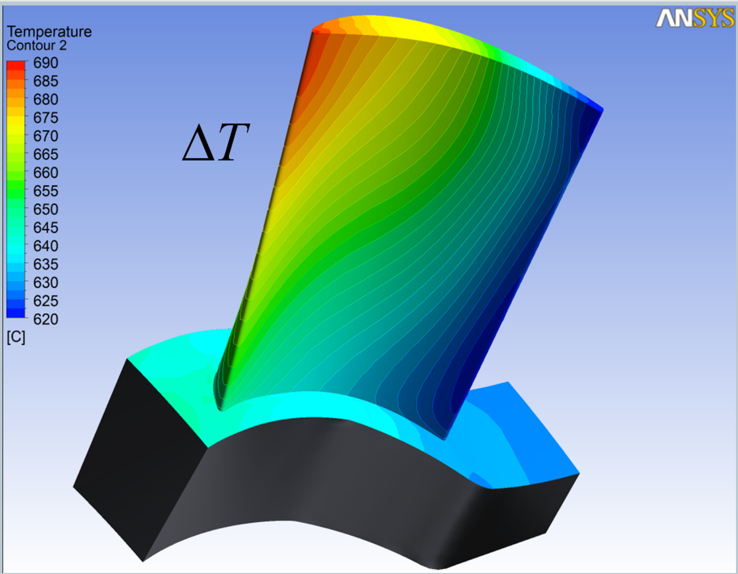
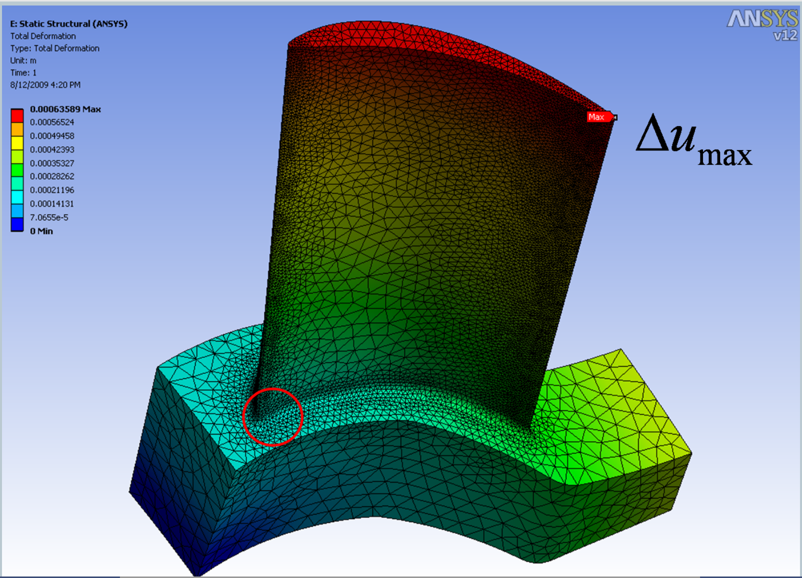
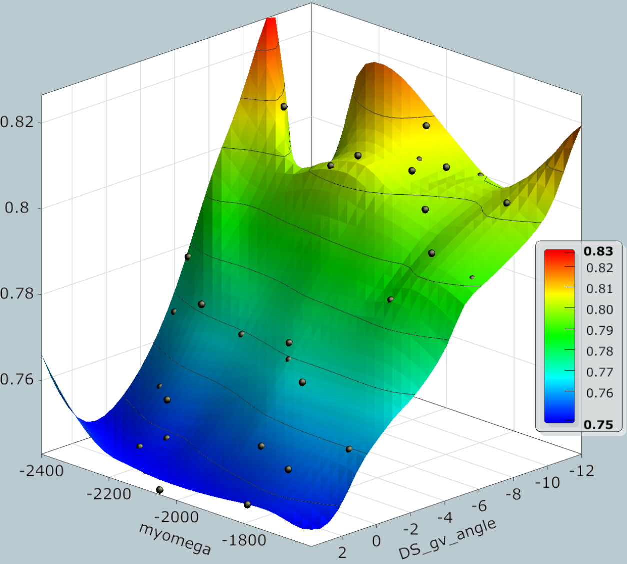
Stochastic constraint: maximal temperature

