Robust Design Optimization of a Centrifugal Compressor concerning Fluid-Structure Interaction and Manufacturing Tolerances

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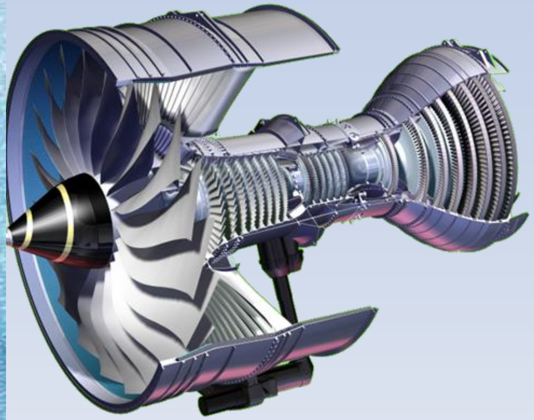
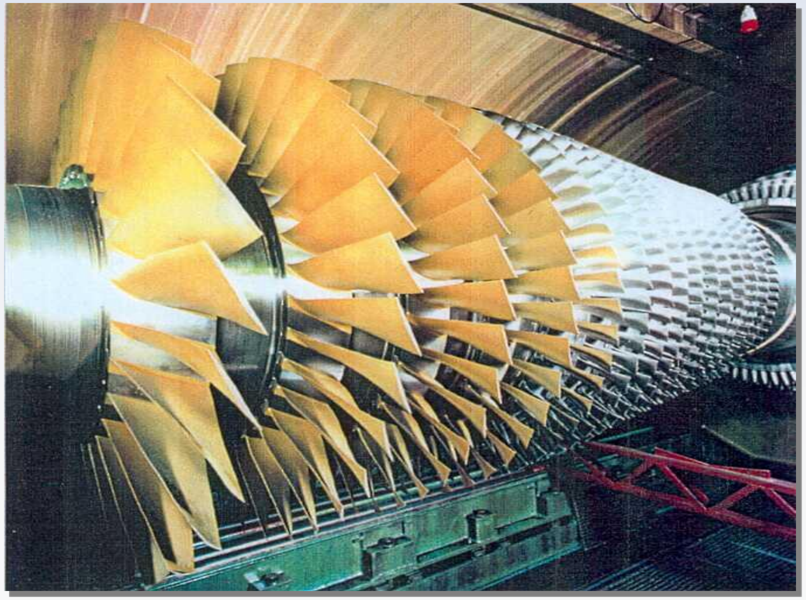
Motivation

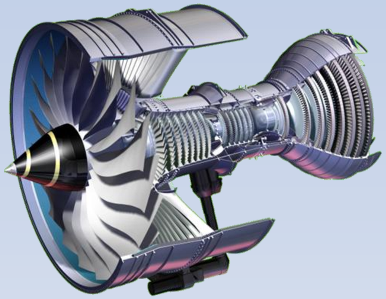



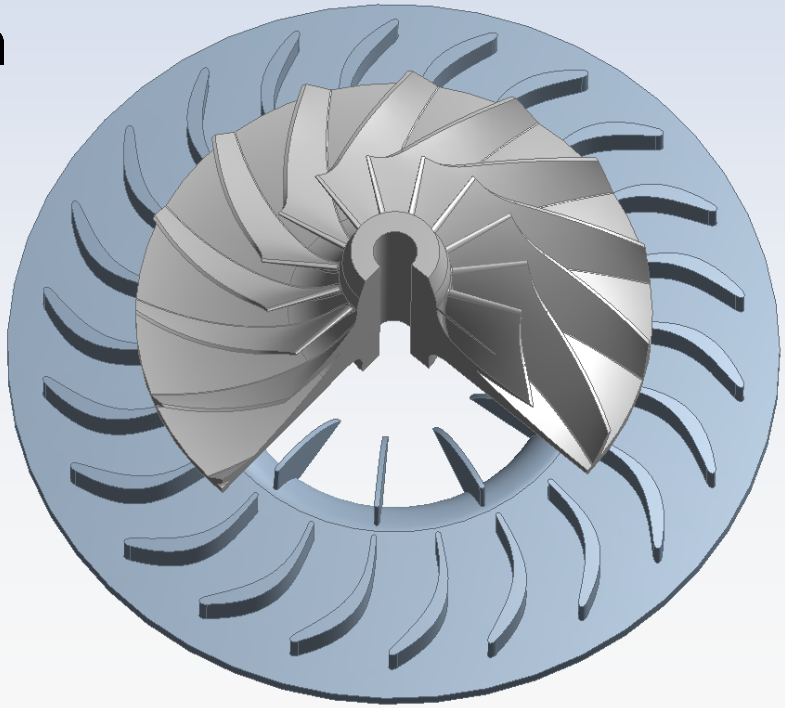



Power Plant	1000 MW
Efficiency	50 %
Increase of 1%	+20 MW
=Electricity for	120 000 Inhabitants

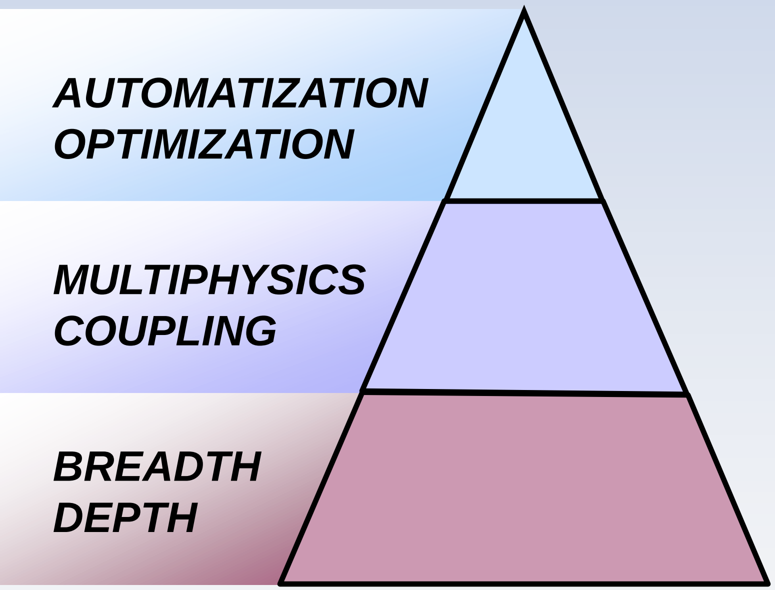
Outline

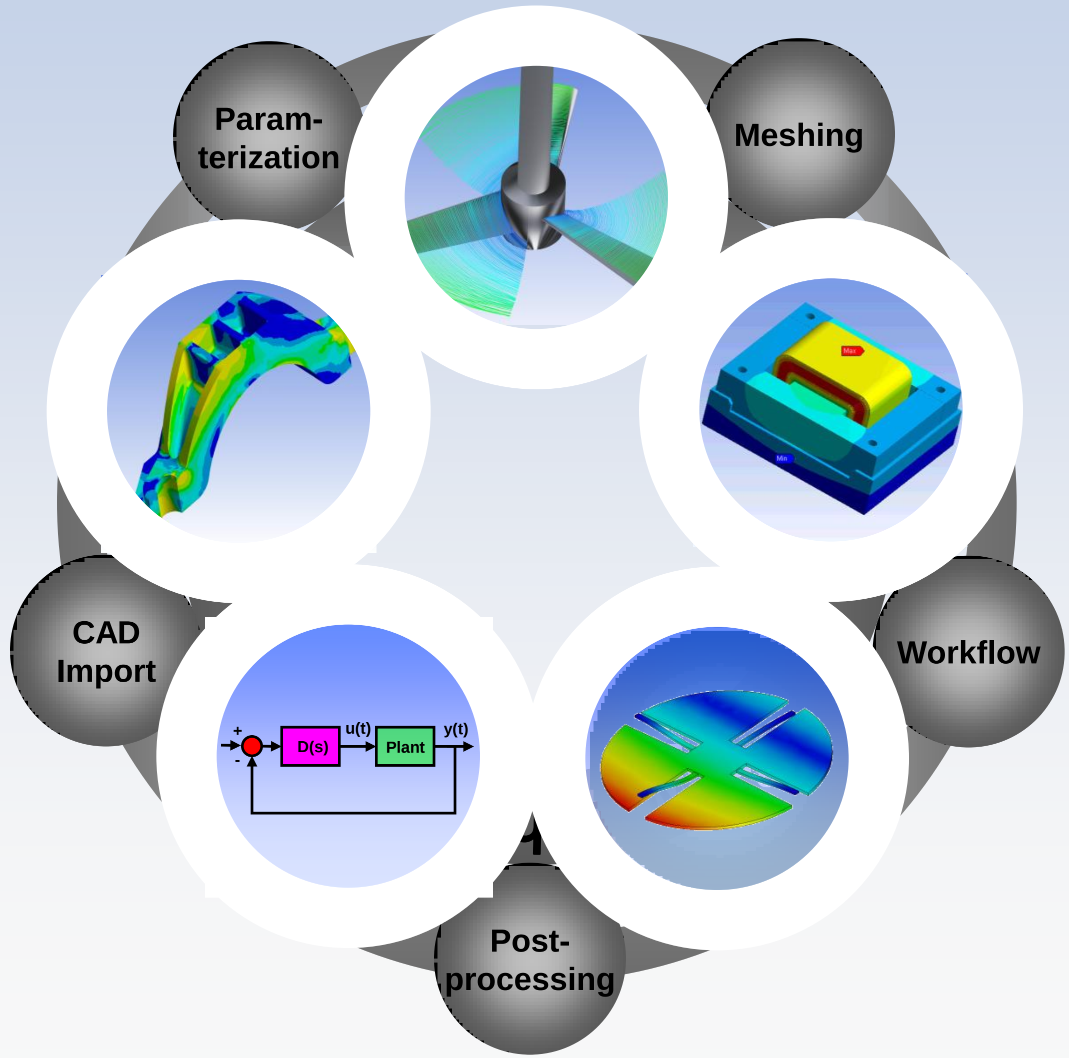
- Workbench Platform
- Parameterization of the Geometry
- Parameterization of the CFD Simulation
- Parameterization of the Mechanical Simulation
- Parametric Process Integration
- Sensitivity Analysis
- Design Optimization
- Robustness Evaluation
- Random Fields
- Robust Design Optimization
- Reliability Analysis





Workbench Parametric Process





Workbench Platform & optiSLang

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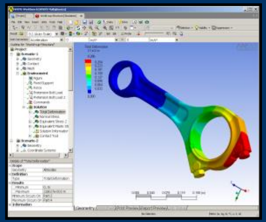
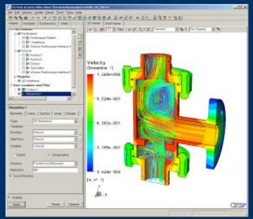
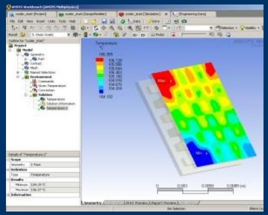
CATIA SolidWorks UGS The PLM Company **CAD / PDM** PTC autodesk

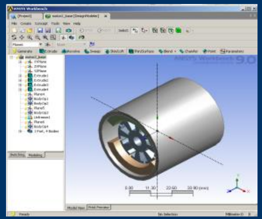
ANSYS Workbench

Structural Mechanics - Fluid Dynamics - Heat Transfer - Electromagnetic









A Multi-Physics Design and Analysis System

Sensitivity | **Optimization** | **Robustness** | **Reliability** | **Robust Design**

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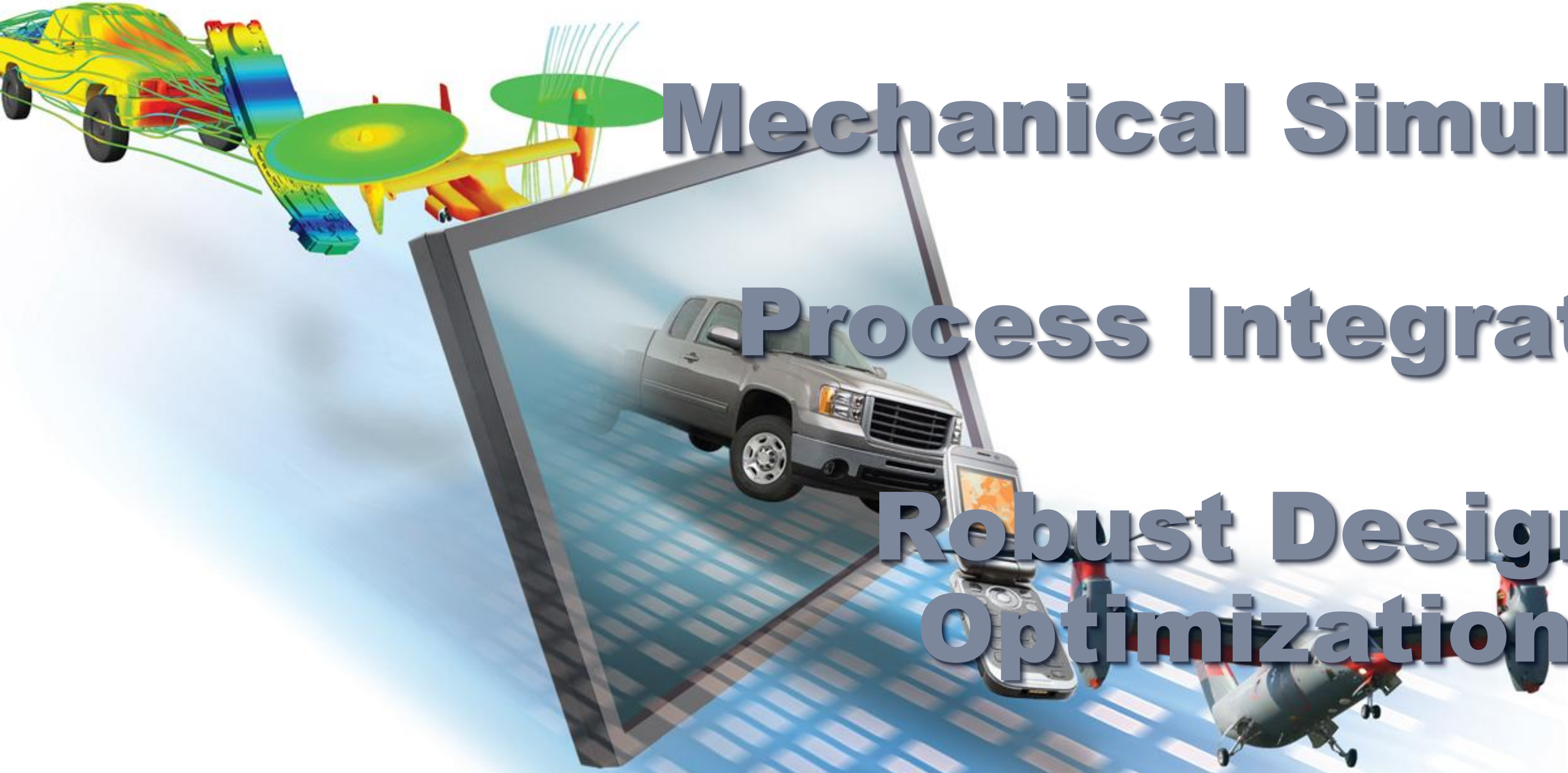
Geometry