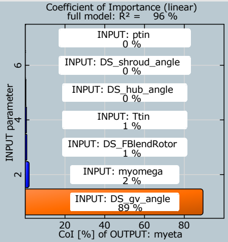


Stochastic constraint: max. displacement



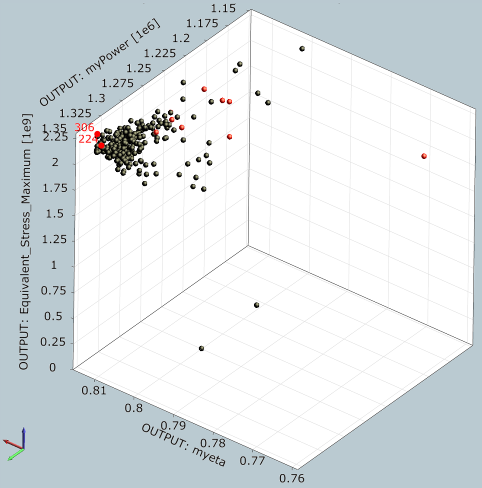
Sensitivity analysis: efficiency



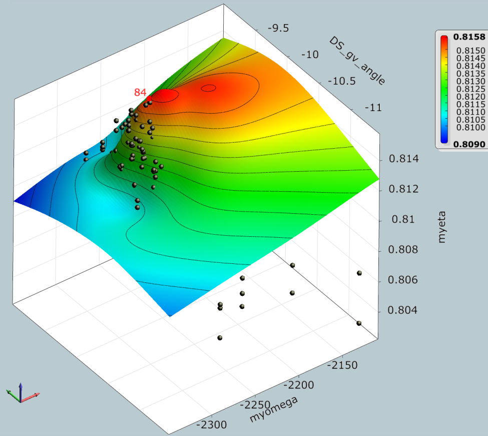


- non-convex objective requires:
- global search strategy (EA) in combination with
- local optimization (ARSM)

Optimization

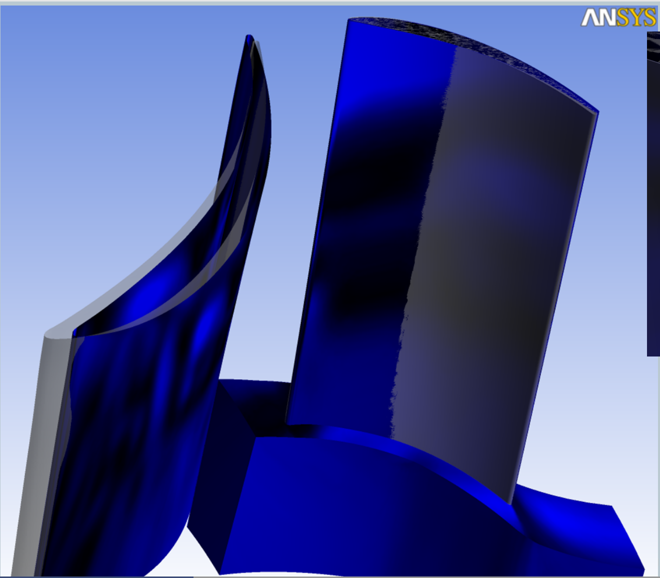


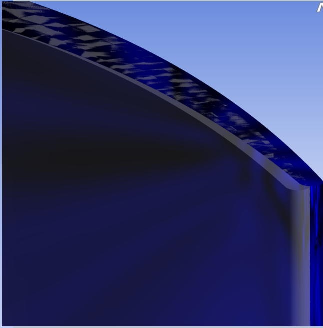
Evolutionary Algorithm ($N = 224$)



Adaptive RSM ($N = 84$)

Optimal deterministic design





- Efficiency [%] 71.54 → 81.54 **(+14%)**
- Power [MW] 1.208 → 1.329 **(+11%)**

No free lunch theorem for optimization

(Wolpert and Macready 1997):

„We show that all algorithms that search for an extremum of a cost function perform exactly the same, when averaged over all possible cost functions.“

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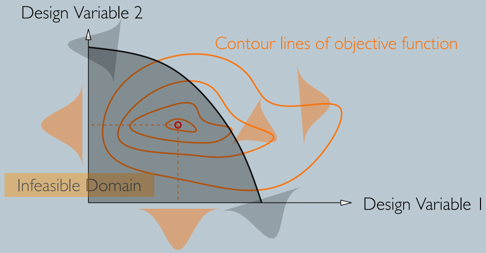
Stochastic optimization problem

$$f(d_1, d_2, \dots d_{n_d}, \sigma^2_{X_1}, \sigma^2_{X_2}, \dots \sigma^2_{X_{n_r}}, P(\mathscr{F})) \rightarrow \min$$

$$g_k(d_1, d_2, \dots d_{n_d}) = 0;\ k = 1, m_e$$
$$h_l(d_1, d_2, \dots d_{n_d}) \geq 0;\ l = 1, m_u$$
$$d_i \in [d_l, d_u] \subset \mathbb{R}^{n_d}$$
$$d_l \leq d_i \leq d_u$$
$$d_i = E[X_i]$$
$$\frac{P(\mathscr{F})}{P^t(\mathscr{F})} - 1 \geq 0; \quad \frac{\sigma^j_L}{\sigma^t_L} - 1 \geq 0$$



$$\sigma^2_{X_i} = \frac{1}{M-1} \sum_{k=1}^{M} \left(x^k_i - \mu_{X_i}\right)^2; \quad P(\mathscr{F}) = P[\{\mathbf{X} : g_j(\mathbf{X}) \leq 0\}]$$

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Sources of uncertainties

Property	SD/Mean %
Metallic materials, yield	**15**
Carbon fiber composites, rupture	17
Metallic shells, buckling strength	14
Junction by screws, rivet, welding	8
Bond insert, axial load	12
Honeycomb, tension	16
Honeycomb, shear, compression	10
Honeycomb, face wrinkling	8
Launch vehicle, thrust	5
Transient loads	**50**
Thermal loads	7.5
Deployment shock	10
Acoustic loads	**40**
Vibration loads	20

(Source: (Klein, Schuëller, Deymarie, Macke, Courrian, and Capitanio 1994))

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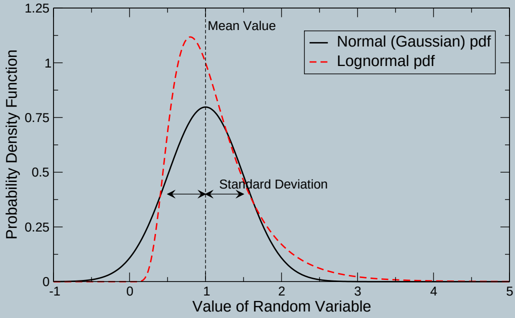
Where to get statistical data?

- From lab testing (always best)
- From measurements of suppliers/manufacturers
- From technical references (mostly only mean values provides)
- Estimation - mostly Gaussian or lognormal distribution



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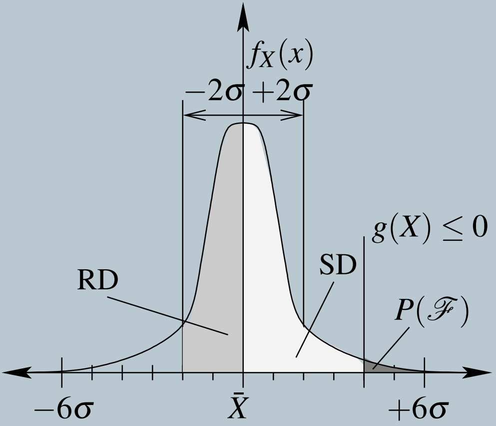
Sigma level



Sigma level

$$\sigma_L = \frac{g(X) - \bar{X}}{\sigma_X}$$

specification limit on $2\sigma_X$ and $6\sigma_X$ level. **Robust design** (RD) ($\geq \pm 2\sigma_X$) and **safety design** (SD) ($\geq \pm 6\sigma_X$) depending on specified limit state function $g(X) \leq 0$, e.g. stress limit state.