CFD Simulation

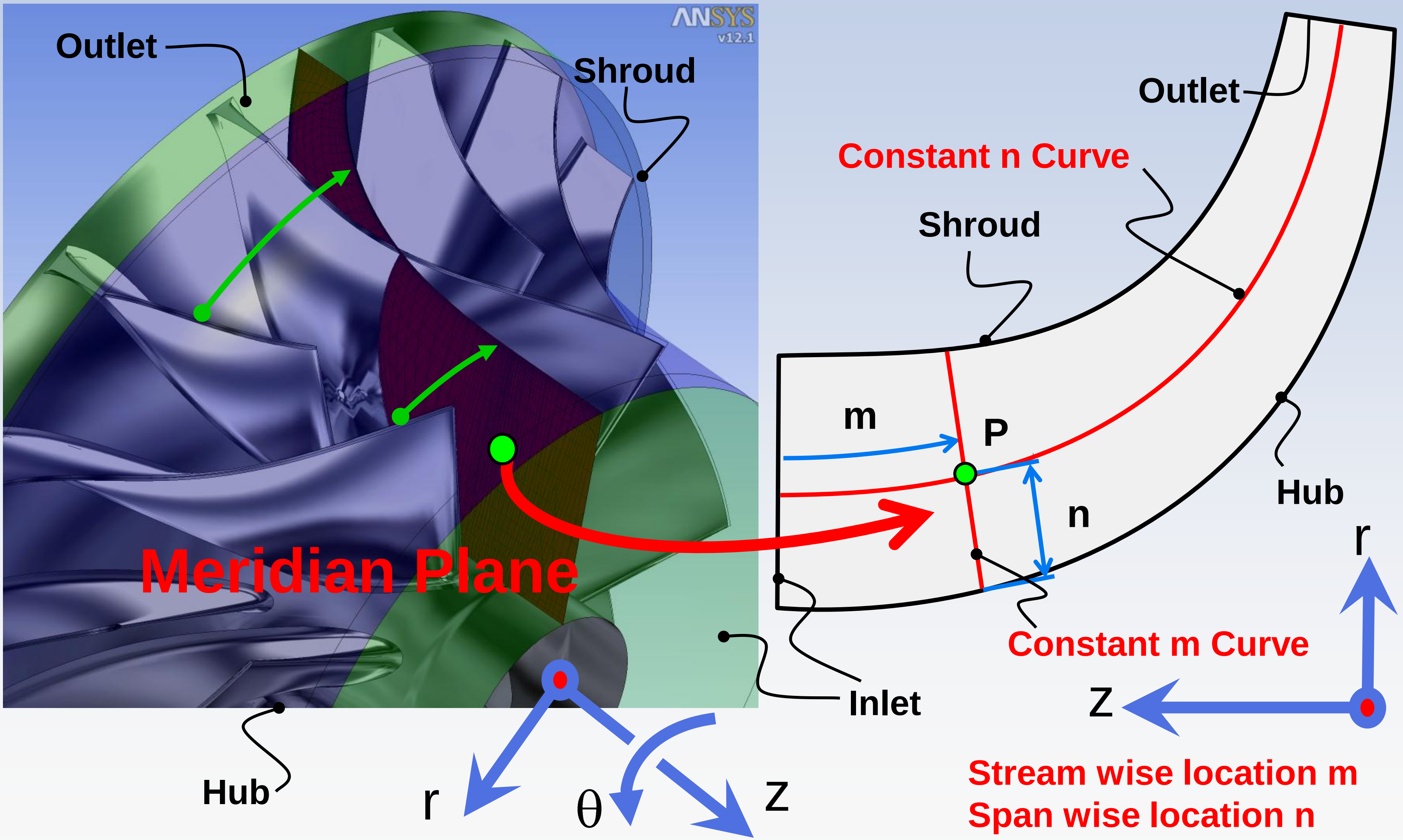
Mechanical Simulation

Process Integration

Robust Design Optimization

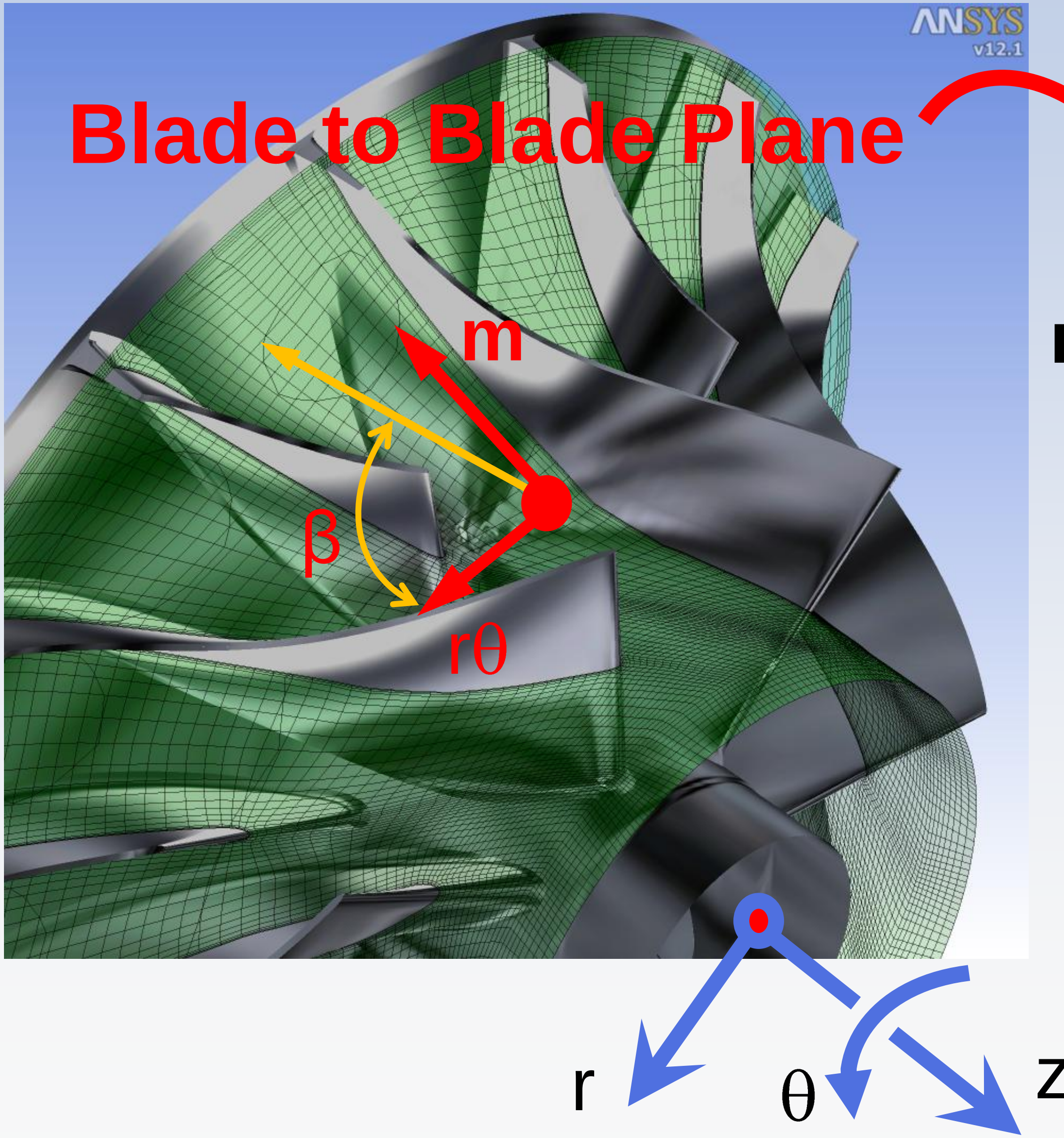


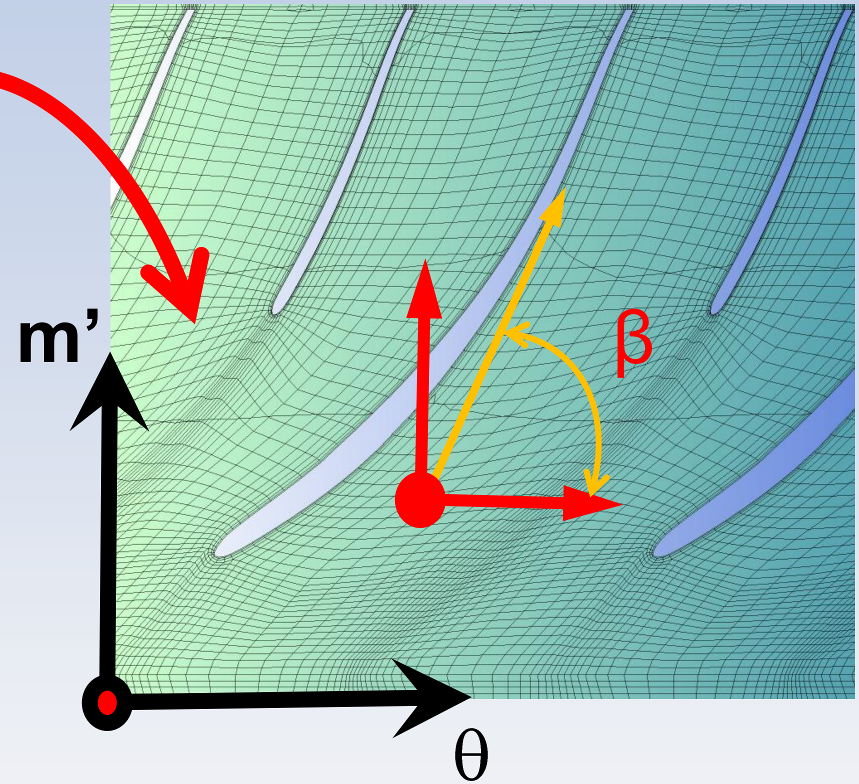
Conformal Mapping



Stream wise location m
Span wise location n

Conformal Mapping



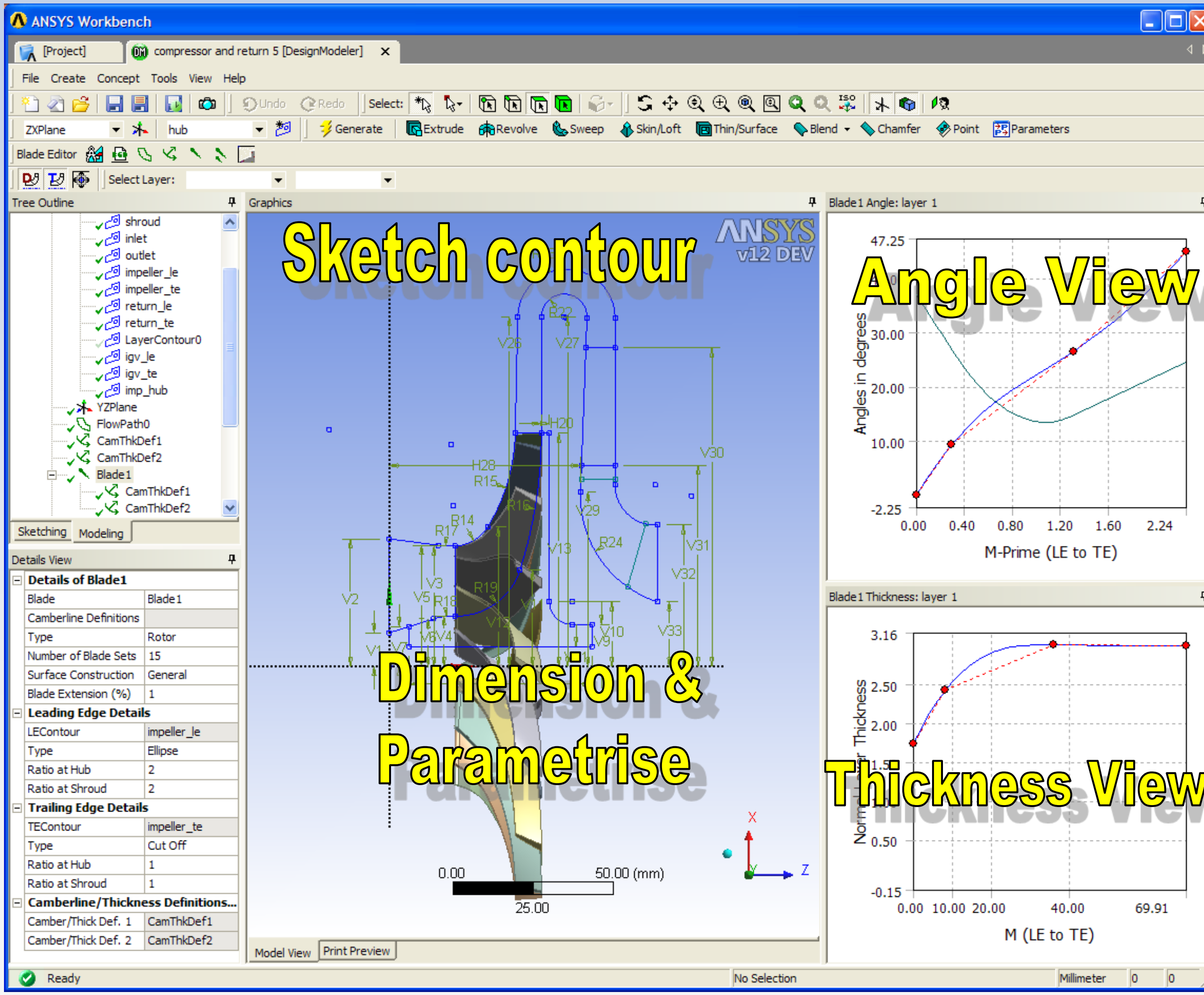


$$\tan(\beta) = \frac{dm}{r \cdot d\theta} \Rightarrow dm' = \frac{dm}{r}$$

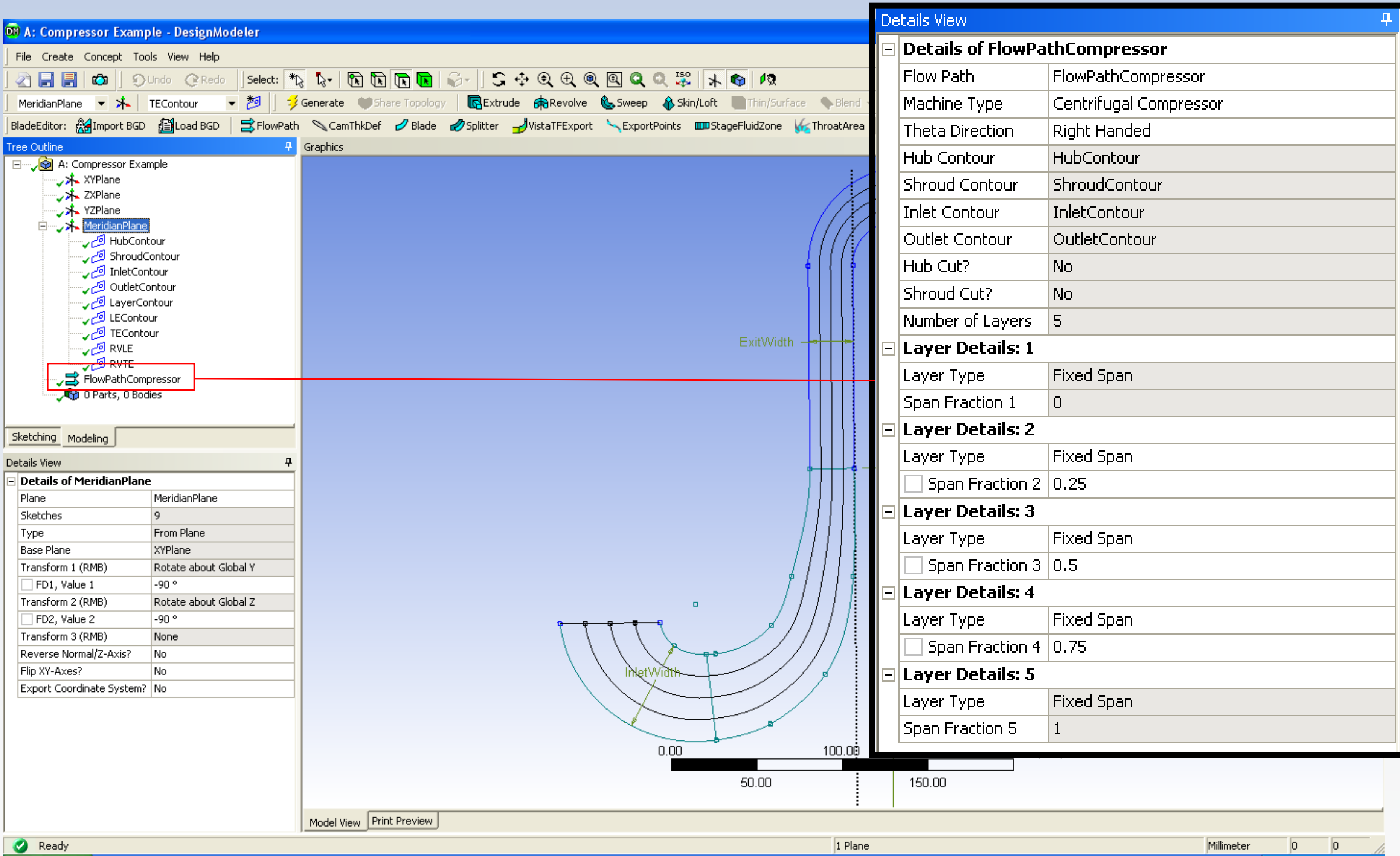
$$\tan(\beta) = \frac{dm'}{d\theta}$$

BladeModeler

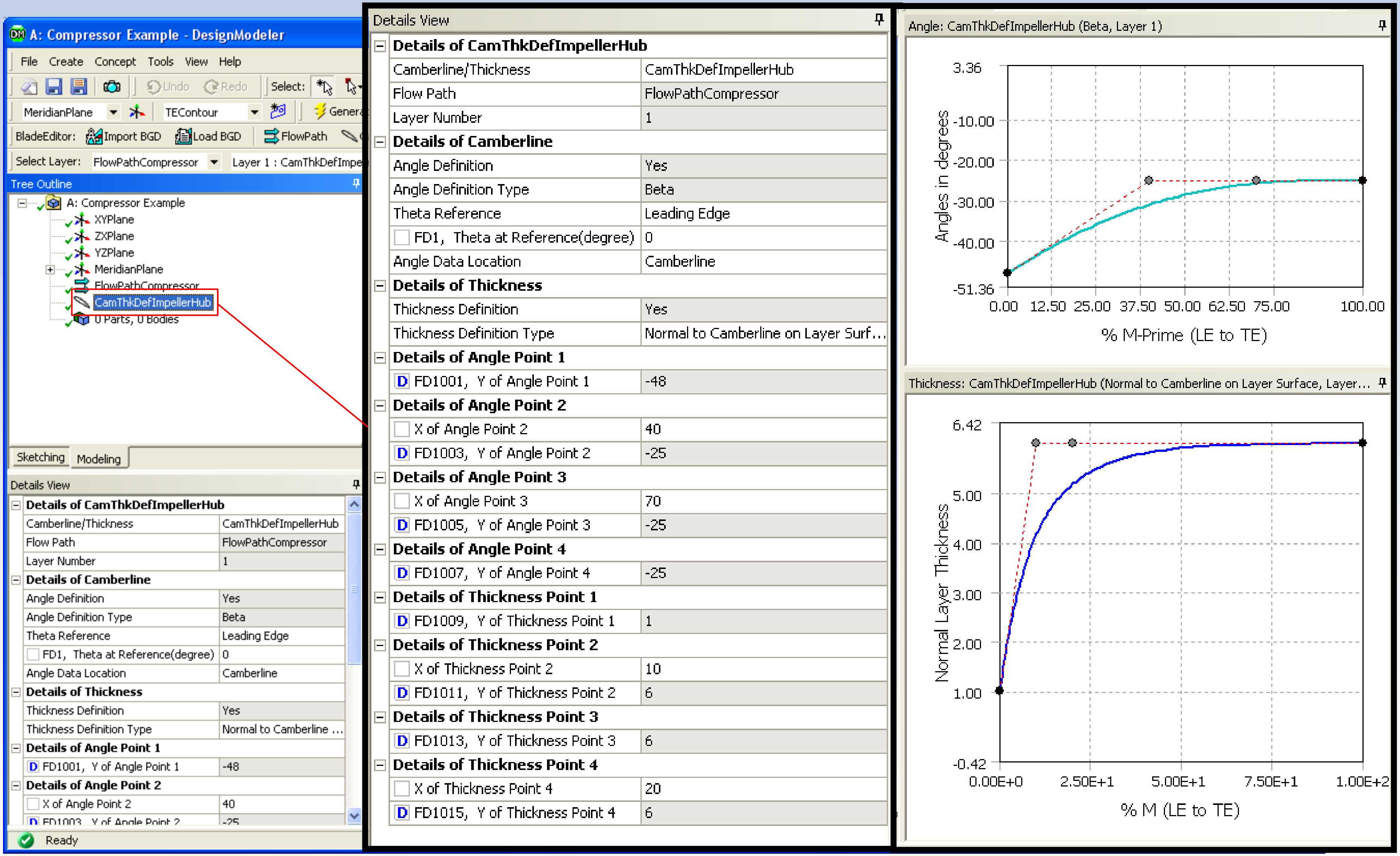
- Blade Design abilities in DesignModeler
- Angle/Thickness modifications in BladeEditor
- Multi-Stage Machines



Meridian Contour

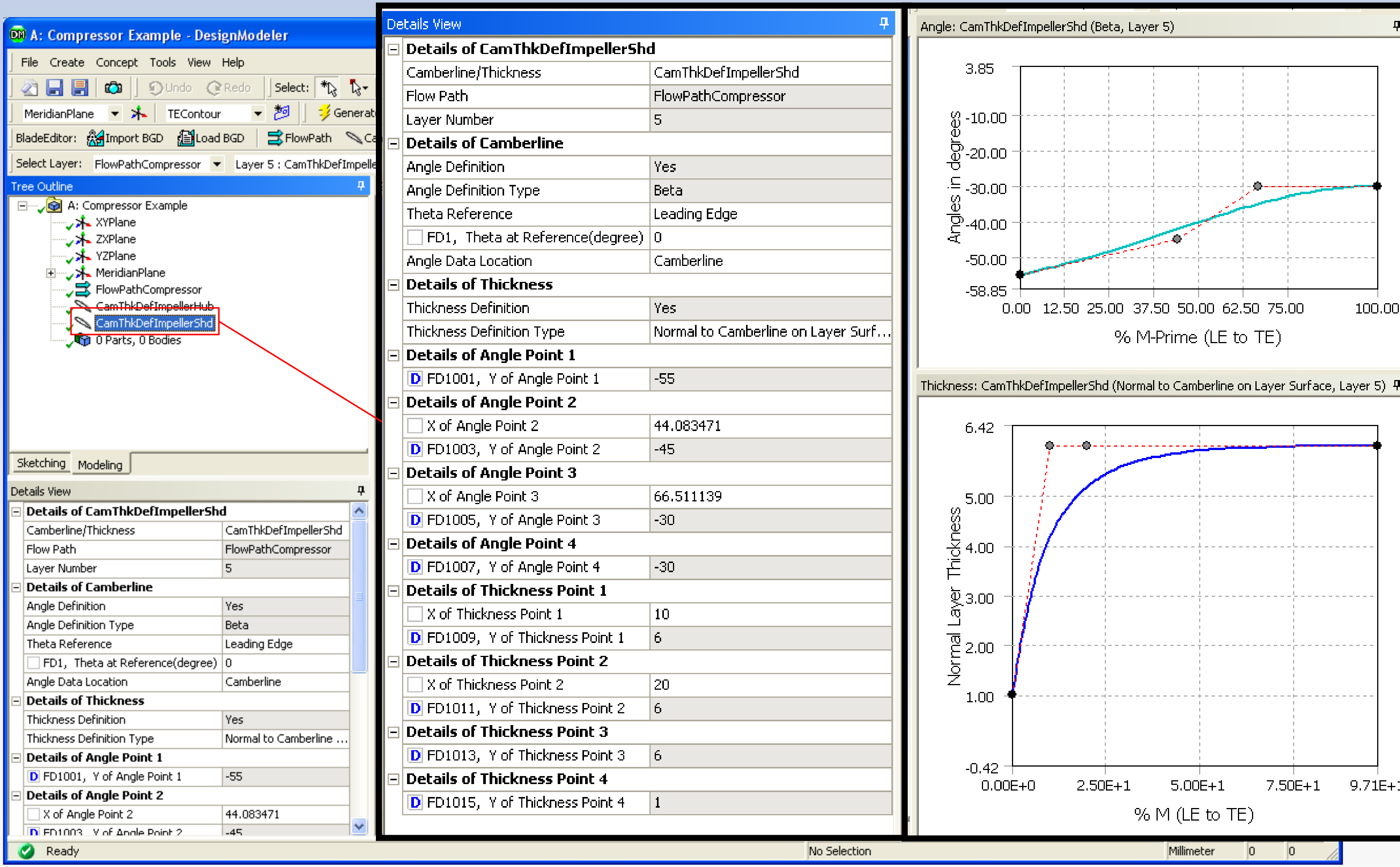


Impeller Hub, CamThk

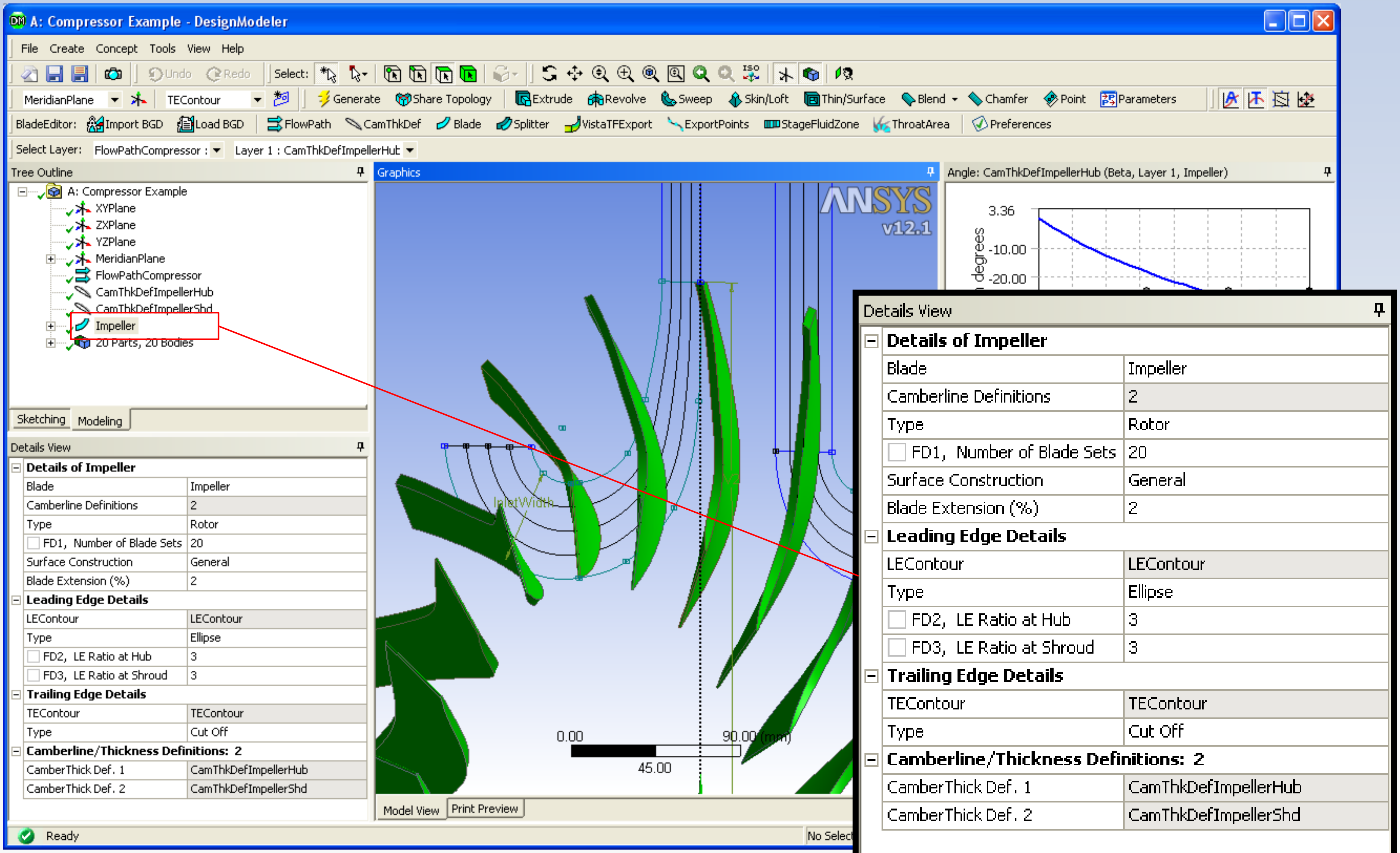


Details of CamThkDefImpellerHub	
Camberline/Thickness	CamThkDefImpellerHub
Flow Path	FlowPathCompressor
Layer Number	1
Details of Camberline	
Angle Definition	Yes
Angle Definition Type	Beta
Theta Reference	Leading Edge
FD1, Theta at Reference(degree)	0
Angle Data Location	Camberline
Details of Thickness	
Thickness Definition	Yes
Thickness Definition Type	Normal to Camberline on Layer Surf…
Details of Angle Point 1	
D FD1001, Y of Angle Point 1	-48
Details of Angle Point 2	
X of Angle Point 2	40
D FD1003, Y of Angle Point 2	-25
Details of Angle Point 3	
X of Angle Point 3	70
D FD1005, Y of Angle Point 3	-25
Details of Angle Point 4	
D FD1007, Y of Angle Point 4	-25
Details of Thickness Point 1	
D FD1009, Y of Thickness Point 1	1
Details of Thickness Point 2	
X of Thickness Point 2	10
D FD1011, Y of Thickness Point 2	6
Details of Thickness Point 3	
D FD1013, Y of Thickness Point 3	6
Details of Thickness Point 4	
X of Thickness Point 4	20
D FD1015, Y of Thickness Point 4	6