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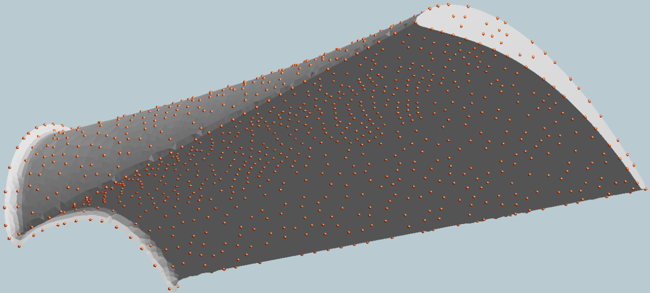
Acceptable probabilities of failure

Values of acceptable annual probabilities of failure $P^t(\mathscr{F})$ and target reliabilities σ_L^t (DNV 1992)[1]

Class of failure	Consequence of failure: Less serious	Consequence of failure: Serious
I - Redundant structure	$P^t(\mathscr{F}) = 10^{-3}$ $\sigma_L^t = 3.09$	$P^t(\mathscr{F}) = 10^{-4}$ $\sigma_L^t = 3.71$
II - Significance warning before the occurrence of failure in a non-redundant structure	$P^t(\mathscr{F}) = 10^{-4}$ $\sigma_L^t = 3.71$	$P^t(\mathscr{F}) = 10^{-5}$ $\sigma_L^t = 4.26$
III - No warning before the occurrence of failure in a non-redundant structure	$P^t(\mathscr{F}) = 10^{-5}$ $\sigma_L^t = 4.26$	$P^t(\mathscr{F}) = 10^{-6}$ $\sigma_L^t = 4.75$

[1] Det Norske Veritas (DNV) is an autonomous and independent Foundation with the objective of safeguarding life, property and the environment at sea and ashore.

Measurements of manufacturing tolerances



- 3D coordinates (observed with 2 cameras) for **1500 points** (independent of the current geometry)
- **Tolerance interpolation** to different meshes of the multi-physics analysis, **ANSYS Workbench** implementation

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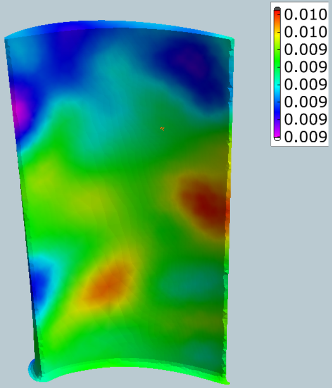
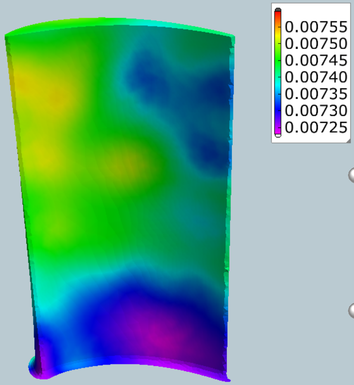
Measurements of manufacturing tolerances



Standard deviation radial direction y



Standard deviation tangential direction z

- Mean values and standard deviation of imperfections
- Calculation of the **covariance matrix** $\mathbf{C}_{XX}$ of the random field

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Modelling of random fields

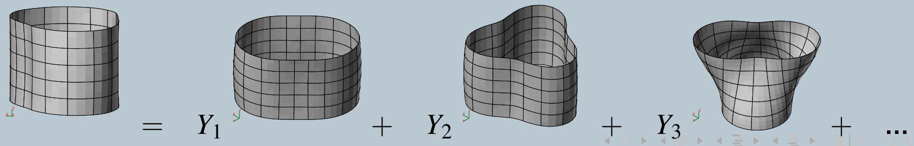
- Decomposition of the covariance matrix:

$$\mathbf{\Psi}^T \mathbf{C}_{XX} \mathbf{\Psi} = \mathrm{diag}\{\lambda_i\}$$

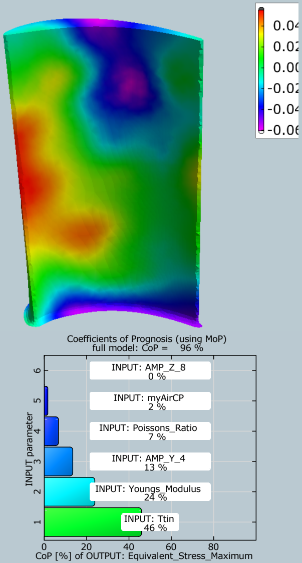
- Simulate **Gaussian** variables $Y_i \sim N(0; \sqrt{\lambda_i}), \rho_{i \neq j} = 0$ with variances λ_i
- Transformation between simulated variables $\mathbf{Y}$ and 'real-world'