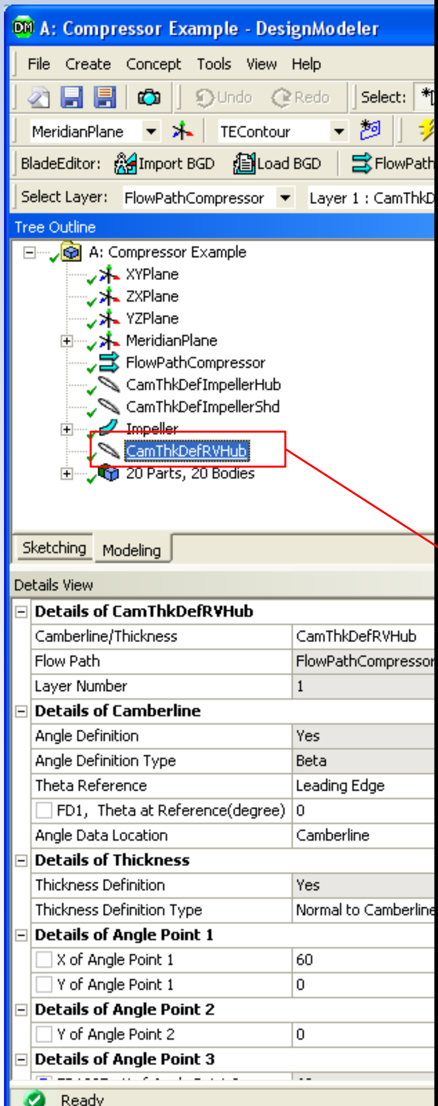
Impeller Shroud, CamThk



Impeller Blade

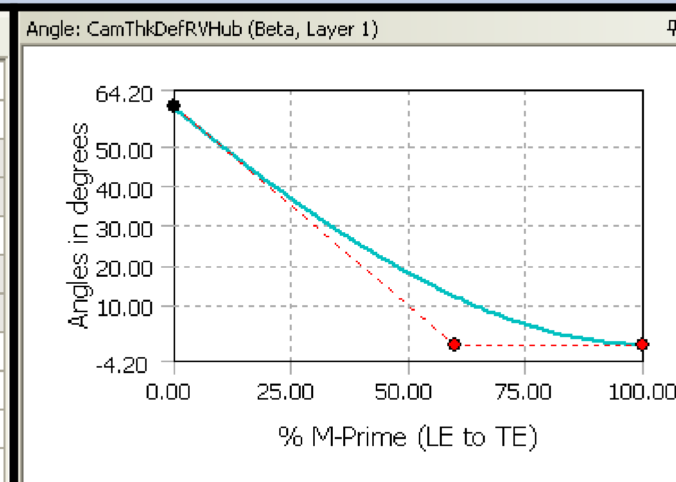


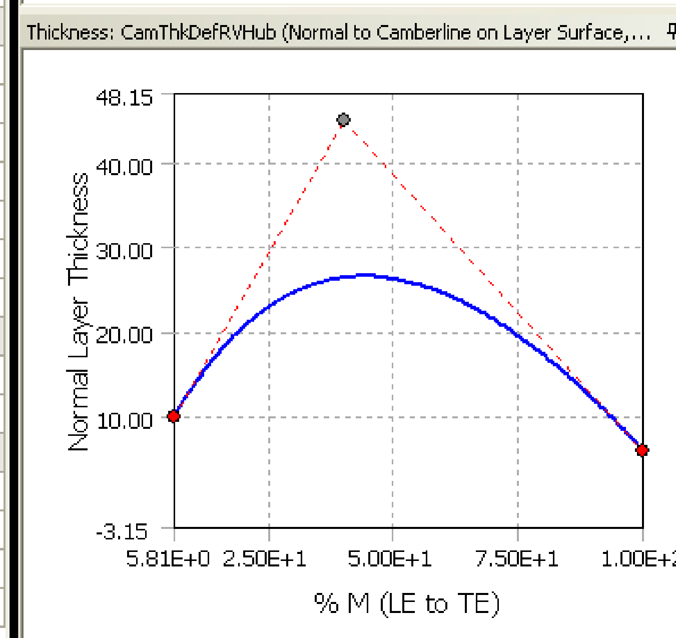
Return Vane Hub, CamThk



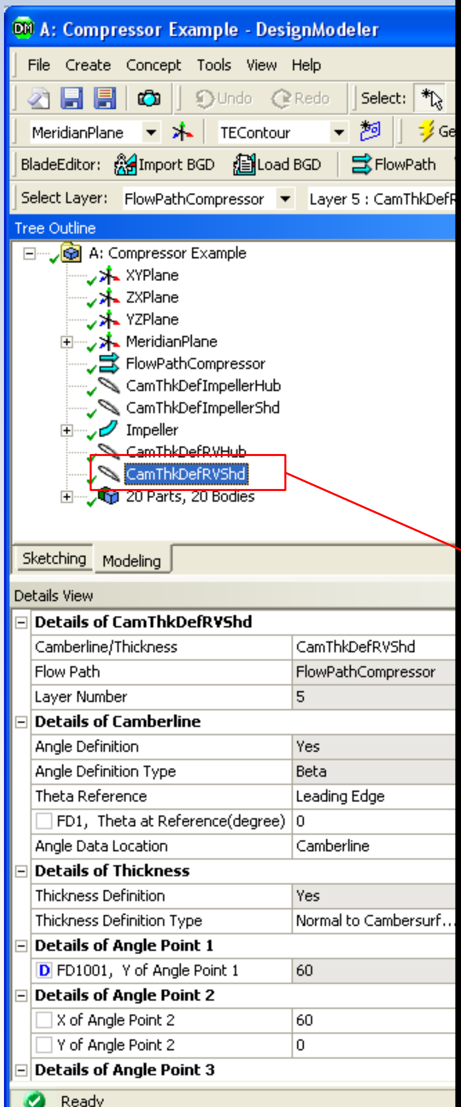
Details View

Details of CamThkDefRVHub	
Camberline/Thickness	CamThkDefRVHub
Flow Path	FlowPathCompressor
Layer Number	1
Details of Camberline	
Angle Definition	Yes
Angle Definition Type	Beta
Theta Reference	Leading Edge
FD1, Theta at Reference(degree)	0
Angle Data Location	Camberline
Details of Thickness	
Thickness Definition	Yes
Thickness Definition Type	Normal to Camberline on ...
Details of Angle Point 1	
X of Angle Point 1	60
Y of Angle Point 1	0
Details of Angle Point 2	
Y of Angle Point 2	0
Details of Angle Point 3	
D FD1005, Y of Angle Point 3	60
Details of Thickness Point 1	
X of Thickness Point 1	40
D FD1007, Y of Thickness Point 1	45
Details of Thickness Point 3	
Y of Thickness Point 3	6
Details of Thickness Point 4	
Y of Thickness Point 4	10



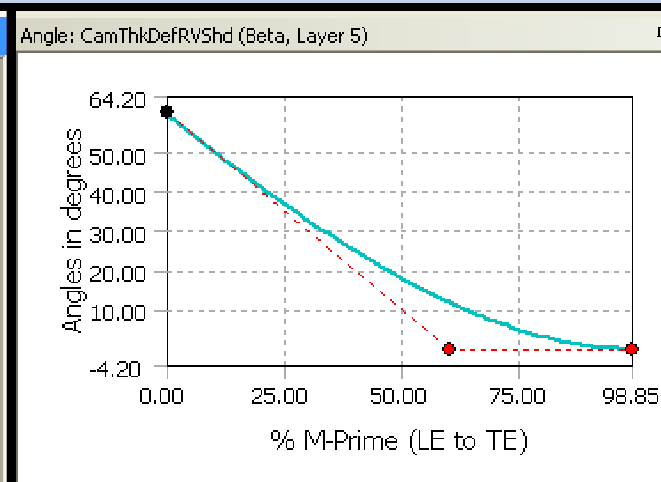


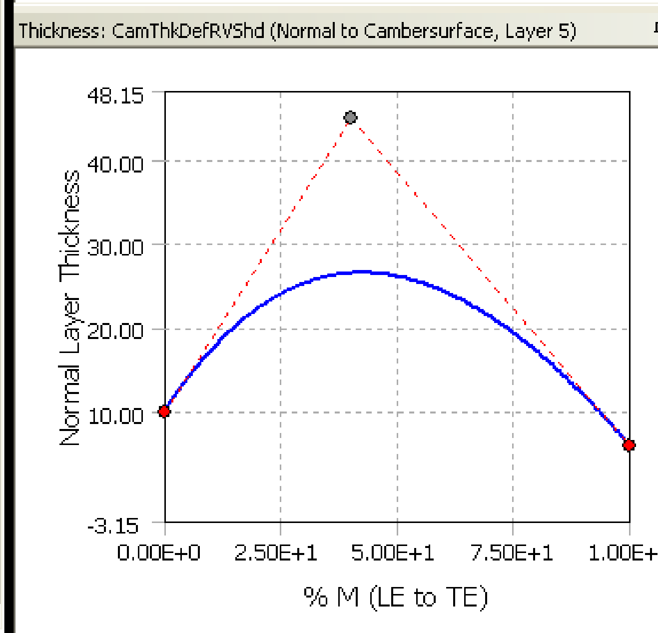
Return Vane Shroud, CamThk



Details View

Details of CamThkDefRVShd	
Camberline/Thickness	CamThkDefRVShd
Flow Path	FlowPathCompressor
Layer Number	5
Details of Camberline	
Angle Definition	Yes
Angle Definition Type	Beta
Theta Reference	Leading Edge
FD1, Theta at Reference(degree)	0
Angle Data Location	Camberline
Details of Thickness	
Thickness Definition	Yes
Thickness Definition Type	Normal to Cambersurface
Details of Angle Point 1	
D FD1001, Y of Angle Point 1	60
Details of Angle Point 2	
X of Angle Point 2	60
Y of Angle Point 2	0
Details of Angle Point 3	
Y of Angle Point 3	0
Details of Thickness Point 1	
Y of Thickness Point 1	10
Details of Thickness Point 2	
X of Thickness Point 2	40
D FD1009, Y of Thickness Point 2	45
Details of Thickness Point 3	
Y of Thickness Point 3	6





Return Vane Blade

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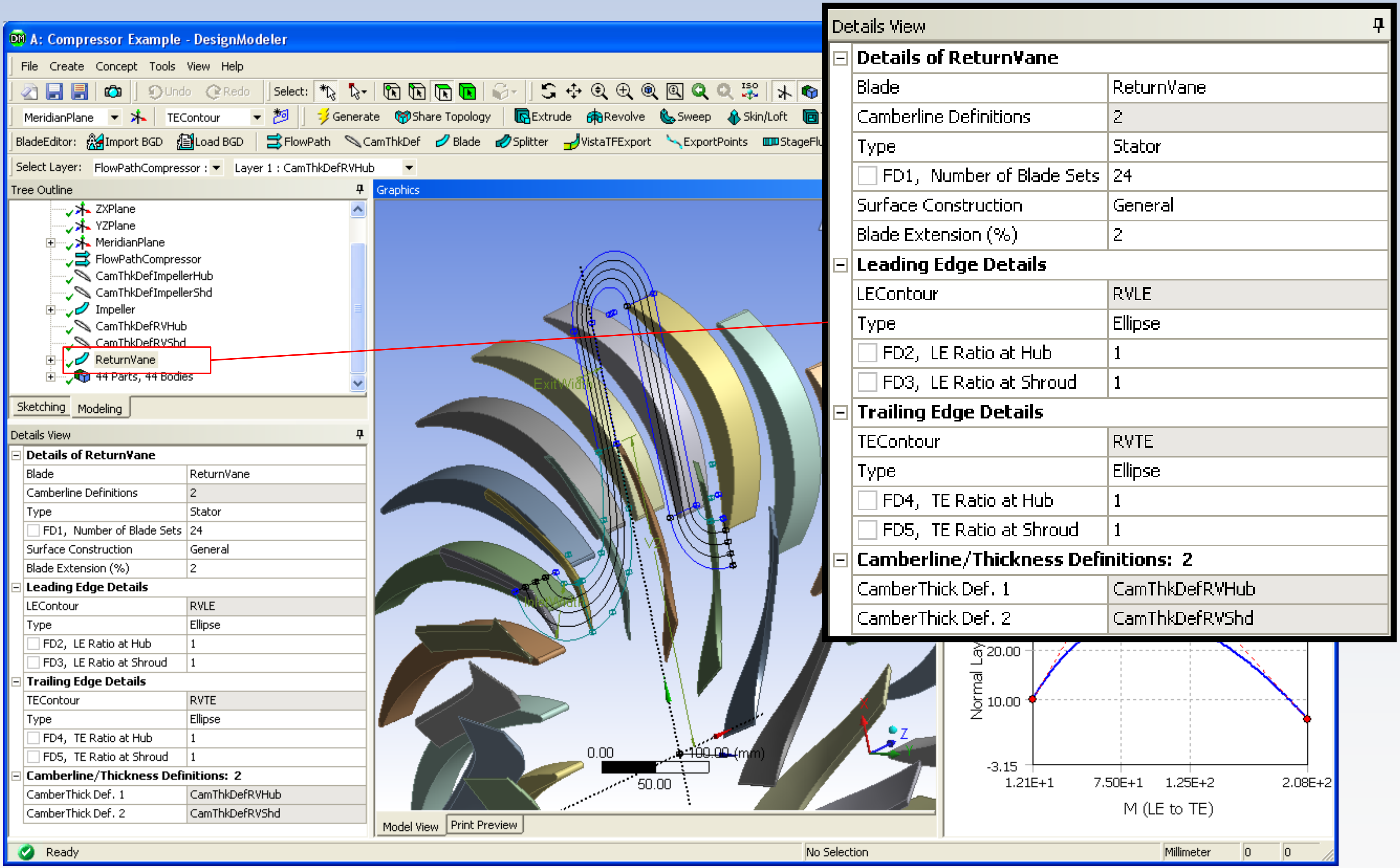
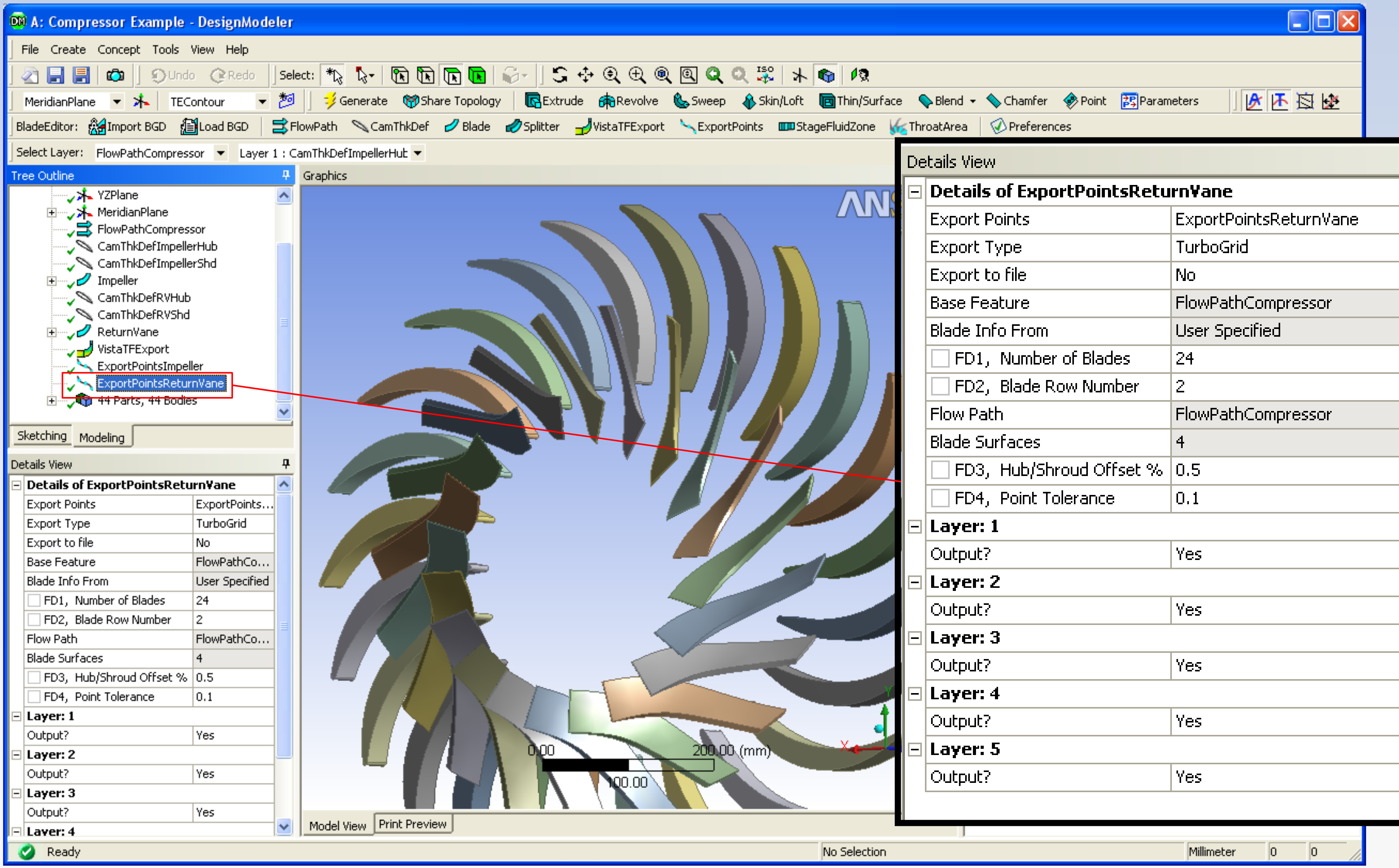


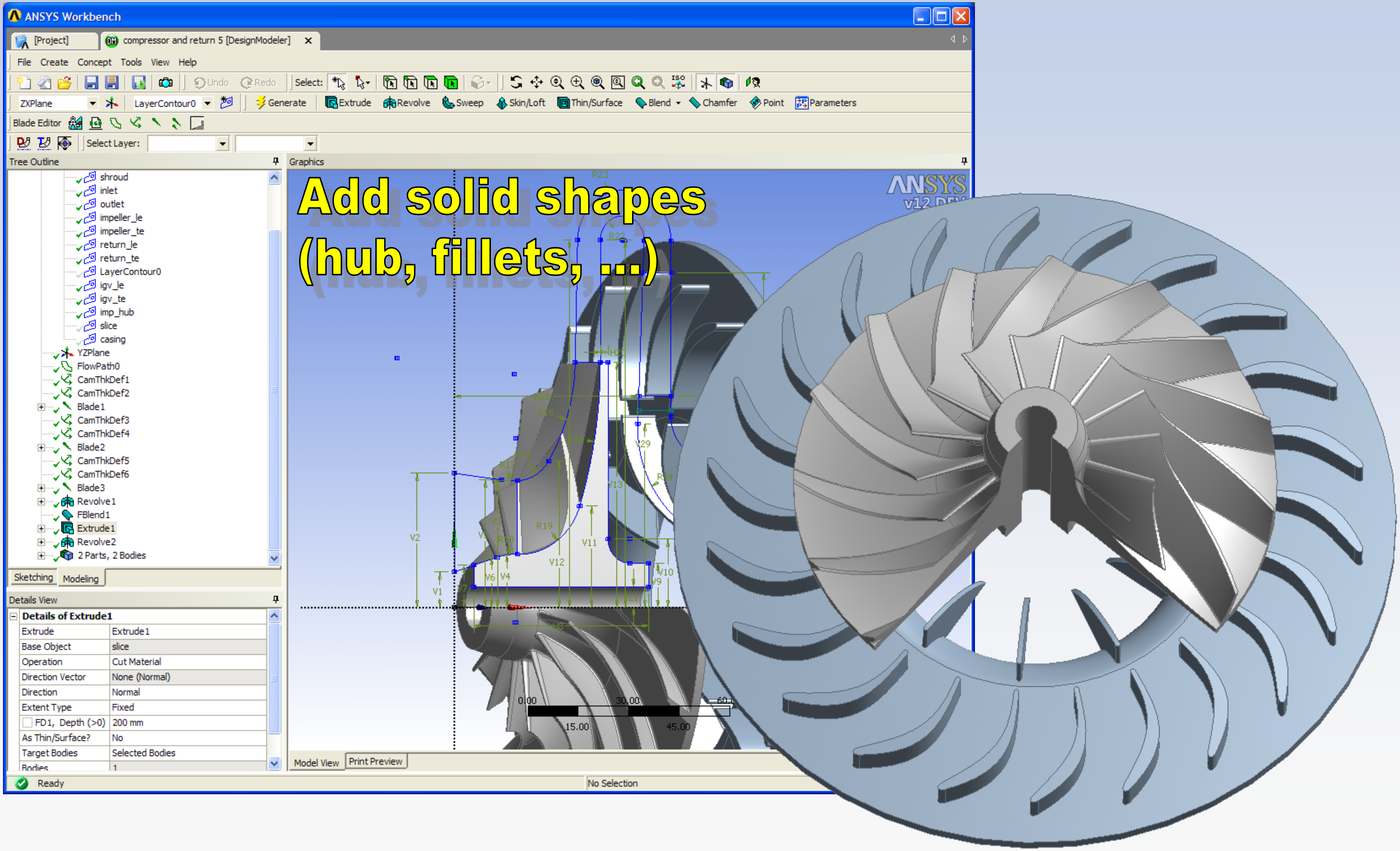
Details View

Details of ReturnVane	
Blade	ReturnVane
Camberline Definitions	2
Type	Stator
FD1, Number of Blade Sets	24
Surface Construction	General
Blade Extension (%)	2
Leading Edge Details	
LEContour	RVLE
Type	Ellipse
FD2, LE Ratio at Hub	1
FD3, LE Ratio at Shroud	1
Trailing Edge Details	
TEContour	RVTE
Type	Ellipse
FD4, TE Ratio at Hub	1
FD5, TE Ratio at Shroud	1
Camberline/Thickness Definitions: 2	
CamberThick Def. 1	CamThkDefRVHub
CamberThick Def. 2	CamThkDefRVShd

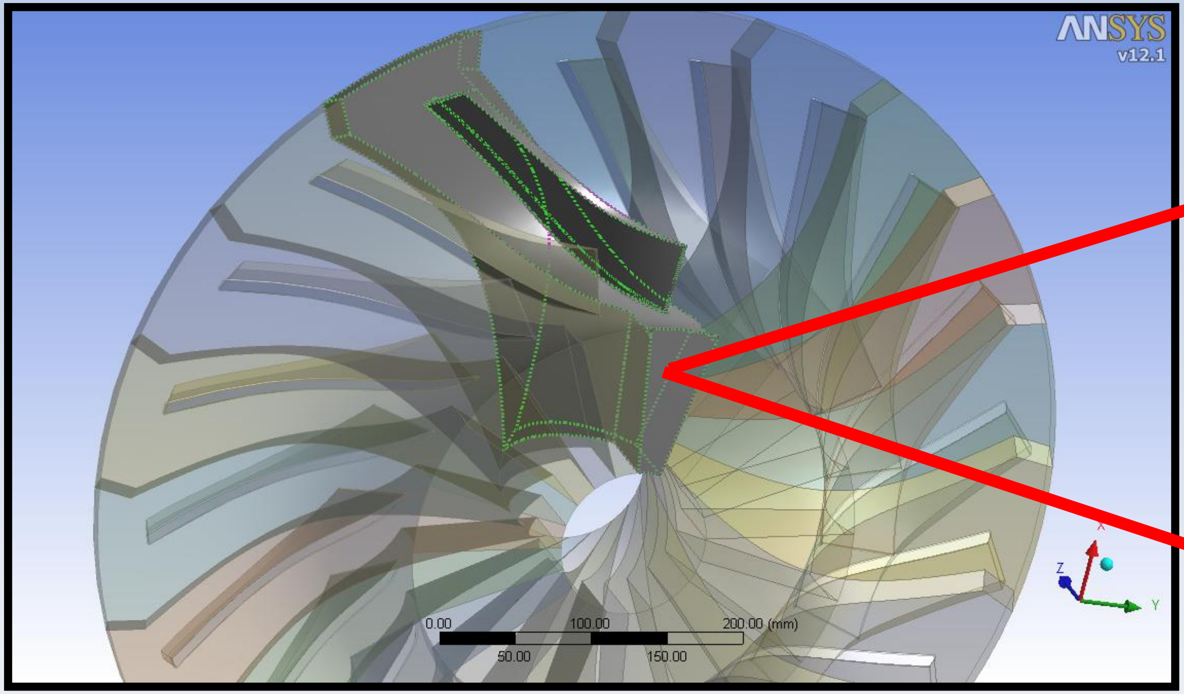
Geometry Export, TurboGrid

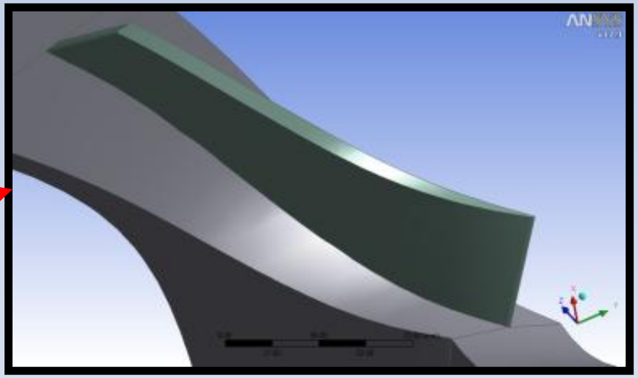


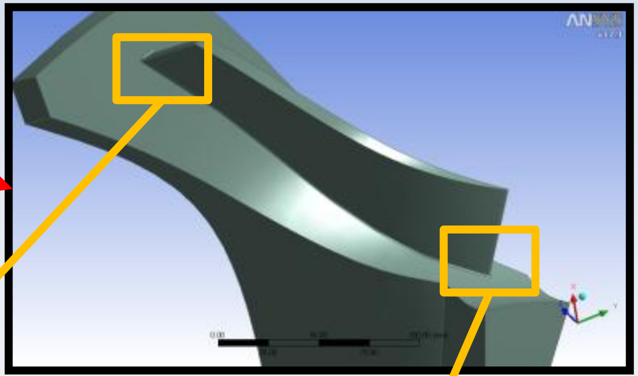
3D Blade Design



Geometry of Impeller

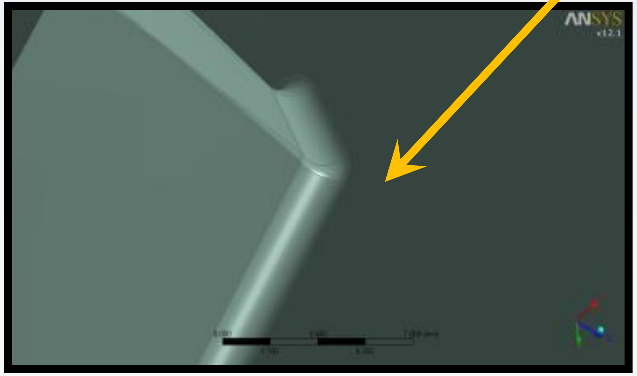


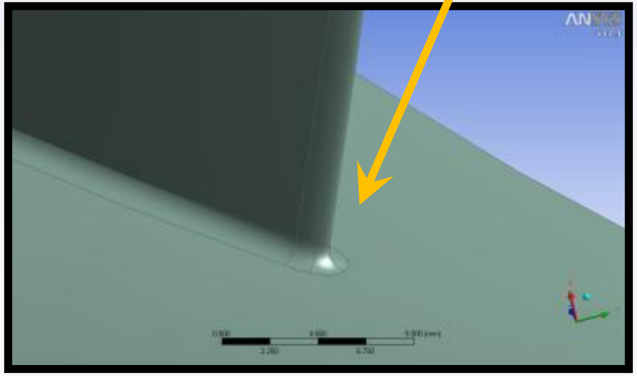




One sector

Model:
A, no blend
B, blend 1 mm





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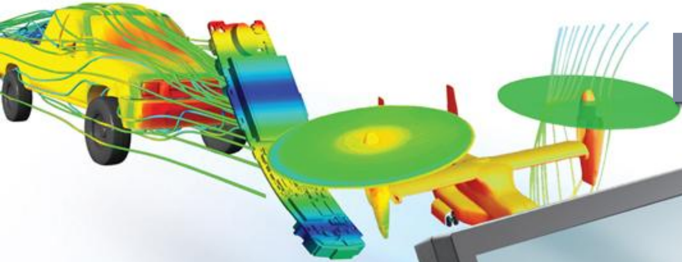
Geometry

CFD Simulation



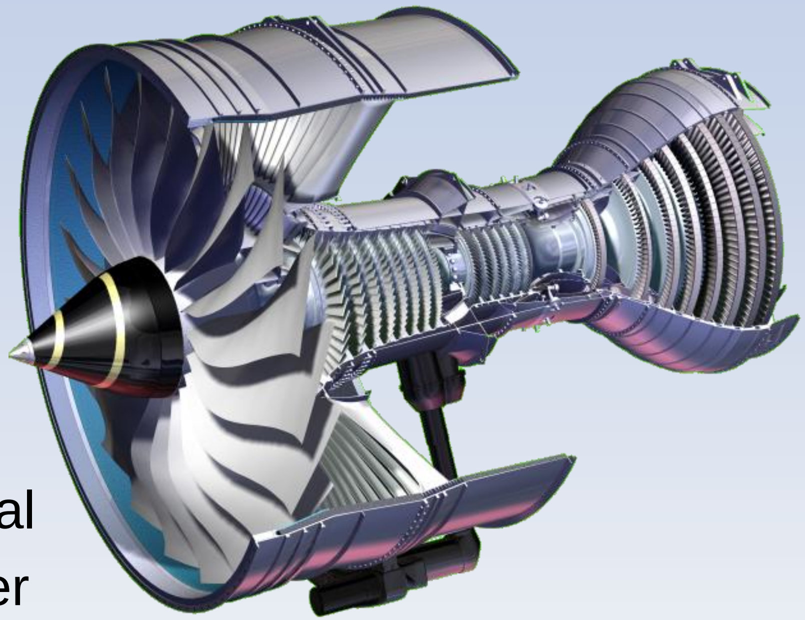
Mechanical Simulation

Process Integration

Robust Design Optimization

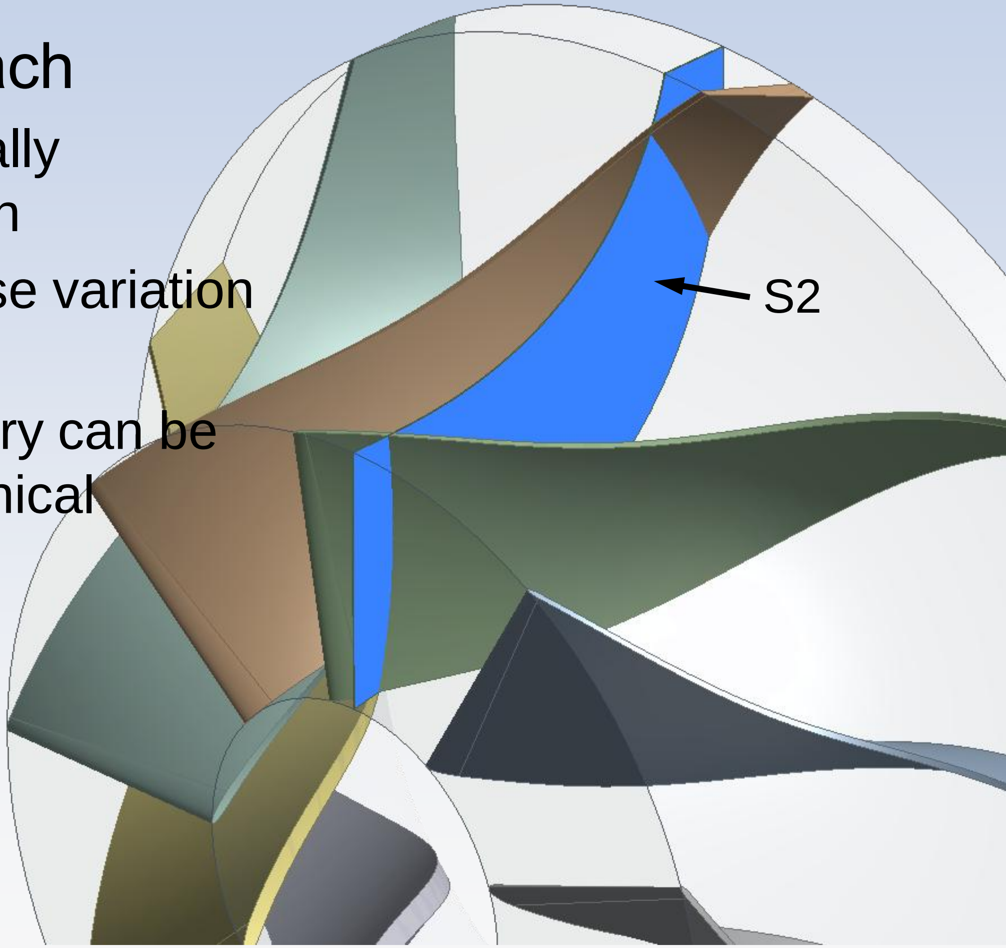
Vista CCD, Overview

- Preliminary Design
  - Mean line, or “1D”
  - Velocity triangles
    - Defines the geometrical envelope within the larger picture
    - Determines the performance potential
    - Errors very expensive to recover later
  - Semi-empirical approach
    - Many calculations, needs to be rapid and robust
    - Can be amenable to optimization



Vista TF, Overview

- Expand into 2D
  - S2 or 'through flow' approach
    - Solution of the circumferentially averaged equations of motion
    - Take account of the span wise variation in design parameters
    - First estimates of 3D geometry can be made, both aero and mechanical aspects
- Refine in 3D
    - Accept longer run times
    - Absolute accuracy important



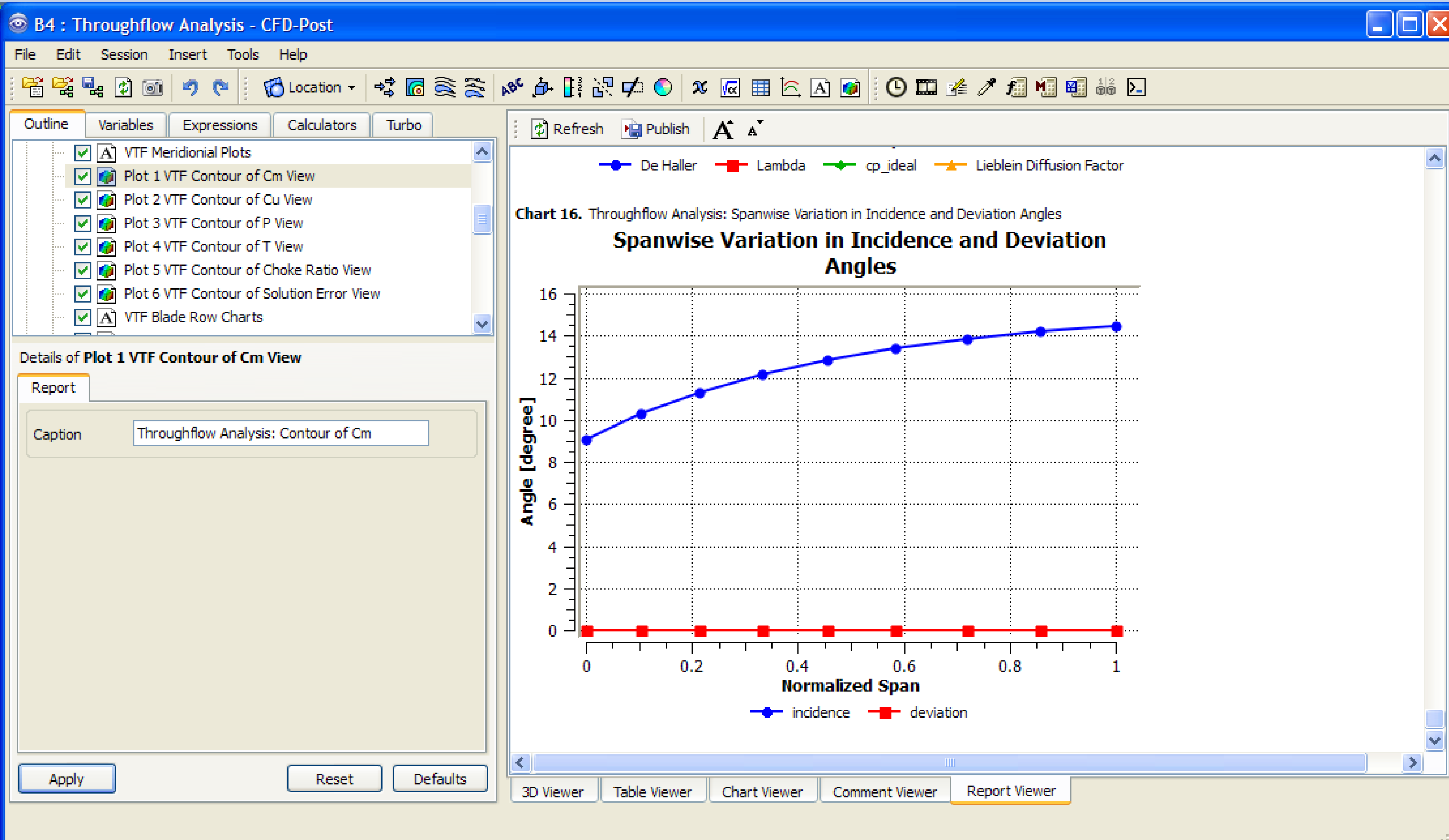
Vista TF, Solution Procedure

- Approximate meridional streamlines by splitting the flow into regions of equal area and make a first guess of all parameters
  – s from loss correlations
  – h_t from Euler equation
  – ρ from h_t and s

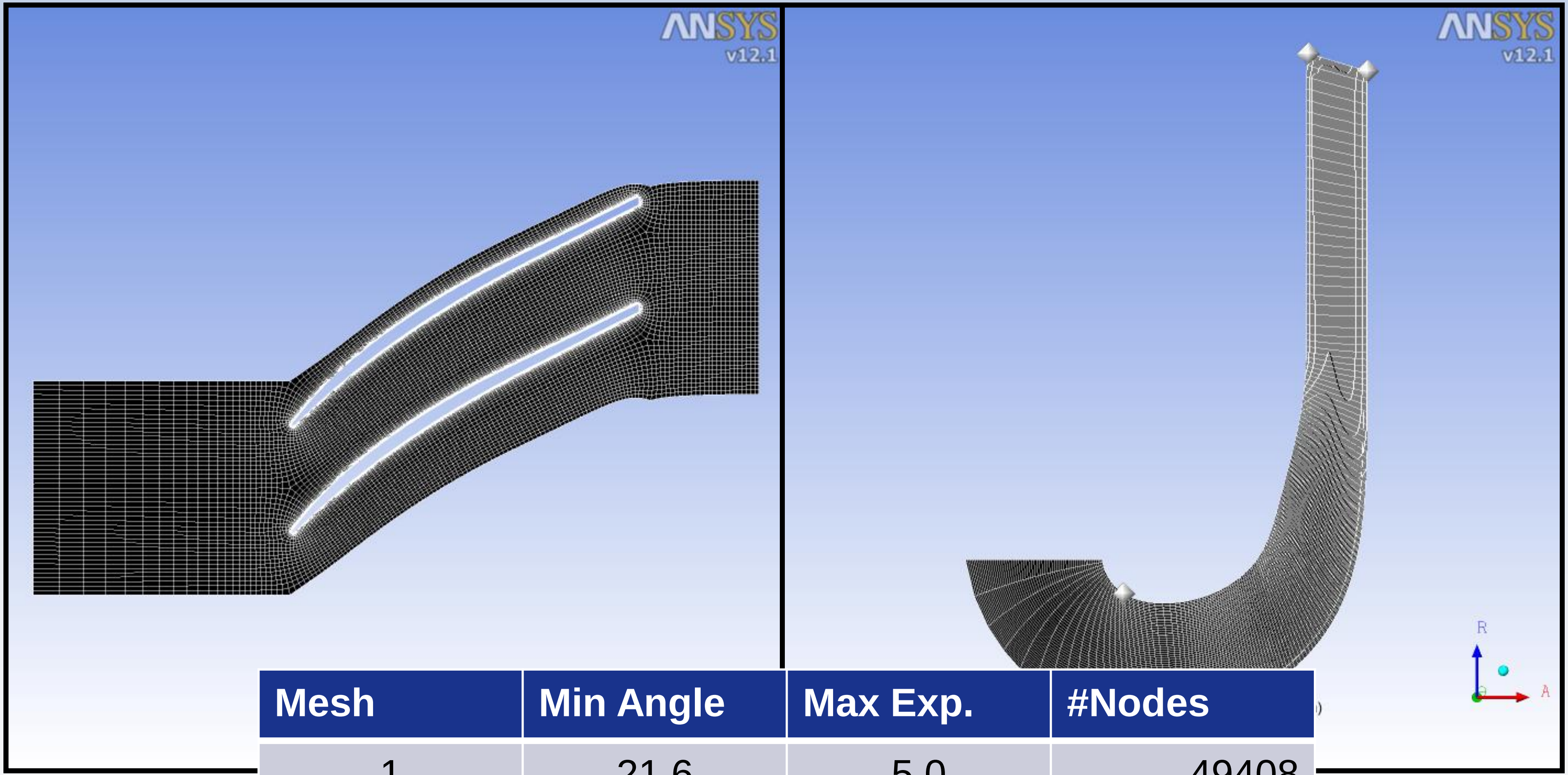
- Determine the distribution of c_m
  – Gradient of the meridional velocity from general radial equilibrium
  – Level of the meridional velocity from the continuity equation
  – Adjust c_m on mean streamline until both gradient and mass flow satisfied

- Improve estimate of the streamline positions and recalculate
- Iterate until c_m and all other parameters are converged

Vista TF Simulation Result

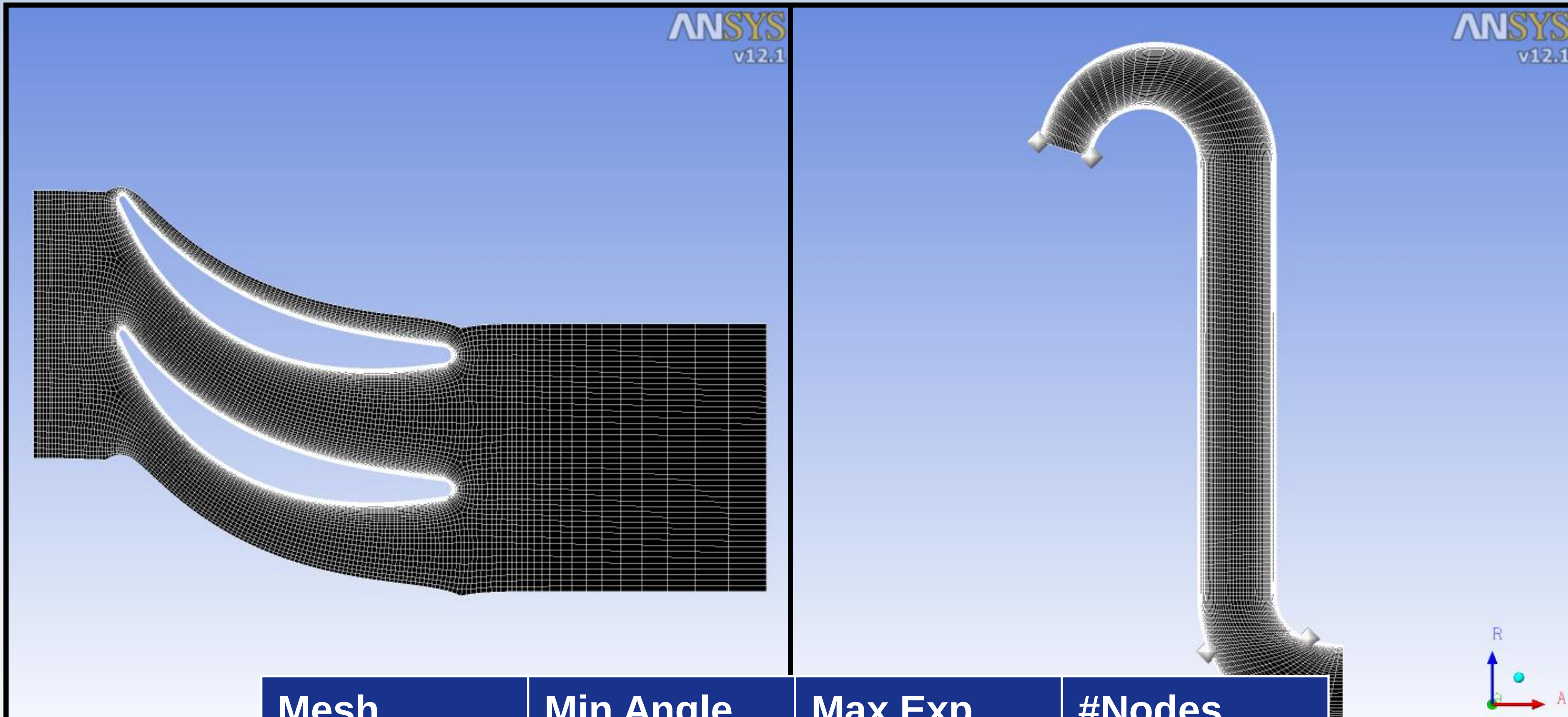


TurboGrid Mesh, Impeller



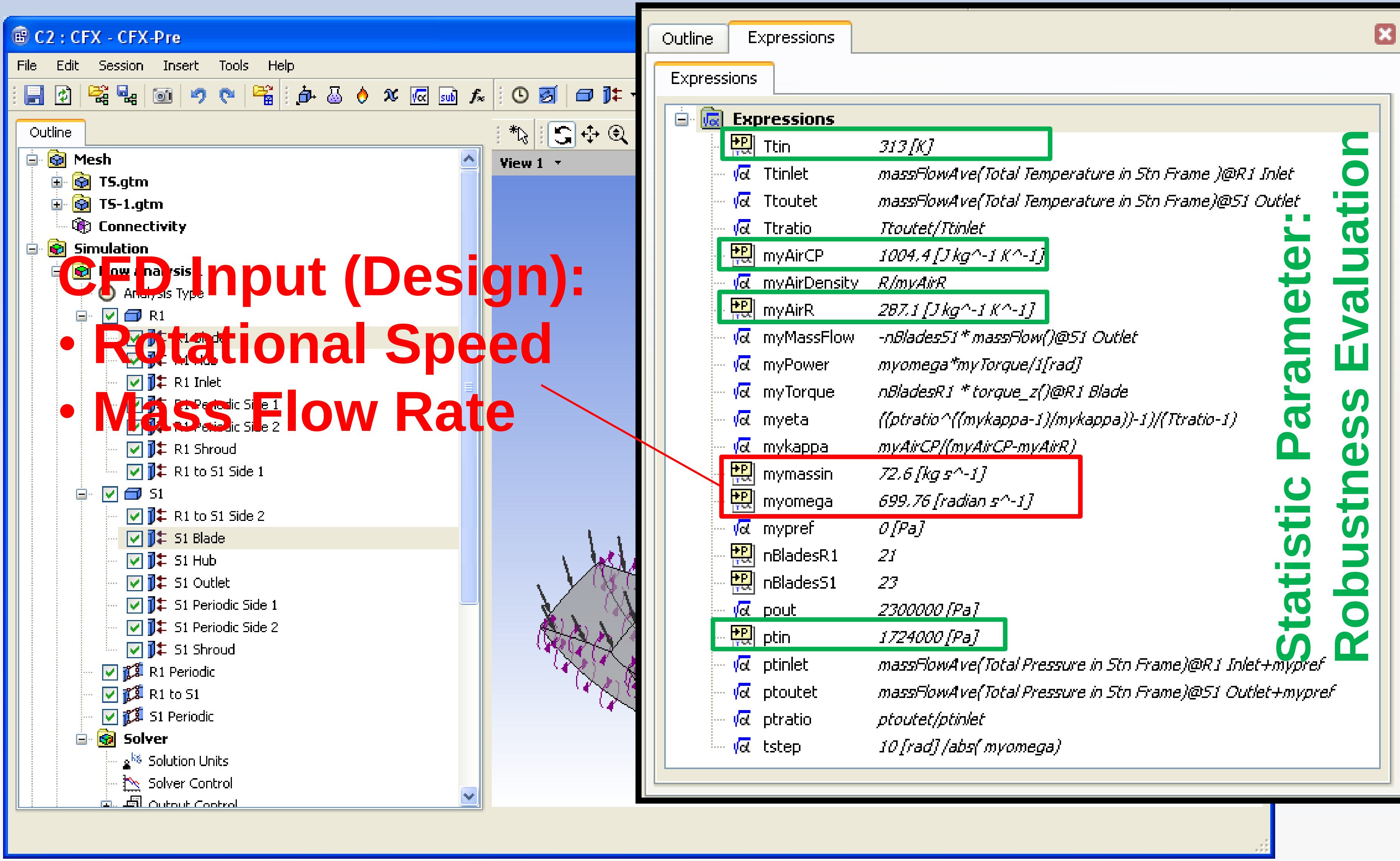
Mesh	Min Angle	Max Exp.	#Nodes
1	21.6	5.0	49408
2	21.9	3.0	174127
3	21.6	3.0	559370