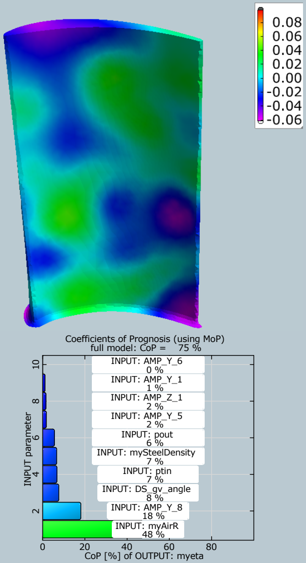
$$\mathbf{Y} = \mathbf{\Psi}^T \mathbf{X} \quad \Leftrightarrow \quad \mathbf{X} = \mathbf{\Psi} \mathbf{Y}$$

with $\mathbf{Y}$: **matrix of the imperfection modes** (Eigenvectors)



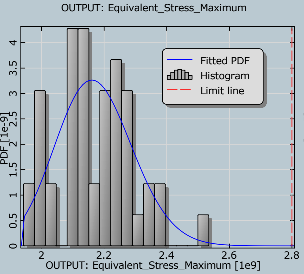
Robustness analysis of an axial turbine

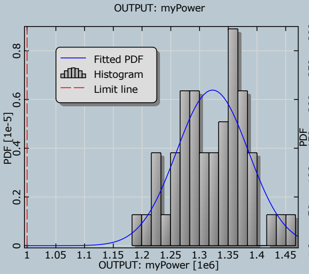


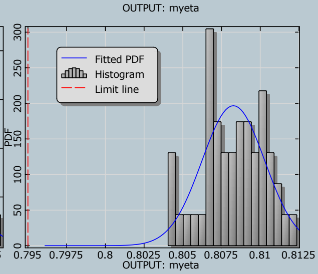


- Identification of $n_r = 9$ random parameters with large prognosis according limit state conditions $g_i, i = 1, \ldots, 3$
- Material an process parameters
- **4th** and **8th imperfection mode** radial direction y
- Angle of the guide vane

Robustness analysis ($\sigma_L \geq 4.5$, $P(\mathscr{F}) \leq 3.4 \cdot 10^{-6}$)







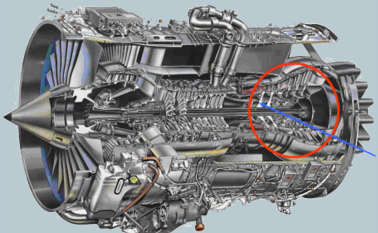
Statistic data			
Min:	1.942e+09	Max:	2.535e+09
Mean:	2.171e+09	Sigma:	1.227e+08
CV:	0.0565		
Skewness:	0.3893	Kurtosis:	3.235
Fitted PDF: Log-Normal			
Mean:	2.171e+09	Sigma:	1.227e+08
Limit x = 2.8e+09			
P_rel =	1	P_fit =	0.999997
Probability P(X<x) = 0.95			
x_rel =	2.37151e+09	x_fit =	2.37861e+09

Statistic data			
Min:	1.183e+06	Max:	1.467e+06
Mean:	1.323e+06	Sigma:	6.246e+04
CV:	0.04722		
Skewness:	-0.03409	Kurtosis:	2.646
Fitted PDF: Normal			
Mean:	1.323e+06	Sigma:	6.246e+04
Limit x = 1e+06			
P_rel =	0	P_fit =	1.19045e-07
Probability P(X<x) = 0.95			
x_rel =	1.43062e+06	x_fit =	1.42543e+06

Statistic data			
Min:	0.8041	Max:	0.8124
Mean:	0.8083	Sigma:	0.002028
CV:	0.002509		
Skewness:	-0.1842	Kurtosis:	2.345
Fitted PDF: Normal			
Mean:	0.8083	Sigma:	0.002028
Limit x = 0.795			
P_rel =	0	P_fit =	3.07755e-11
Probability P(X<x) = 0.95			
x_rel =	0.811073	x_fit =	0.811601

$\sigma_L \approx 5.1$, $P(\mathscr{F}) \approx 5 \cdot 10^{-7} \neq 3 \cdot 10^{-6}$, $\sigma_L \approx 5.2$, $\sigma_L \approx 6.6$

Six sigma analysis & Reliability analysis



- Probabilistic Aerothermal Design of Compressor Airfoils (Garzon 2003)
- Impact of Geometric Variability on Axial Compressor Performance (Garzon and Darmofal 2003)
- Multidisciplinary optimization and reliability analysis of a Two-Stage Turbine (Parchem and Meissner 2009) shows:
- Stochastic analysis should include material, manufacturing uncertainties and process parameters and has to support all sigma levels $\sigma_L^t \leq 4.5 \ldots 6$