TurboGrid Mesh, Return Vane



Mesh	Min Angle	Max Exp.	#Nodes
1	30.3	5.0	49040
2	30.0	3.0	172670
3	29.6	2.0	554185

CFX Preprocessing

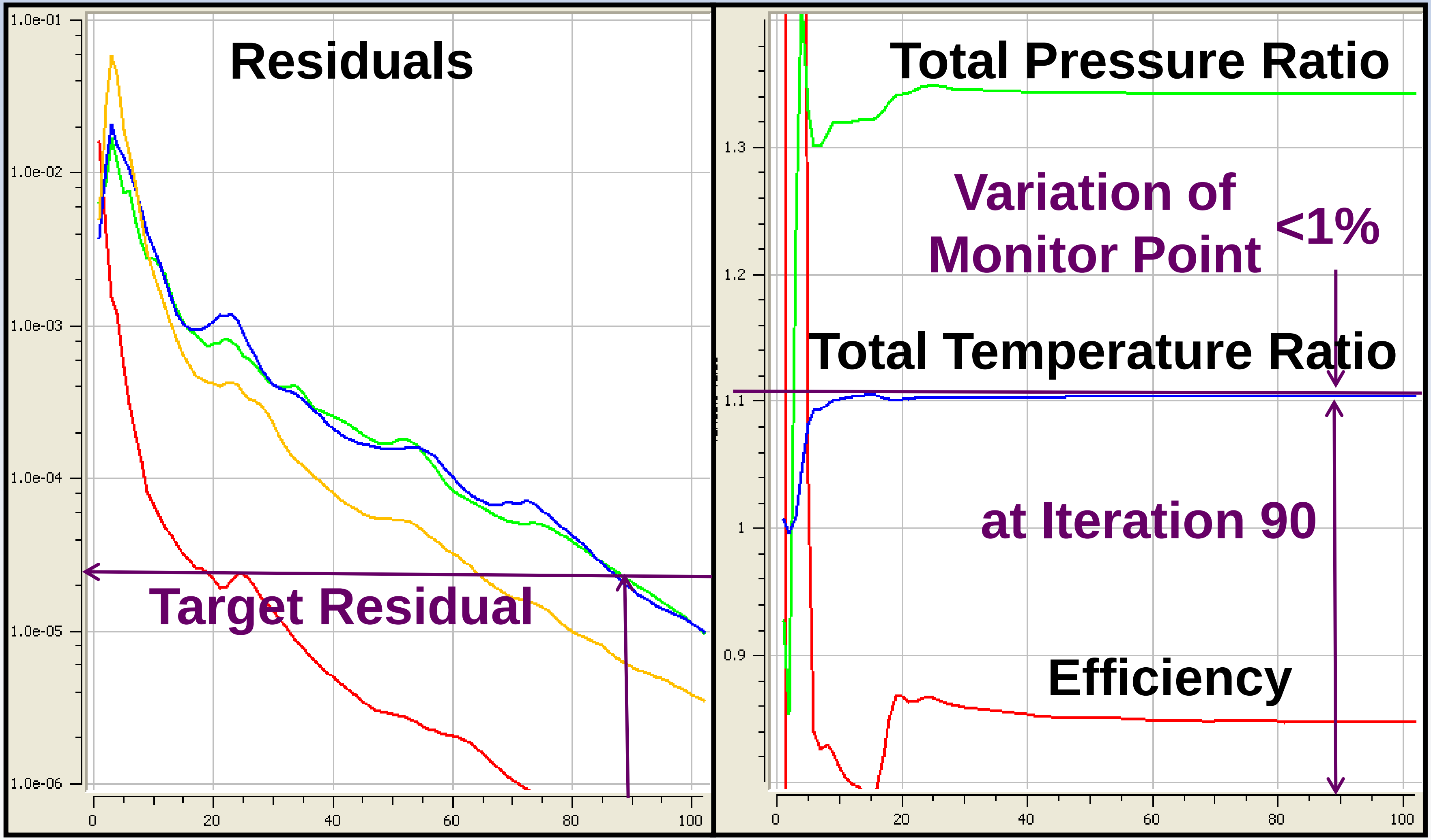


CFD Input (Design):

- **Rotational Speed**
- **Mass Flow Rate**

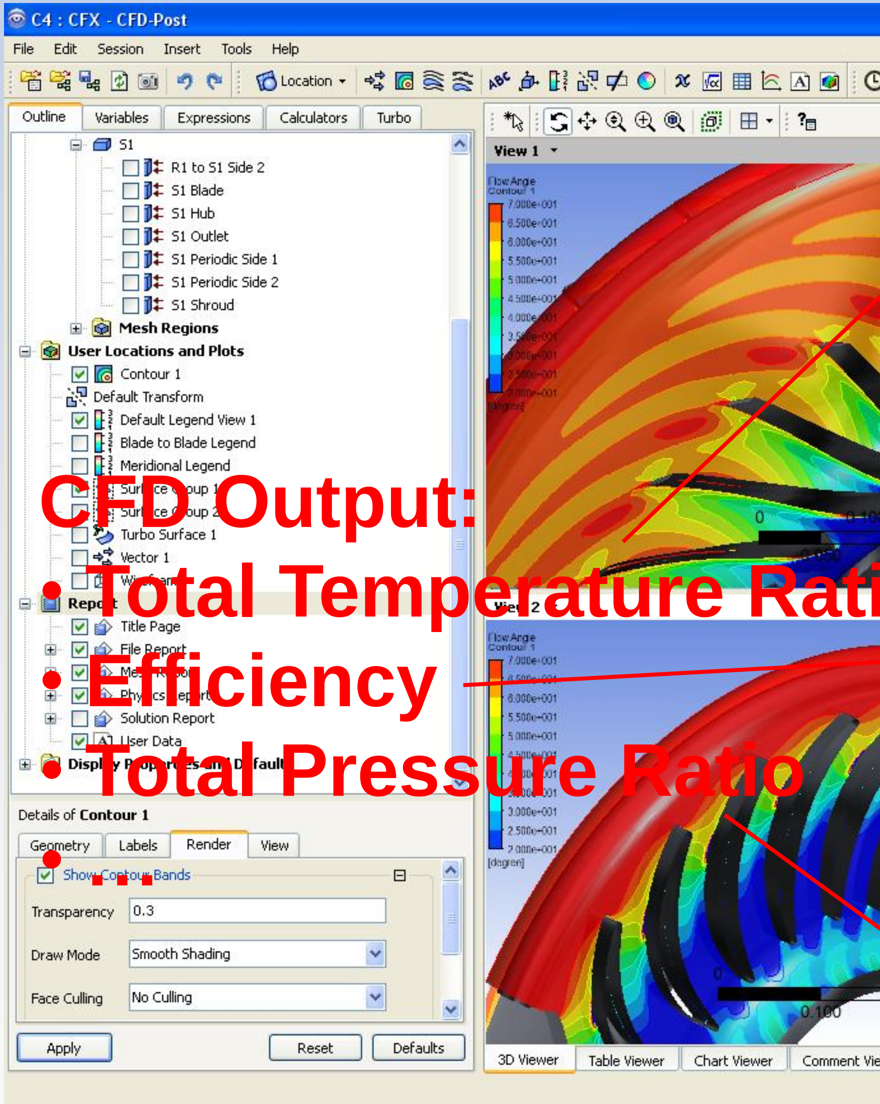
Statistic Parameter: Robustness Evaluation

CFX Solver



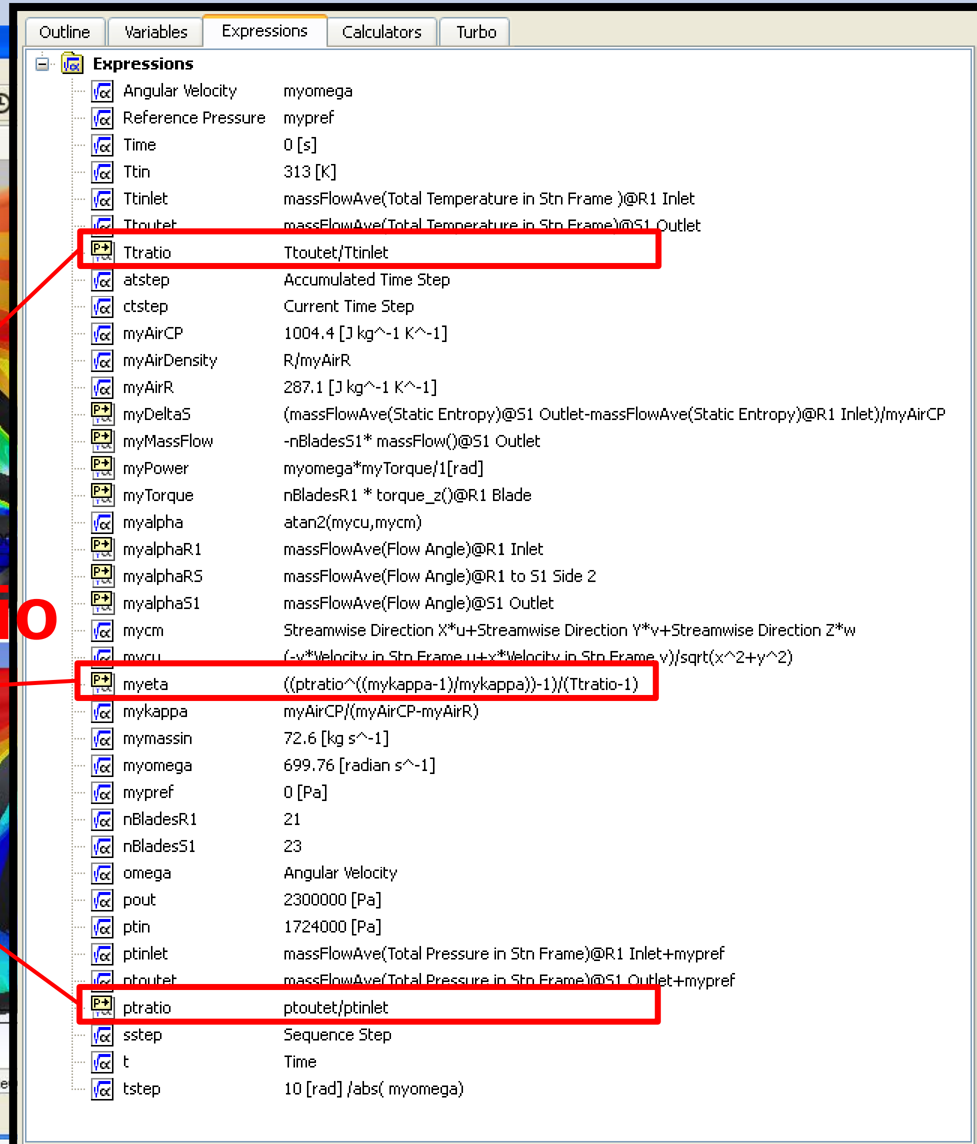
Residuals
Target Residual
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
0
20
40
60
80
100
Total Pressure Ratio
Variation of
Monitor Point
<1%
Total Temperature Ratio
at Iteration 90
Efficiency
1.3
1.2
1.1
1
0.9

CFX Postprocessing

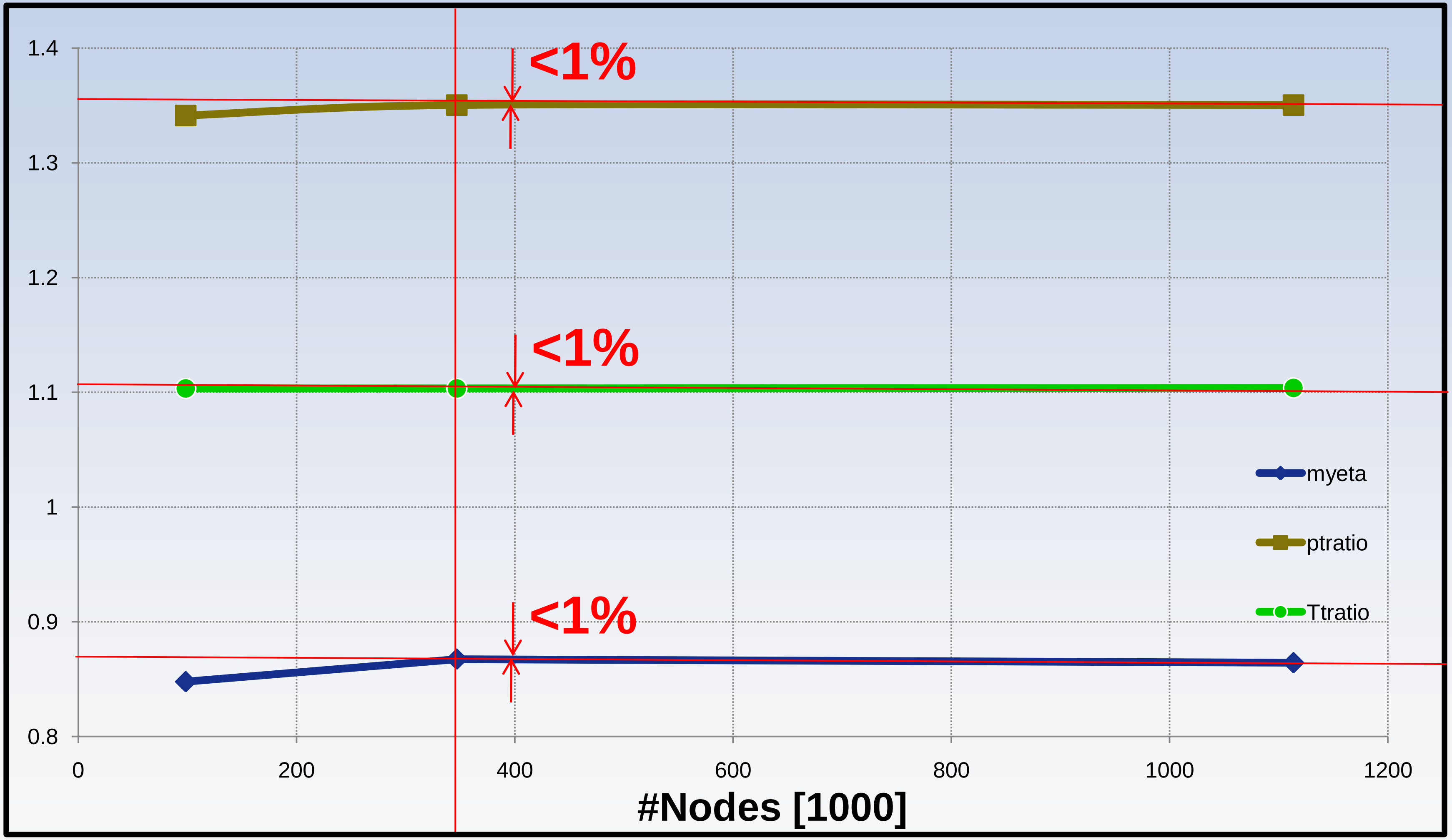


CFD Output:

- Total Temperature Ratio
- Efficiency
- Total Pressure Ratio
- ...