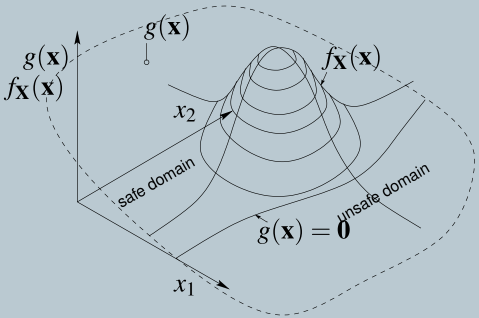
Formula of the failure probability $P(\mathscr{F})$

State function of response

$$g(\mathbf{x})$$

Failure state condition:
limit state function

$$g(\mathbf{x}) = g(x_1, x_2, \ldots, x_n) \leq 0$$



Joint probability density function $f_{\mathbf{X}}(\mathbf{x})$

$$P(\mathscr{F}) = P[\mathbf{X} : g(\mathbf{X}) \leq 0] = \int \overset{n}{\cdots} \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

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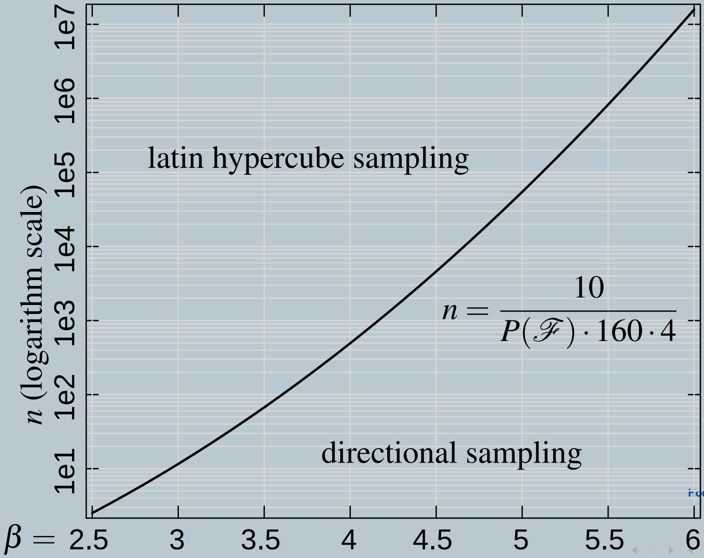
Directional sampling vs. latin hypercube sampling



- (Bjerager 1988; Ditlevsen and Bjerager 1989)
- limited application area

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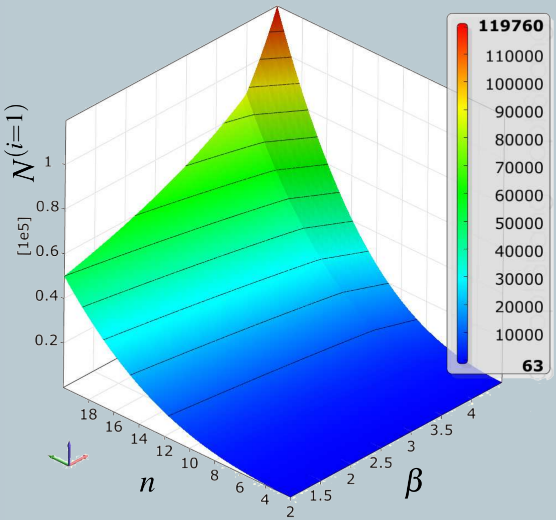
Adaptive importance sampling



- (Bucher 1988)
- minimal initial sample size $N^{(i=1)}$ depending on the
- expected β and the
- number of random variables n
- statistical error: $\sigma_{\bar{P}(\mathscr{F})}/E[\bar{P}(\mathscr{F})]$ = 10 %