CFX, Best Practice



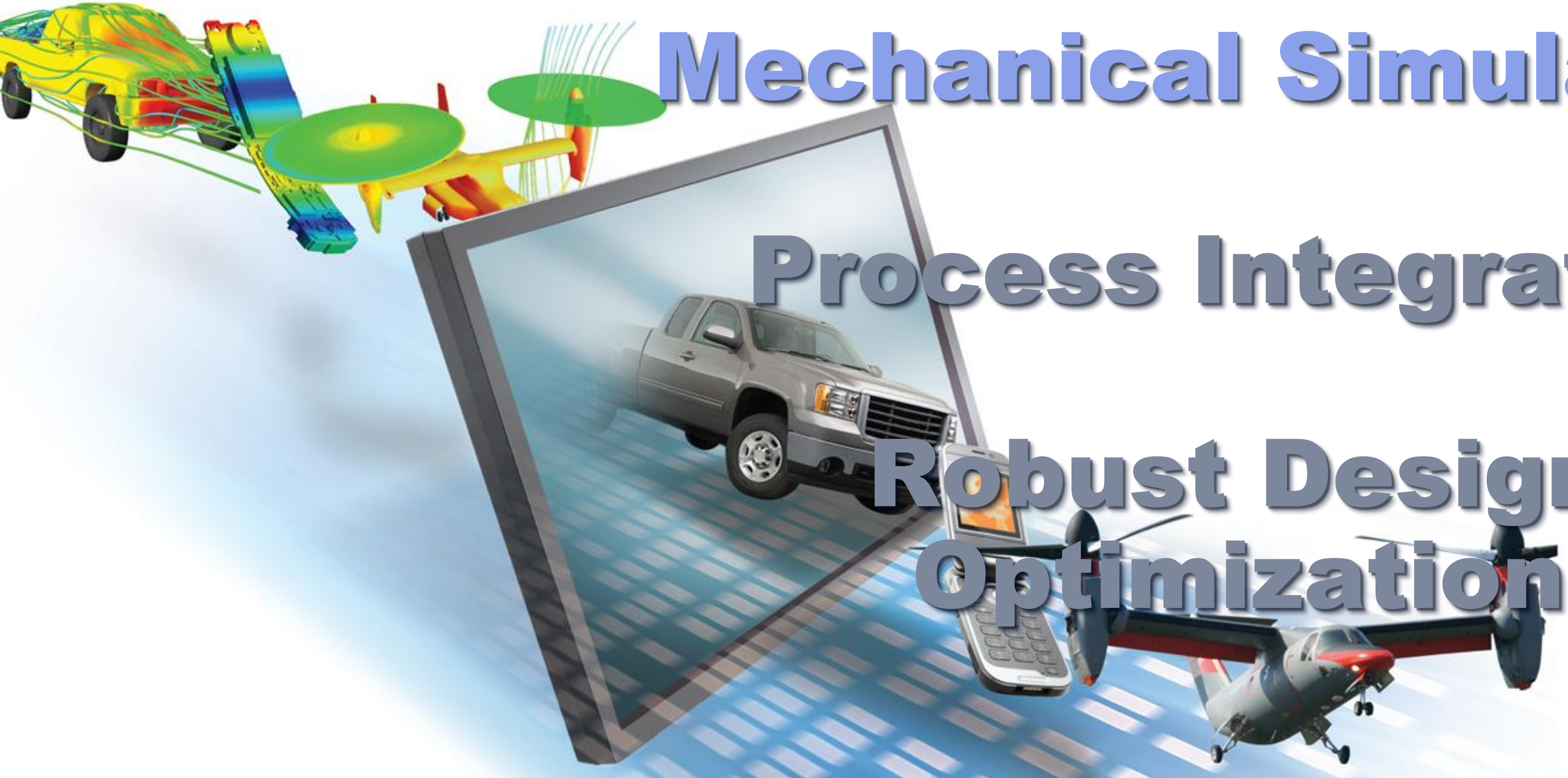
Geometry

CFD Simulation

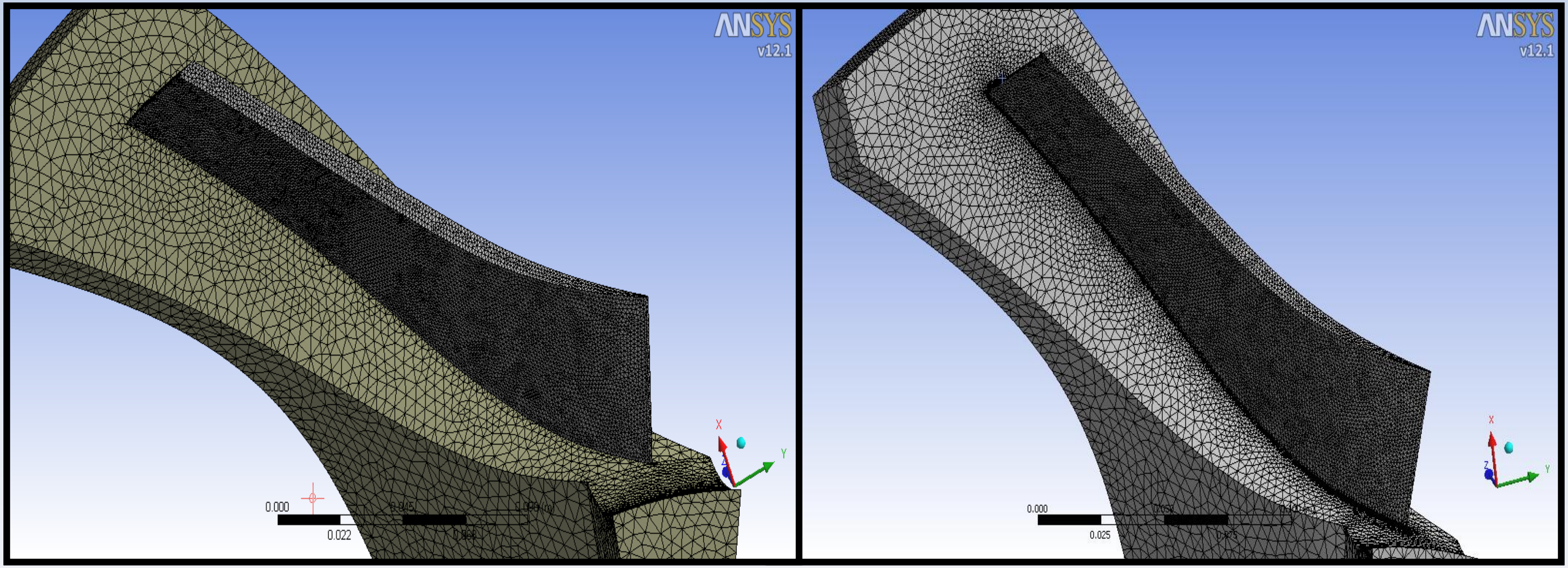
Mechanical Simulation

Process Integration

Robust Design Optimization



Structural Meshing

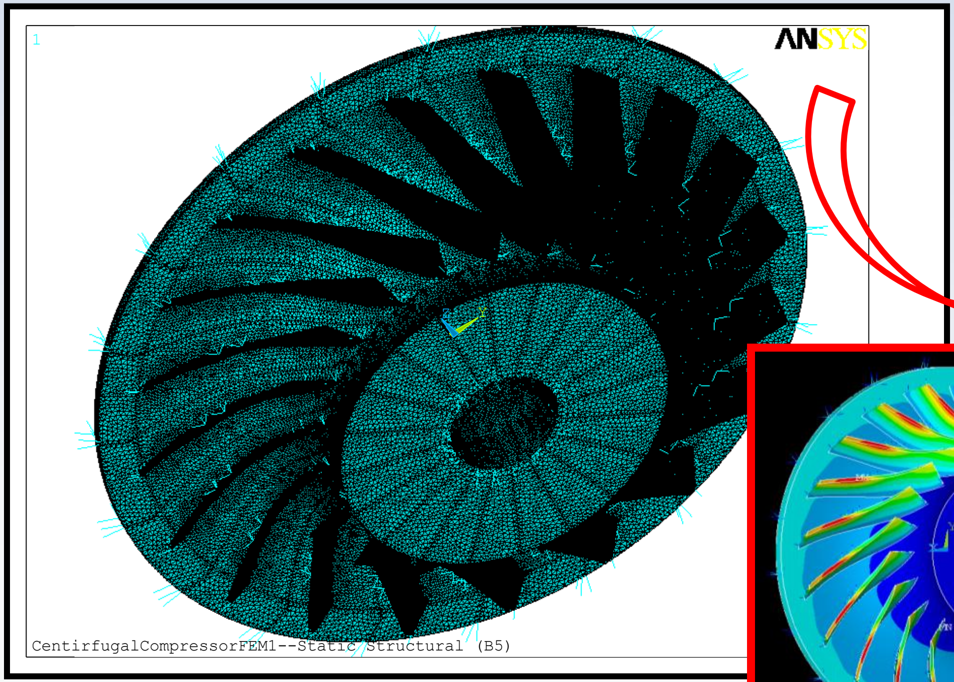


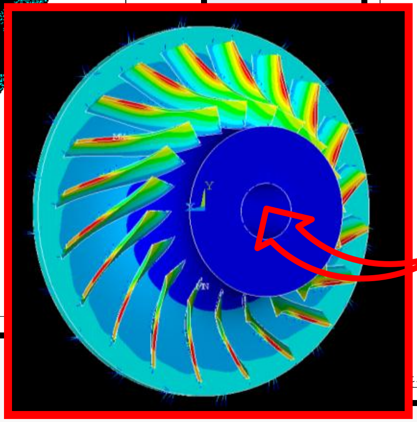
Mesh	#Nodes A	#Nodes B
1	10612	18911
2	54356	60103
3	236883	334242

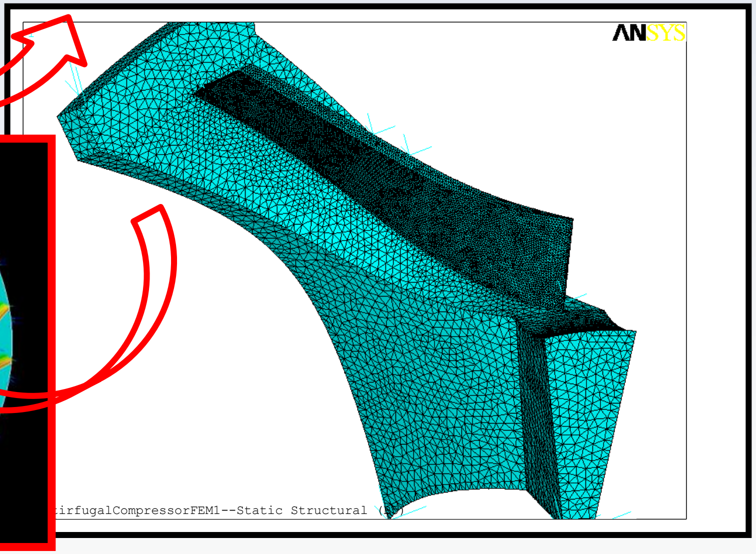
Cyclic Symmetry Structures

$$\begin{pmatrix} M_{11} & \dots & M_{1n} \\ \dots & \dots & \dots \\ M_{n1} & \dots & M_{nn} \end{pmatrix} \begin{pmatrix} \ddot{u}_1 \\ \dots \\ \ddot{u}_n \end{pmatrix} + \begin{pmatrix} K_{11} & \dots & K_{1n} \\ \dots & \dots & \dots \\ K_{n1} & \dots & K_{nn} \end{pmatrix} \begin{pmatrix} u_1 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} f_1(t) \\ \dots \\ f_n(t) \end{pmatrix}$$

$$M_n \ddot{U}_n + K_n U_n = F_n(t)$$



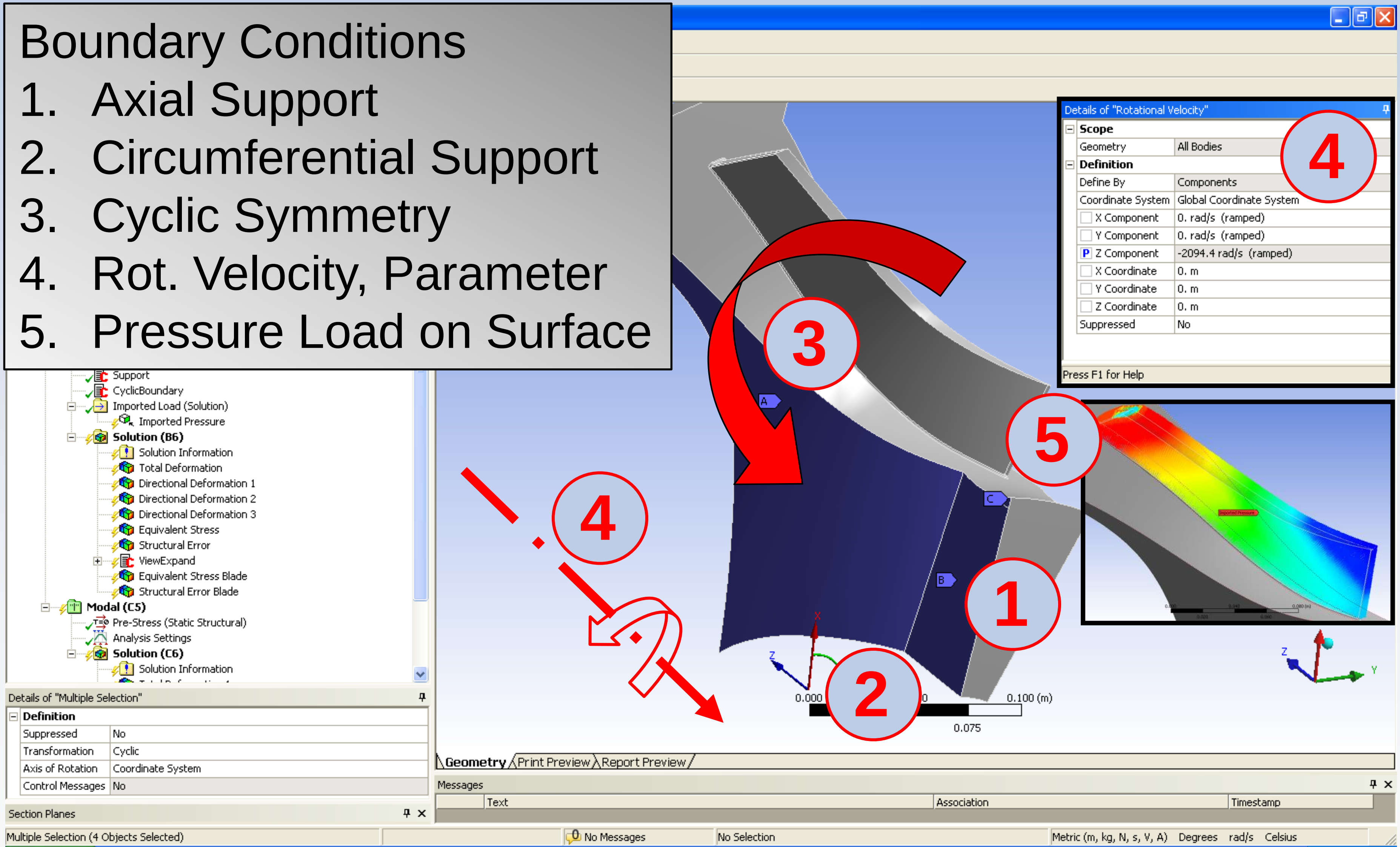




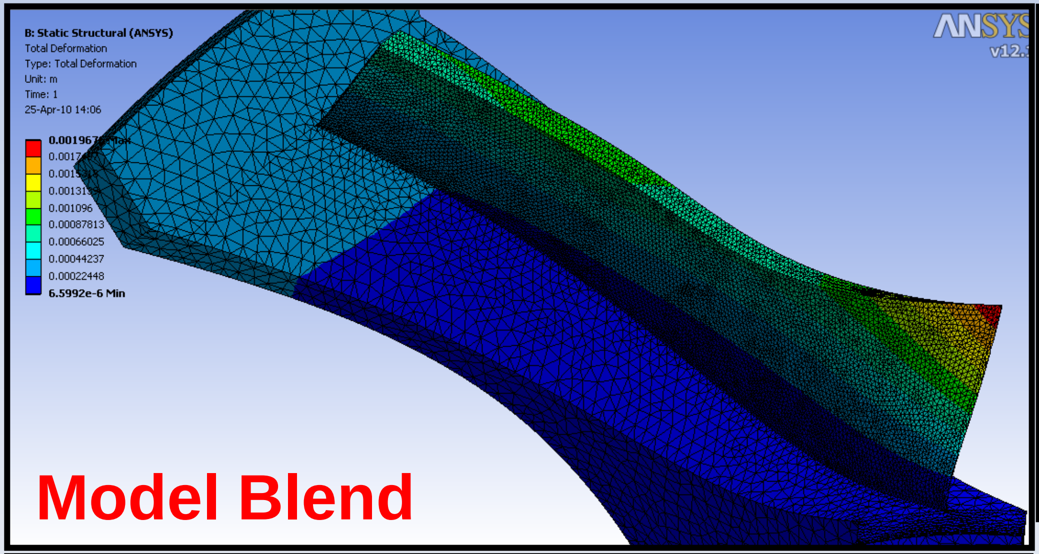
Boundary Conditions and Loads

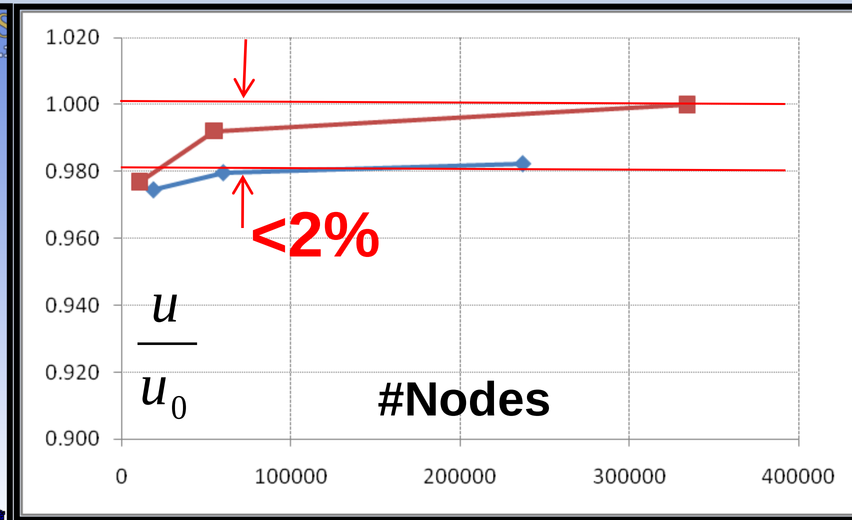
Boundary Conditions

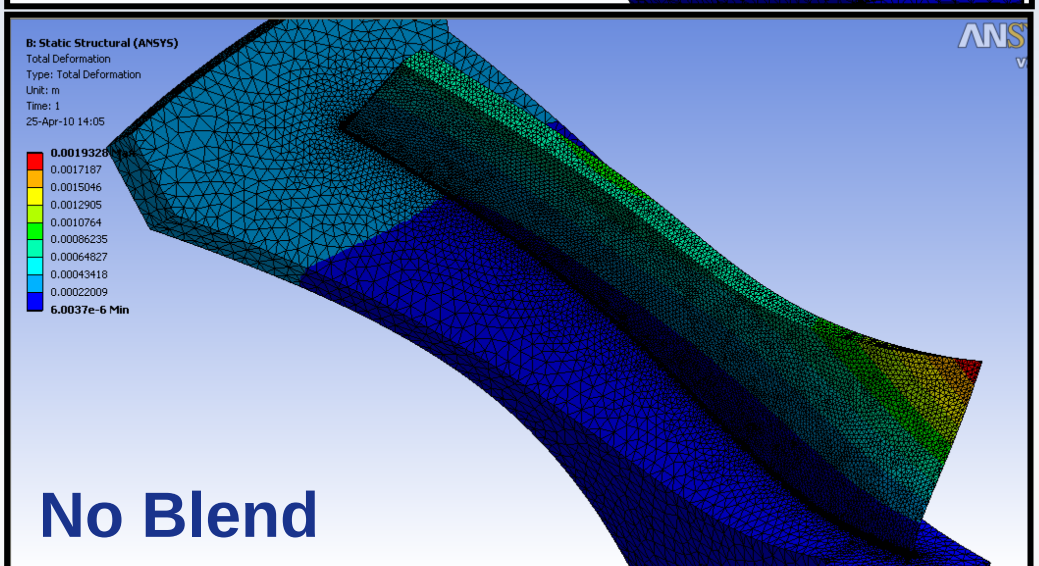
1. Axial Support
2. Circumferential Support
3. Cyclic Symmetry
4. Rot. Velocity, Parameter
5. Pressure Load on Surface