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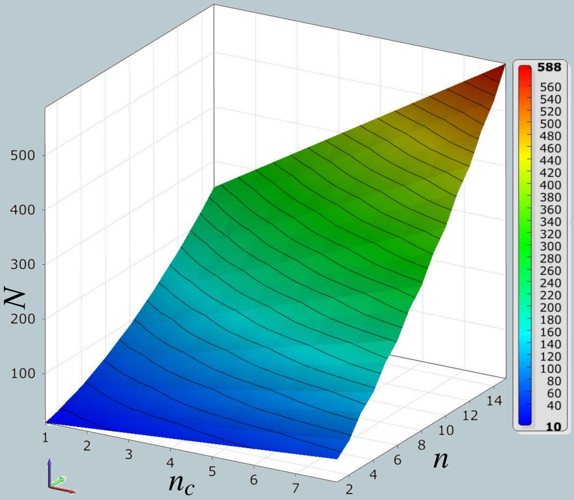
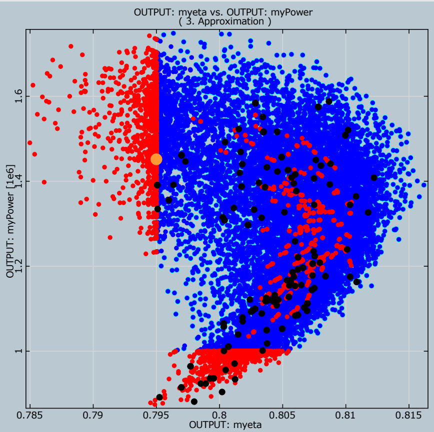
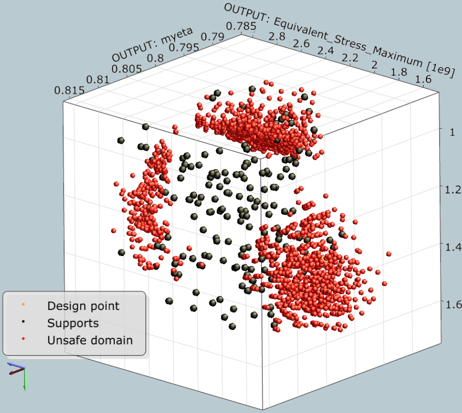
Multi-domain adaptive surrogate models



- Simulation method: directional sampling
- **independent** of sigma level $(\sigma_L, P(\mathscr{F}))$
- $P(\mathscr{F}|\mathbf{a})$-weighted Centroid Cluster Analysis
- D-optimal DOE supports **multiple** failure domains n_c
- (Roos 2011)

Multi-domain adaptive surrogate models





- Number of designs evaluations: $N = 174$
- **Six Sigma Design**: $\bar{P}(\mathscr{F}) = 2.6 \cdot 10^{-7} \leq 3.4 \cdot 10^{-6}$
- $\beta = \sigma_L = 5.7 > 4.5$ point: estimation of **partial safety factors**
- FORM: $\bar{P}_\beta(\mathscr{F}) = 7 \cdot 10^{-9}$

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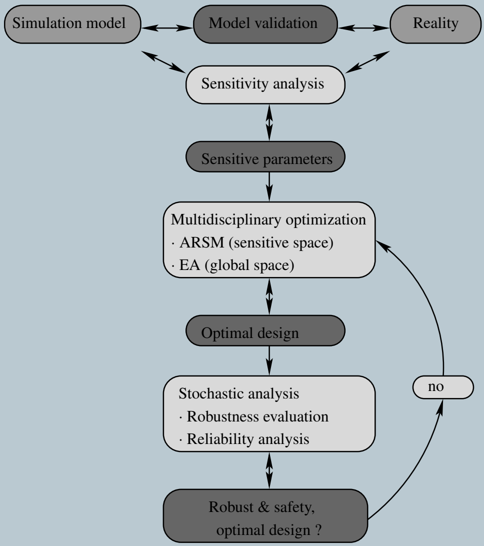
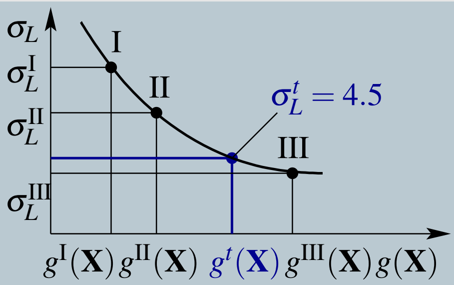
Sequential stochastic design optimization





- Within stochastic optimization
- **constrains (including partial safety factors)** $h(\mathbf{X})$ can be formulated as
- **limit state conditions** $g(\mathbf{X})$
- additional optimization effort

Summary

- **Stochastic design optimization**
- Parameters $n = 33$
  - $n_d = 7$ design geometry parameters and process parameters
  - $n_r = 26$ Material parameters and imperfection modes
- Nonlinear multi-physics analysis
- $N = 36 + 224 + 84 + 47 + \mathbf{2} \cdot (\mathbf{90} + \mathbf{40}) + 174 \approx \mathbf{800}$ design evaluations
- **ANSYS Workbench**: 5 Parallel Tasks
- Calculation time: **48 hours** (2 Intel® Xeon® X5680 Six Core, 3.33 GHz, 12MB Cache)
- **Acknowledgement**: thanks to Ulrike Adams and Daniela Ochsenfahrt of the DYNARDO GmbH (method implementation into the optiSLang software package)

Forschungspartner gesucht für

„Robust-Design-Optimierung in der Fluid-Struktur-Analyse von Gasturbinen und Strahltriebwerken unter Berücksichtigung von Fertigungstoleranzen“

- wissenschaftliche Problemstellungen:
  - Effizienzsteigerung der stochastischen Design-Optimierung
  - effiziente Modellierung räumlich korrelierter Oberflächenabweichungen
  - Anwendung auf rechenzeitintensive, komplexe FSI-Simulationen im Turbomaschinenbau
- Projektpartner:
  - Prof. Dr.-Ing. Dirk Roos, IMH (Robust-Design-Optimierung)
  - Prof. Dr.-Ing. Peter Farber, IMH (Strömungsmechanik)
  - Dr.-Ing. Johannes Einzinger, ANSYS Germany GmbH
  - Prof. Dr.-Ing. M. Geller, Institut für Konstruktion und Simulation, Fachhochschule Dortmund (Fluid-Struktur-Interaktion)
  - ???

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*Contact

Prof. Dr.-Ing. Dirk Roos

Computer Simulation and Design Optimization
IMH - Institute of Modelling and High-Performance Computing
Faculty of Mechanical and Process Engineering
Niederrhein University of Applied Sciences
Germany

dirk.roos@hs-niederrhein.de

Hochschule Niederrhein
University of Applied Sciences

IMH
Institut für Modellbildung und Hochleistungsrechnen
Institute of Modelling and High-Performance Computing

Literature I

Bjerager, P. (1988).
Probability integration by directional simulation.
Journal of Engineering Mechanics, ASCE Vol. 114(No. 8), 1285 – 1302.

Bucher, C. G. (1988).
Adaptive Sampling - An Iterative Fast Monte Carlo Procedure (Vol. 5, No. 2 ed.).
Structural Safety.

Ditlevsen, O. and P. Bjerager (1989).
Plastic reliability analysis by directional simulation.
Journal of Engineering Mechanics, ASCE Vol. 115(No. 6), 1347 – 1362.

DNV (1992).
Structural reliability analysis of marine structure.
Technical Report Classification Notes, No. 30.6, Det Norske Veritas Classification AS, Computer Typesetting by Division Ship and Offshore, Norway.

Literature II

Garzon, V. E. (2003).
Probabilistic Aerothermal Design of Compressor Airfoils.
Ph. D. thesis, Massachusetts Institute of Technology.

Garzon, V. E. and D. L. Darmofal (2003).
Impact of geometric variability on axial compressor performance.
Journal of Turbomachinery 125, 692–703.

Klein, M., G. Schuëller, P. Deymarie, M. Macke, P. Courrian, and R. S. Capitanio (1994, S).
Probabilistic approach to structural factors of safety in aerospace.
In *In Proceedings of the International Conference on Spacecraft Structures and Mechanical Testing*, Paris, France, Cepadues-Edition, pp. 679 – 693.

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Literature III

Parchem, R. and B. Meissner (2009).
Engine multidisciplinary optimization deployed on a two-stage turbine.
In E. Kesseler (Ed.), *Advances in Collaborative Civil Aeronautical Multidisciplinary Design Optimization*, pp. 289 – 331. Amsterdam, The Netherlands: AIAA.

Roos, D. (2011).
Multi-domain adaptive surrogate models for reliability analysis.
In P. Budelmann, Holst (Ed.), *Proceedings of the 9th International Probabilistic Workshop.* Braunschweig, Germany, November 17-18: Technical University Braunschweig.

Wolpert, D. H. and W. G. Macready (1997).
No free lunch theorems for optimization.
IEEE Trans, Evolutionary Computation 1(1), 67 – 82.

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