Mechanical, Displacement

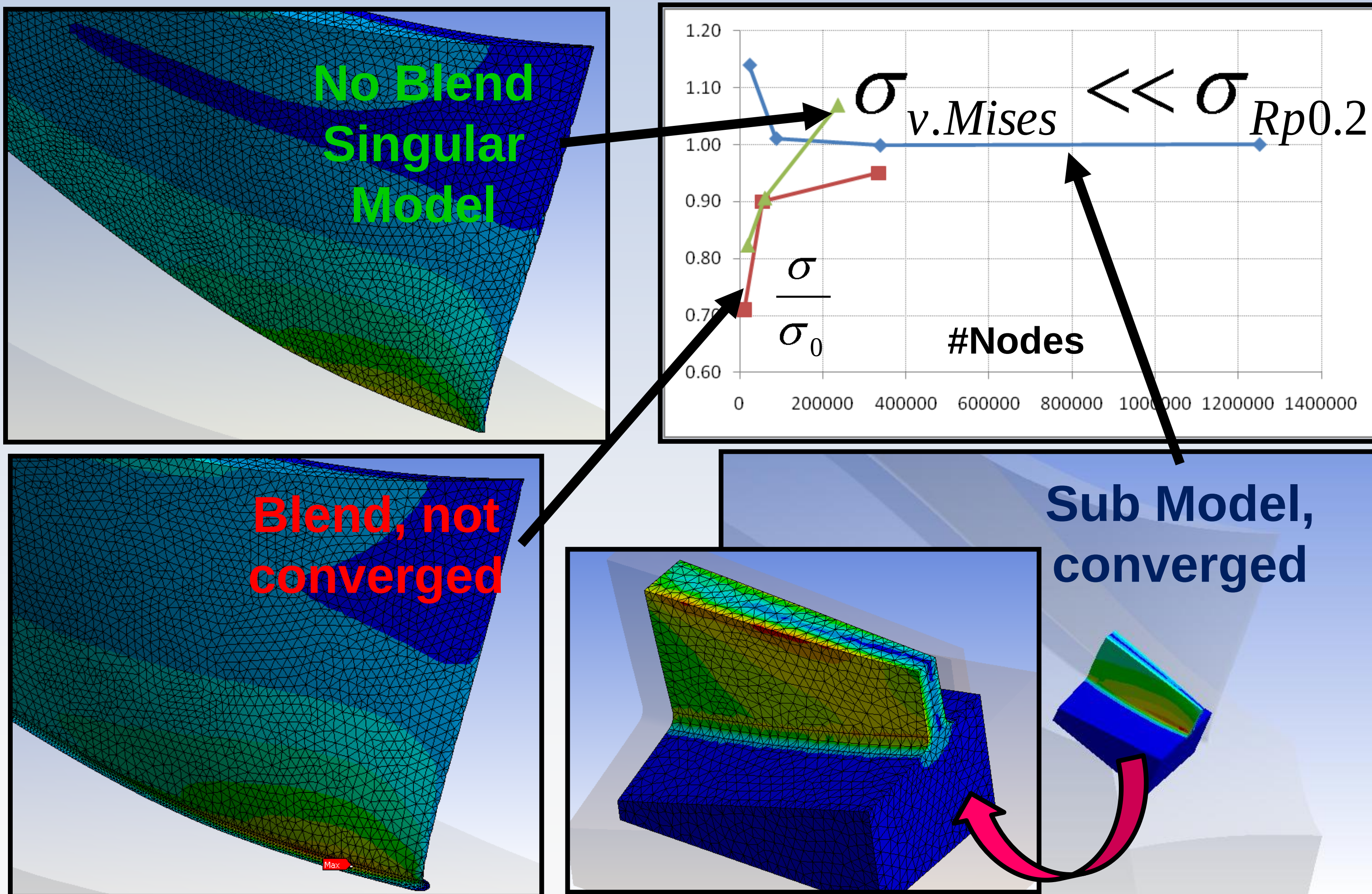




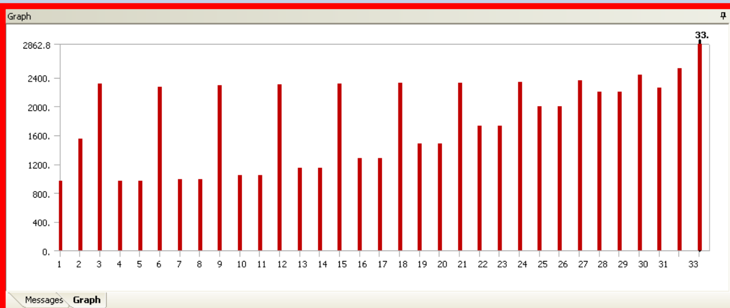


Blend has minor influence on displacement

Mechanical, Stress

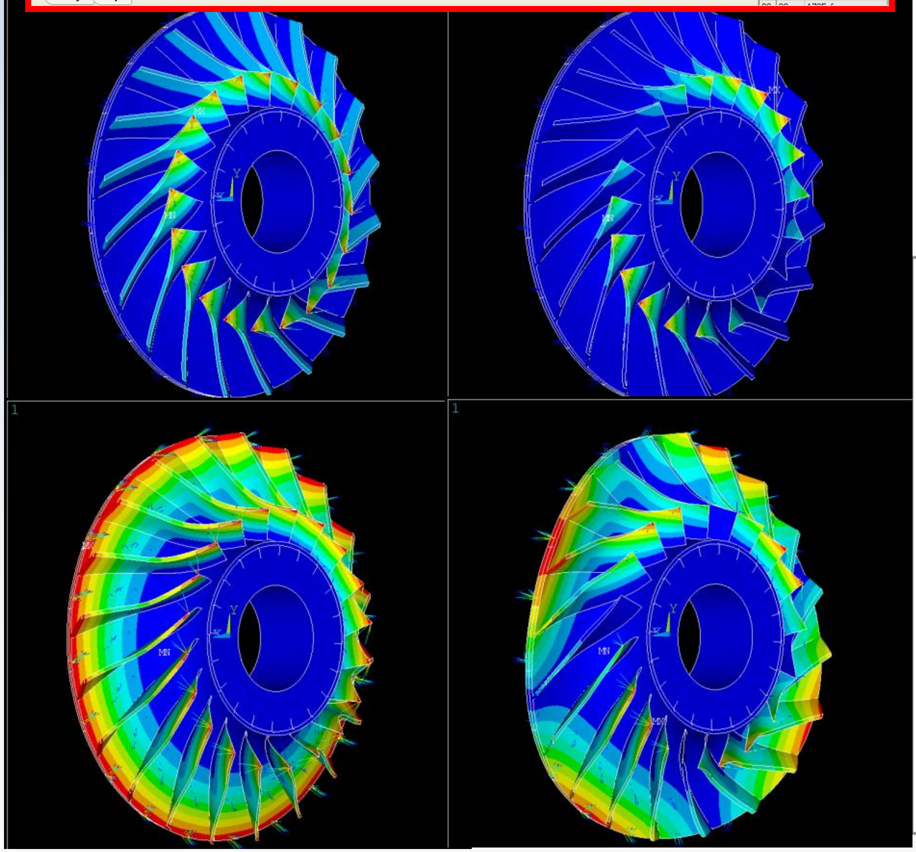


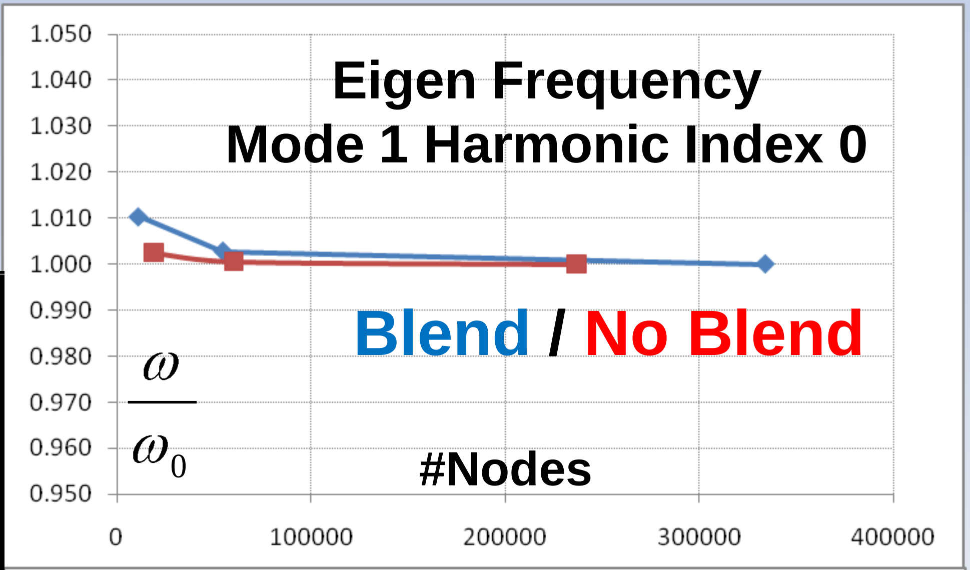
Mechanical, Modal Analysis

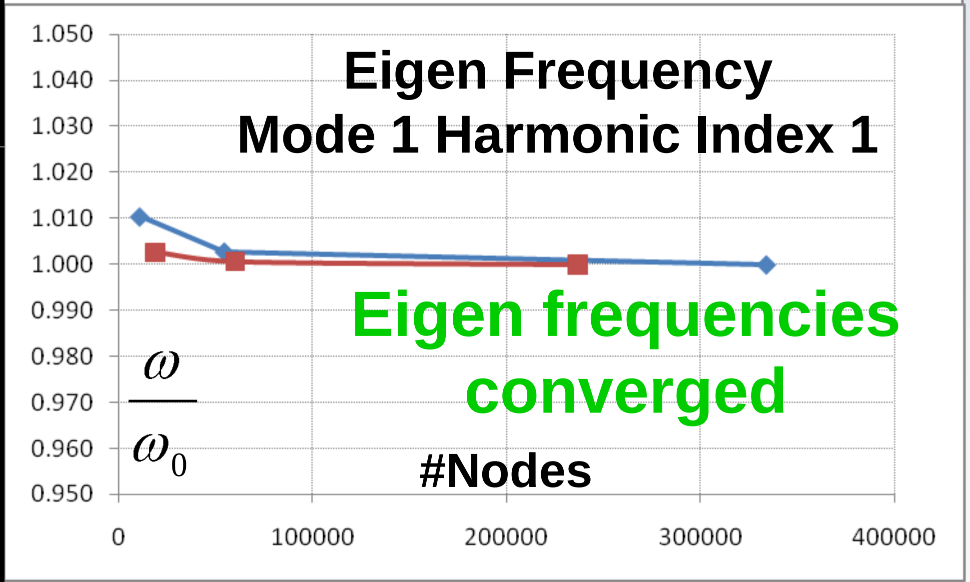


Tabular Data

	Mode	Frequency [Hz]
1	1.	963.57
2	2.	1554.6
3	3.	2317.7
4	4.	964.02
5	5.	964.02
6	6.	2268.7
7	7.	984.63
8	8.	984.63
9	9.	2289.2
10	10.	1041.6
11	11.	1041.6
12	12.	2301.9
13	13.	1139.8
14	14.	1139.8
15	15.	2311.6
16	16.	1285.4
17	17.	1285.4
18	18.	2319.3
19	19.	1482.1
20	20.	1482.1
21	21.	2326.4
22	22.	1725.6

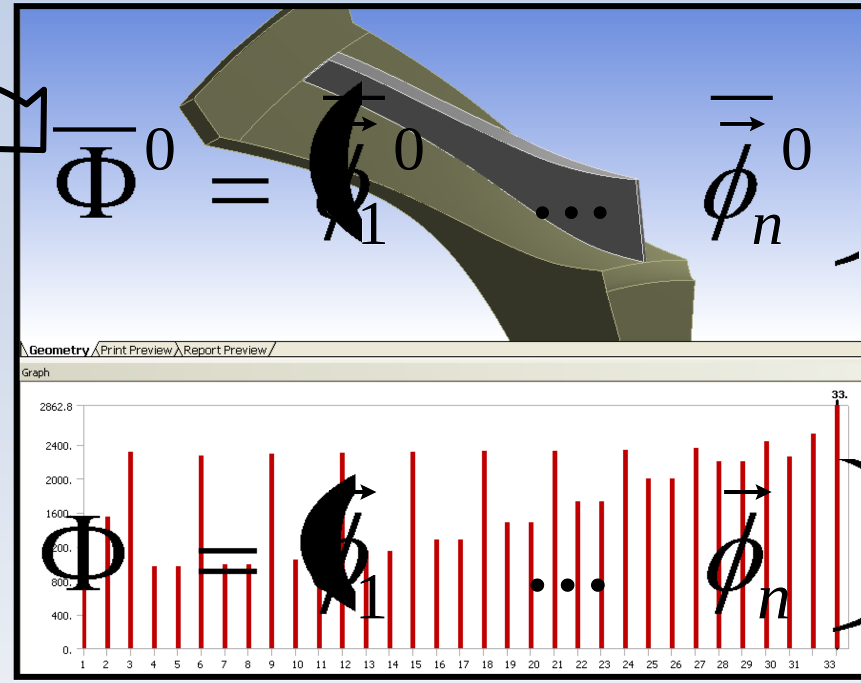






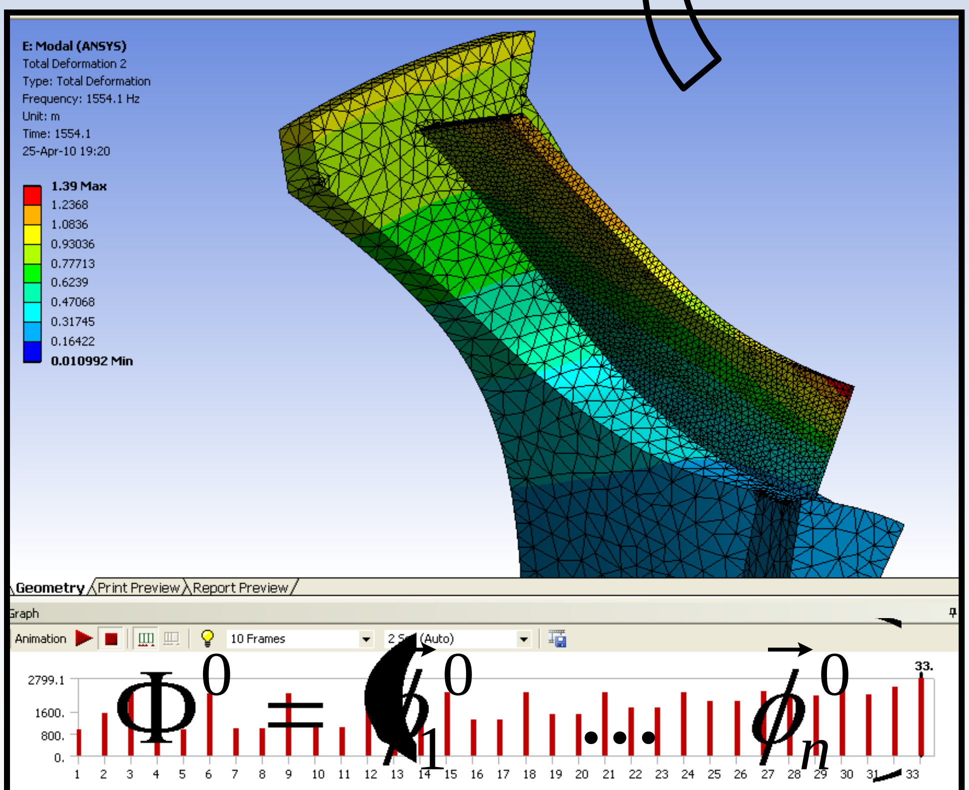
Mode Tracking

Interpolation: Reference Solution to current Design



$$\Phi^0 = \vec{\phi}_1^{\,0} \cdots \vec{\phi}_n^{\,0}$$

$$\Phi = \vec{\phi}_1 \cdots \vec{\phi}_n$$



$$\Phi^0 = \vec{\phi}_1^{\,0} \cdots \vec{\phi}_n^{\,0}$$

Tracking by Mode Projection

$$\Phi^T \cdot \overline{\Phi}^0 = \begin{pmatrix} \approx 1 & \approx 0 & \approx 0 \\ \approx 0 & \approx 0 & \approx 1 \\ \approx 0 & \approx 1 & \approx 0 \end{pmatrix}$$

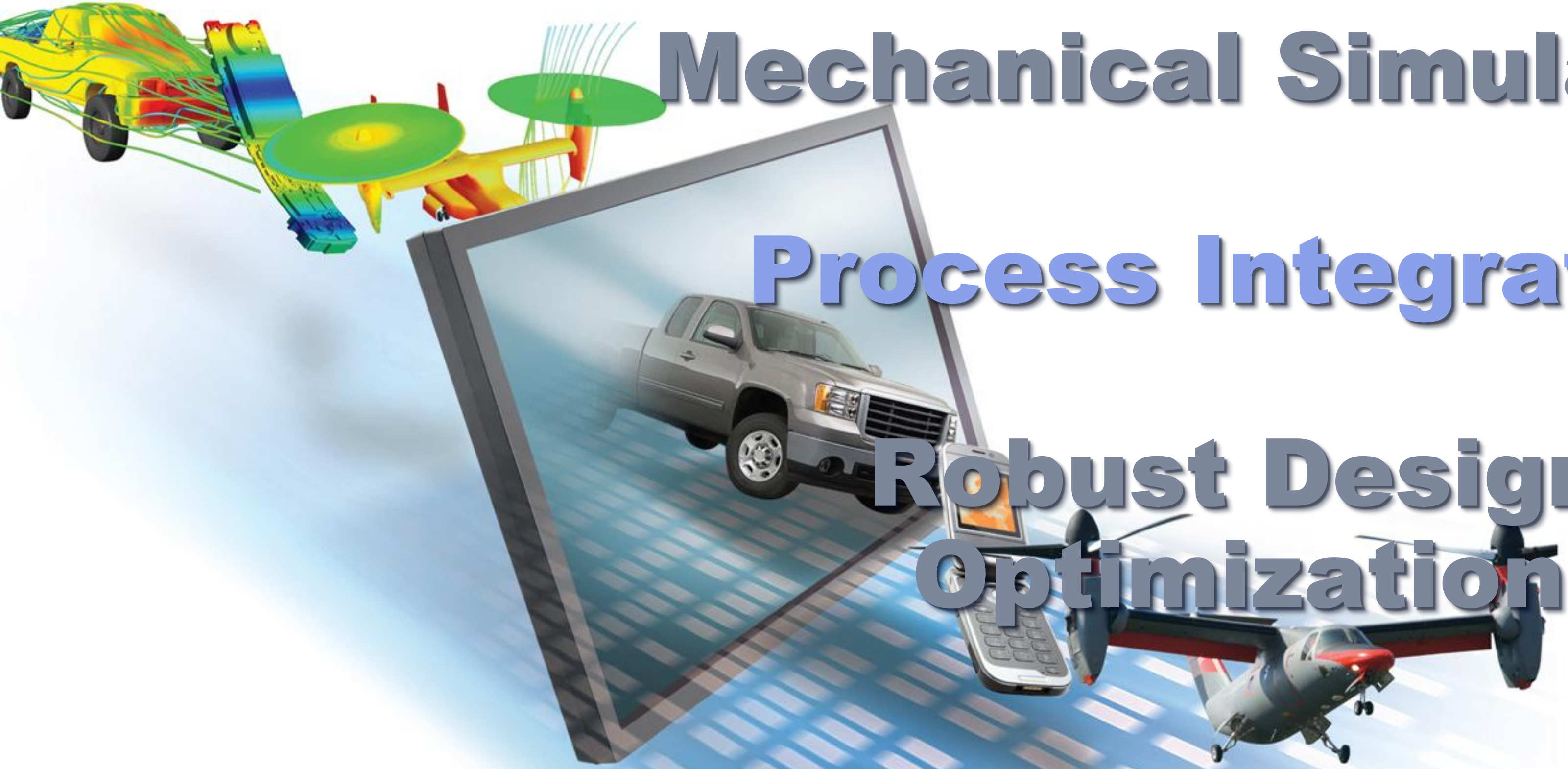
Geometry

CFD Simulation

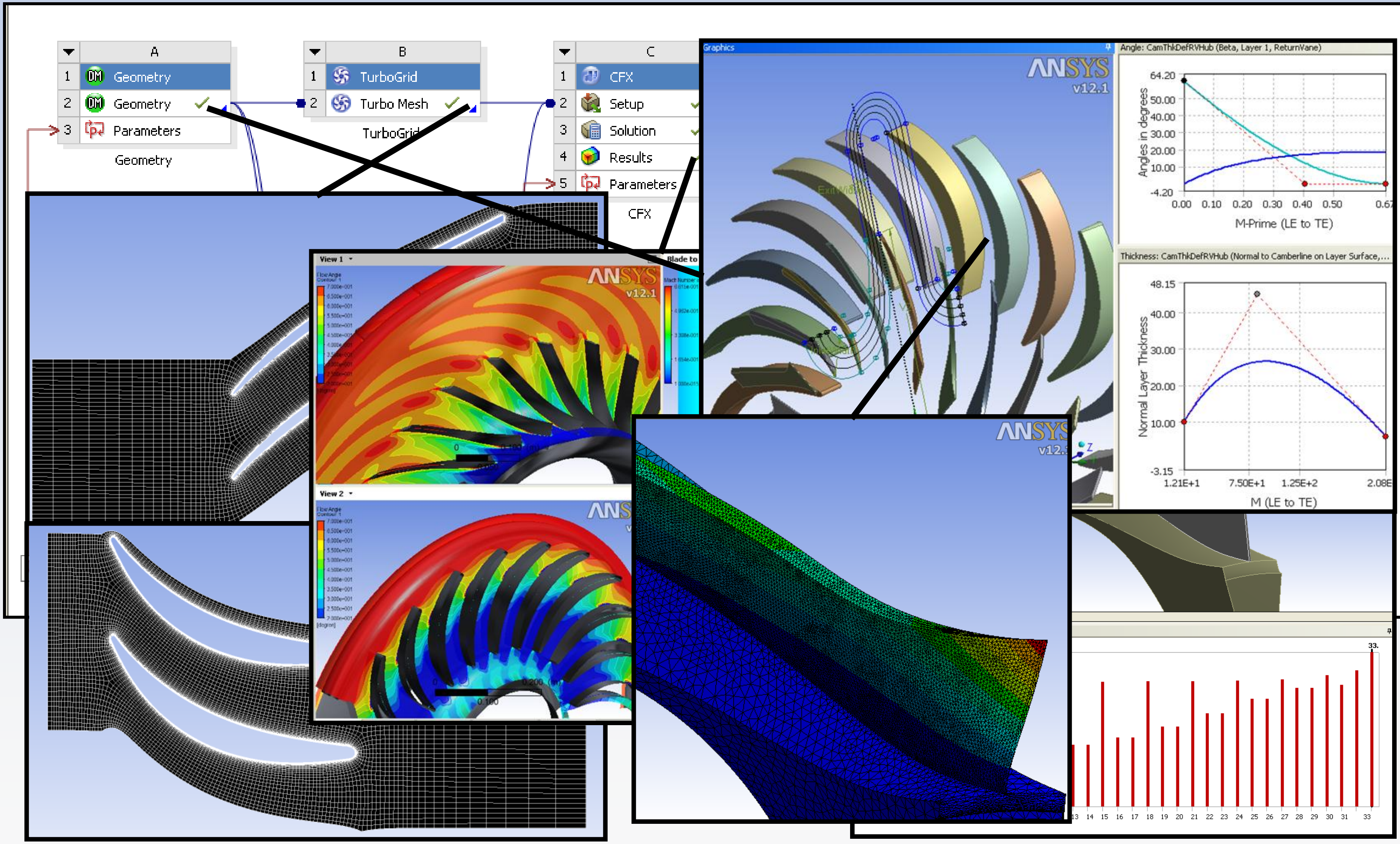
Mechanical Simulation

Process Integration

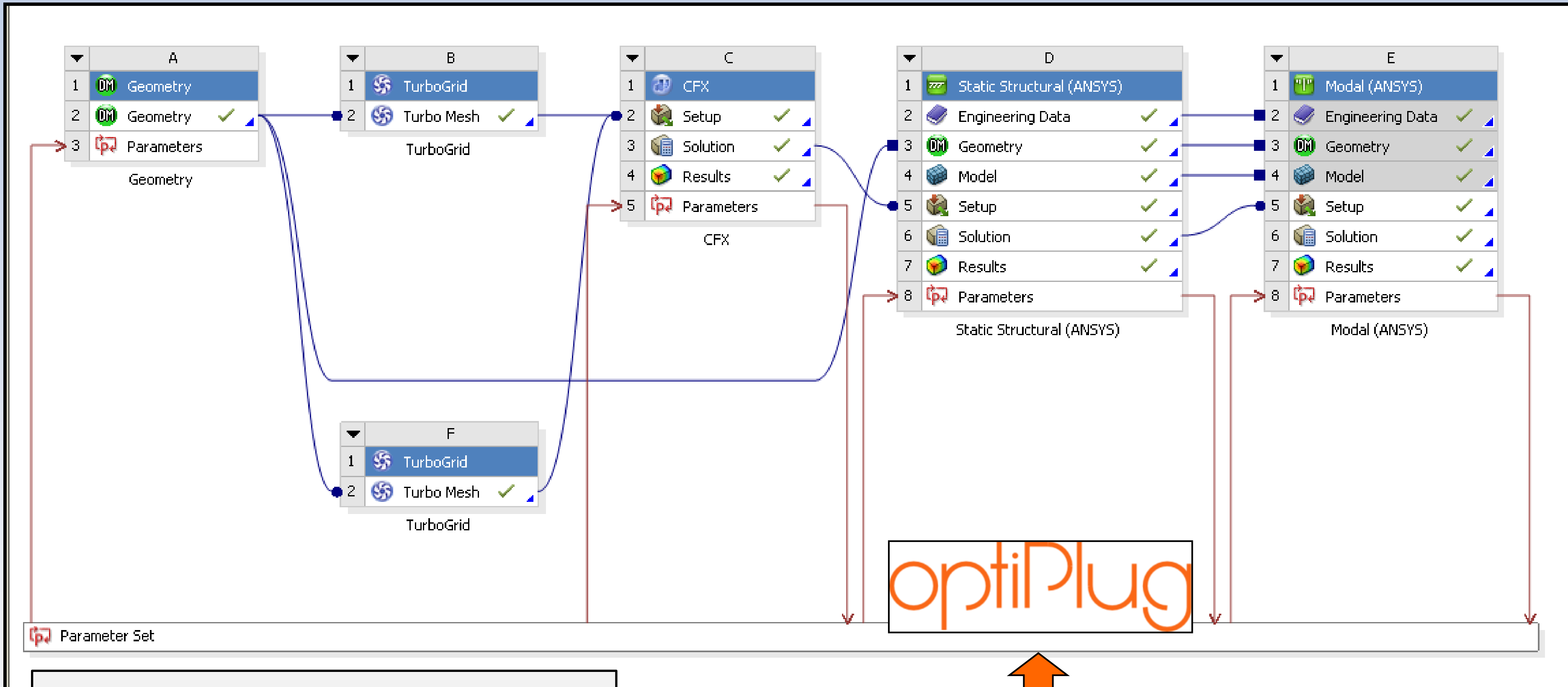
Robust Design Optimization



Parametric Process



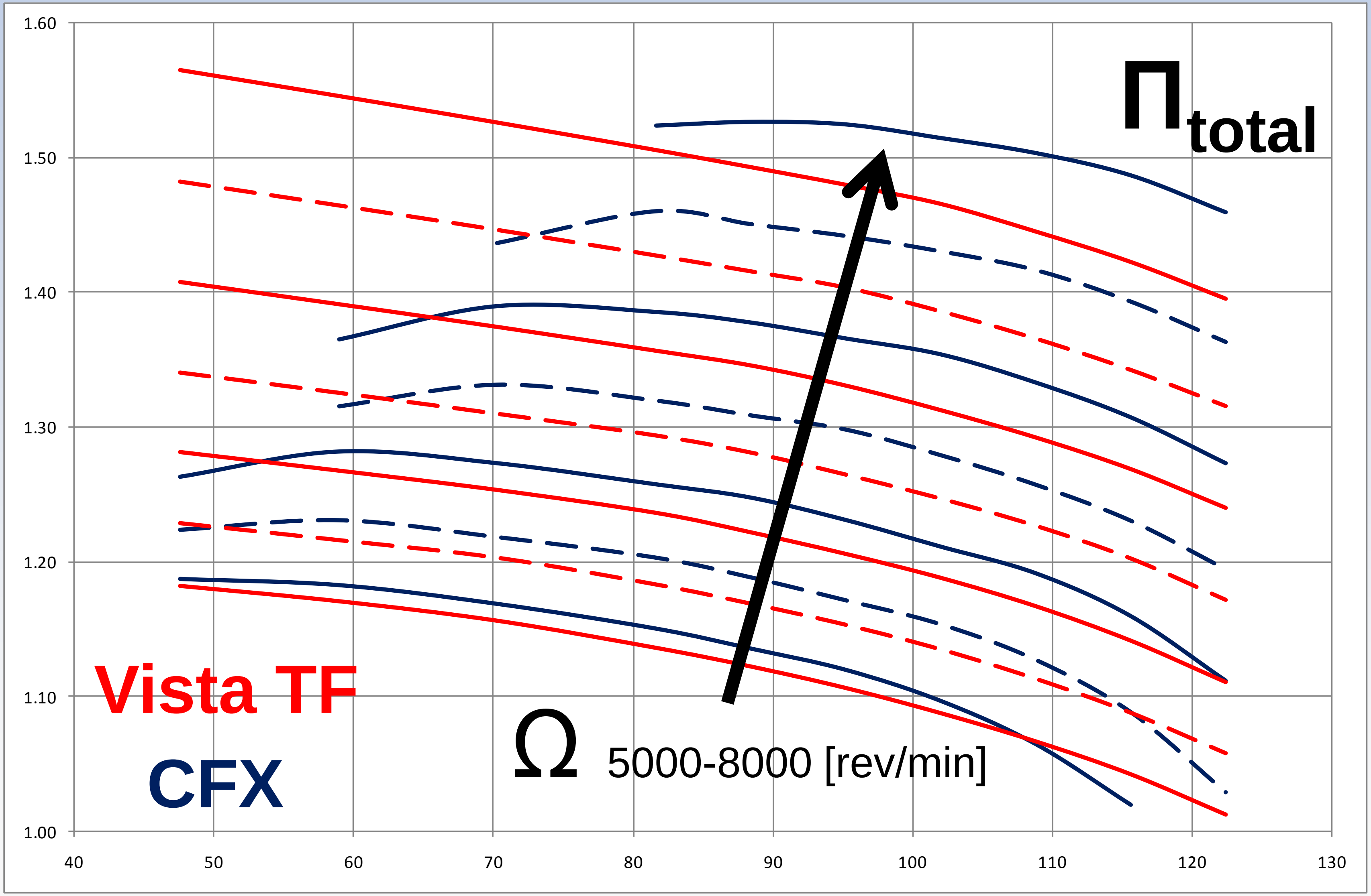
Parametric Process



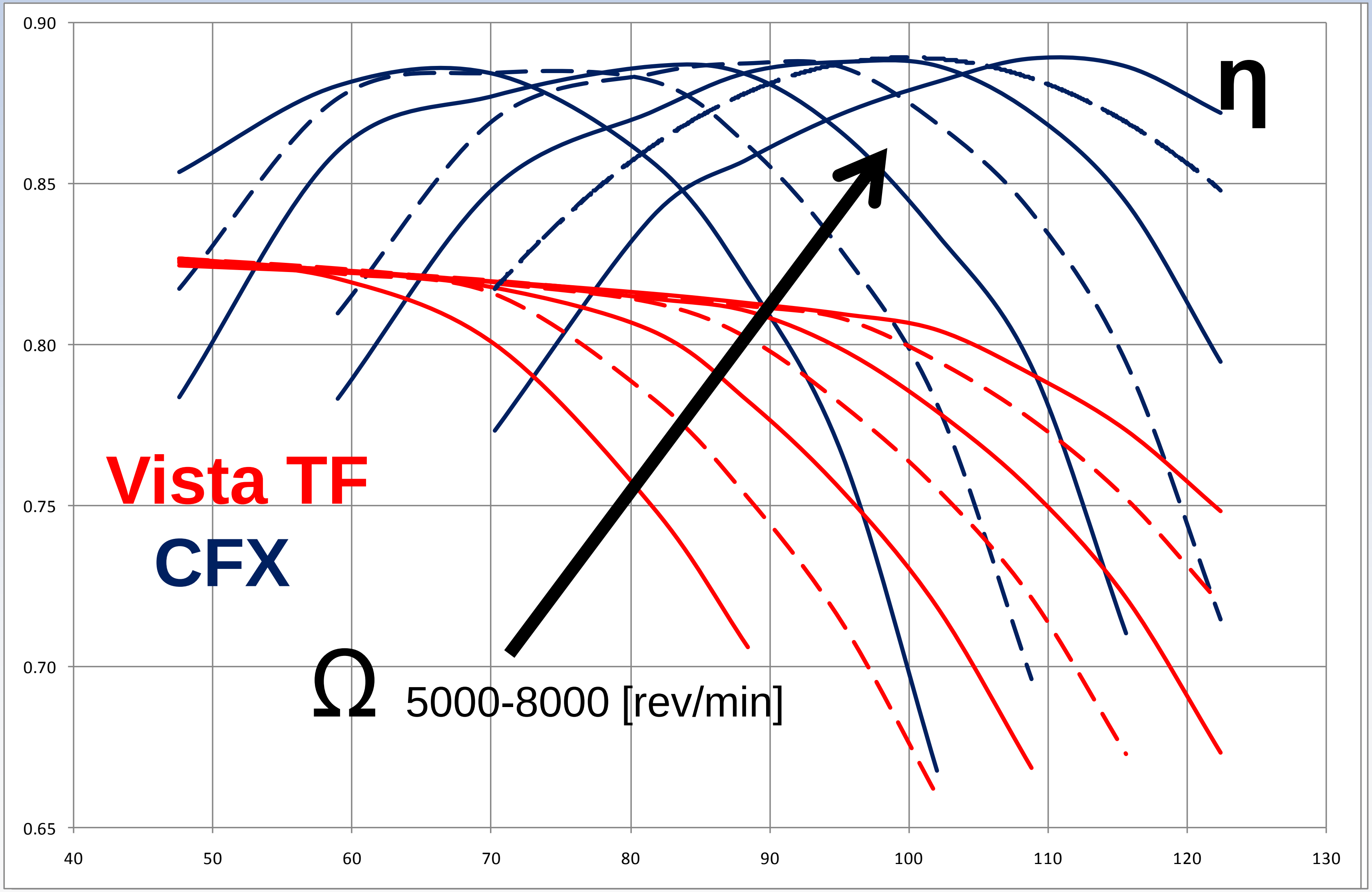
Parametric Workflow for Multi Physic Application

- Extended solver support
- Embedded analysis tool
- Without user interaction

Vista TF vs. CFX, Characteristic



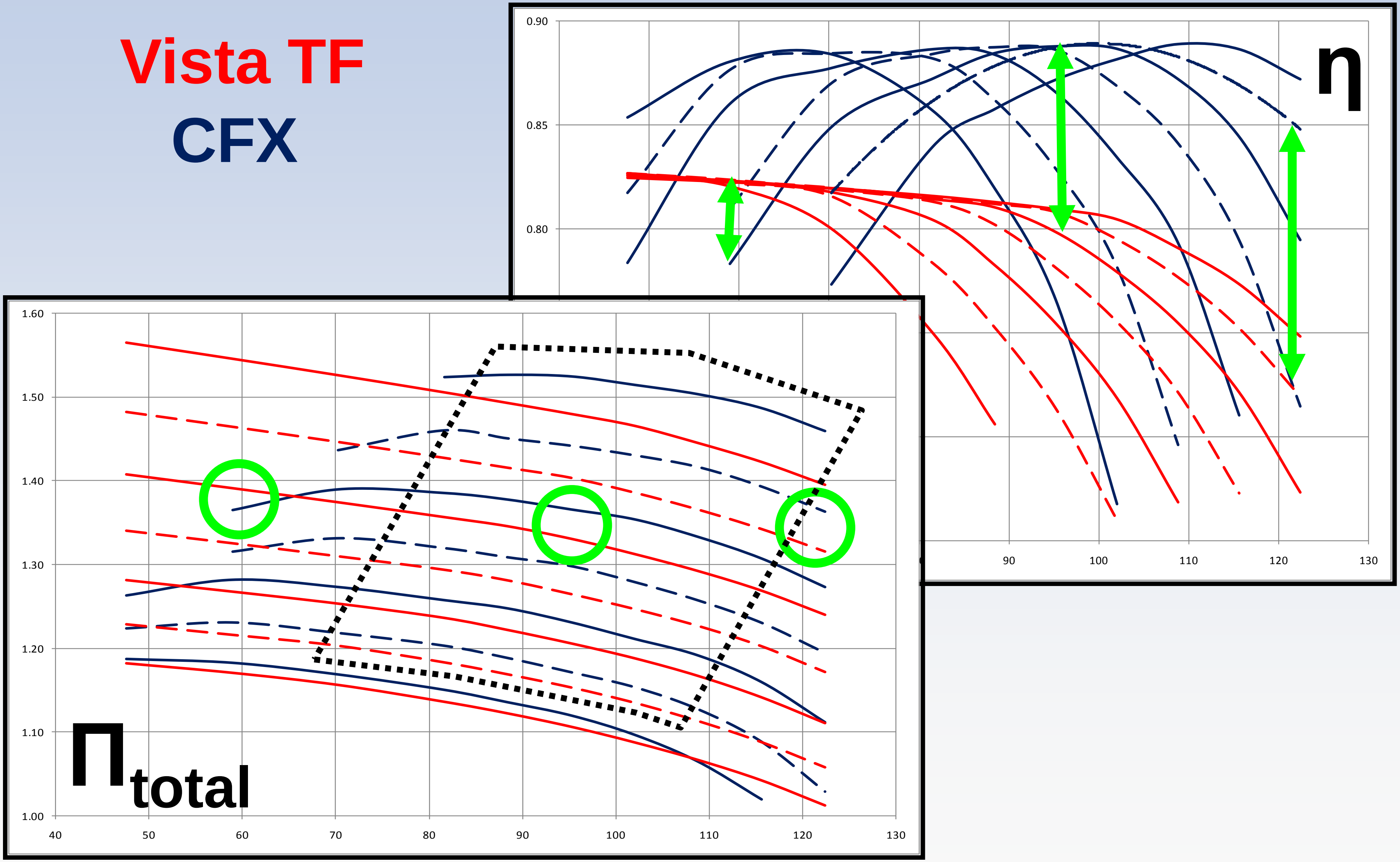
Vista TF vs. CFX, Characteristic



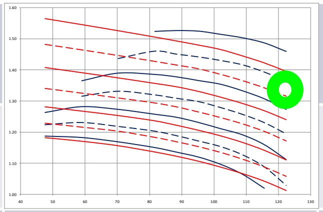
Vista TF vs. CFX, Characteristic

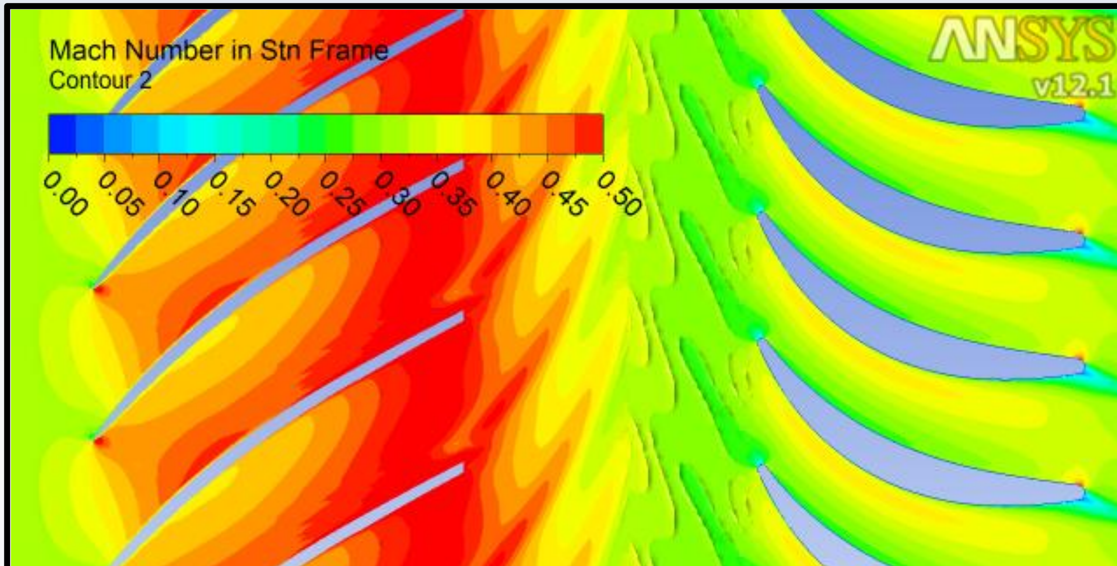
Vista TF

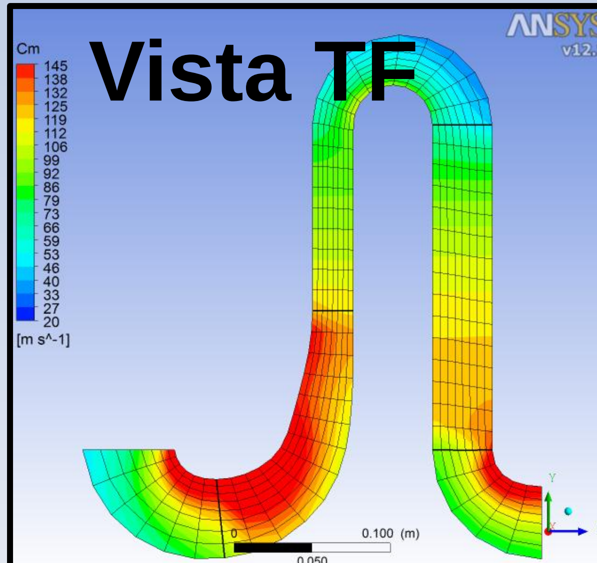
CFX

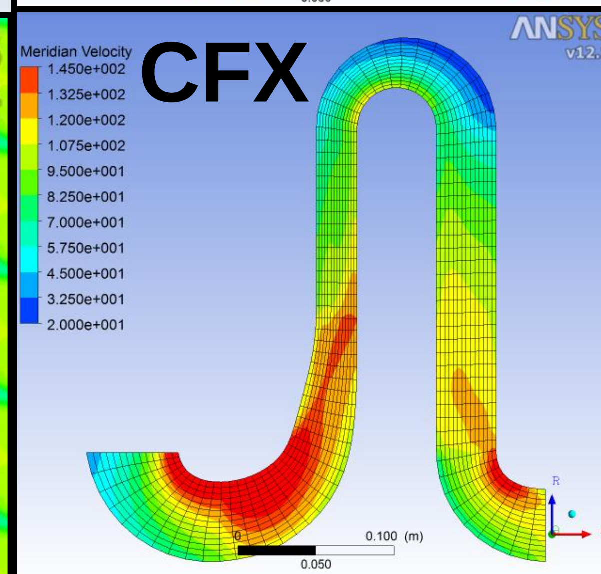


Vista TF vs. CFX, Choke

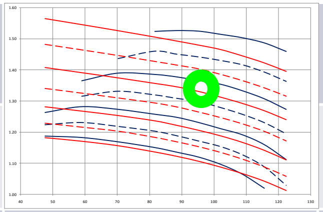
	Vista TF	CFX
m [kg/s]		122.4
Ω [rev/min]		7500
Π_{total} [-]	1.3245	1.3633
η [%]	73.82	84.48

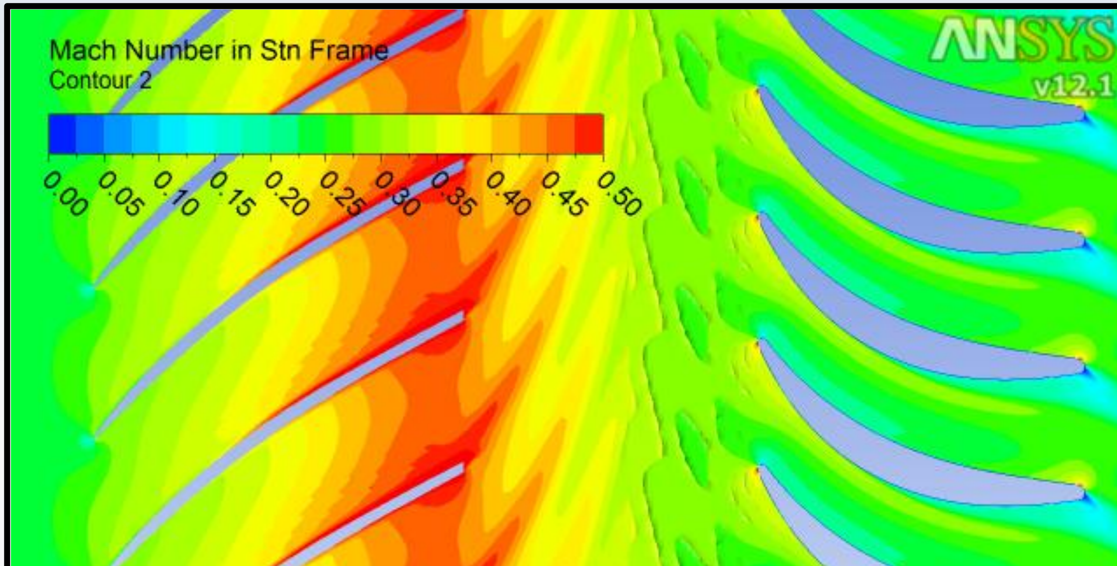


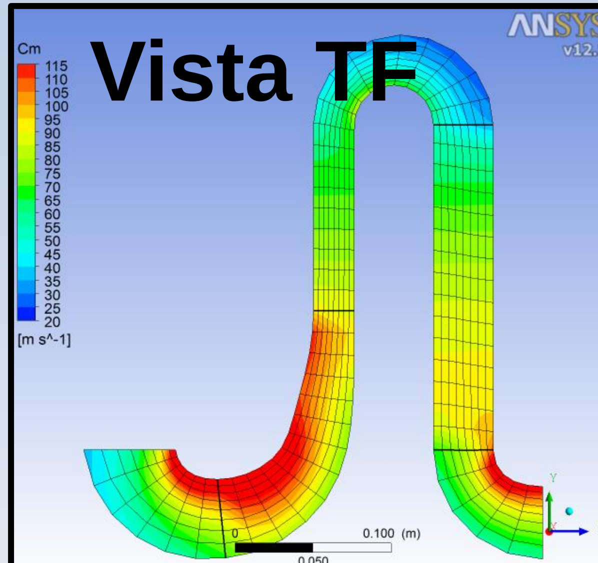


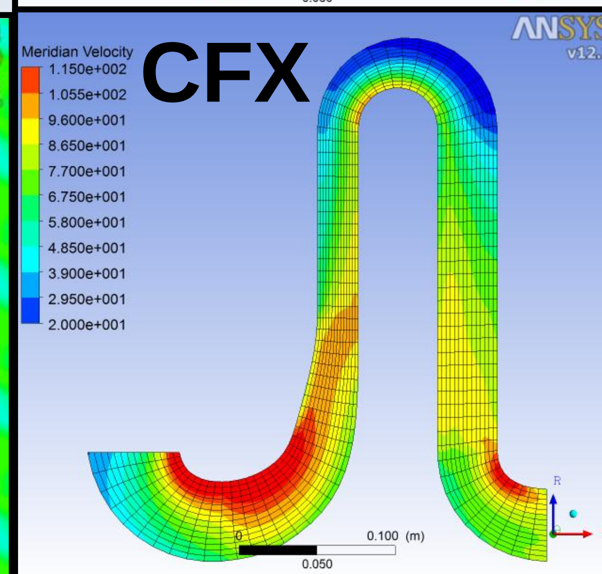


Vista TF vs. CFX, Optimum

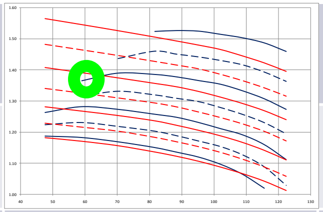
	Vista TF	CFX
m [kg/s]		95.2
Ω [rev/min]		7000
Π_{total} [-]	1.3309	1.3661
η [%]	79.87	88.80

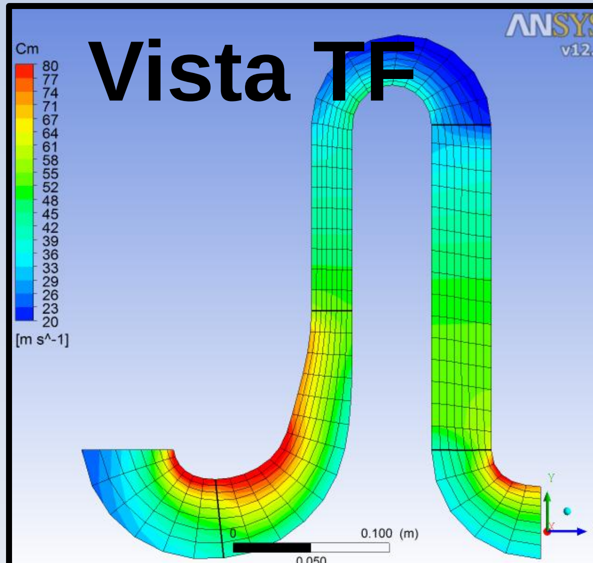


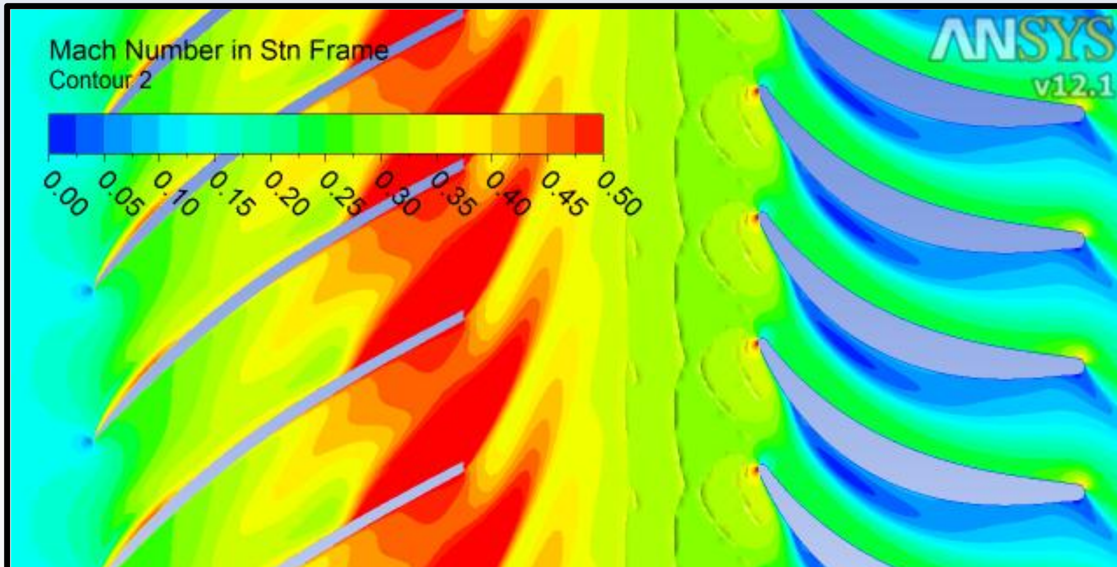


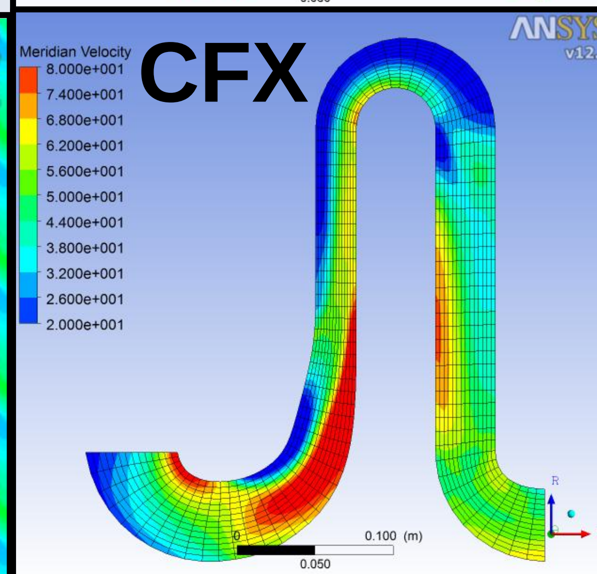


Vista TF vs. CFX, Stall

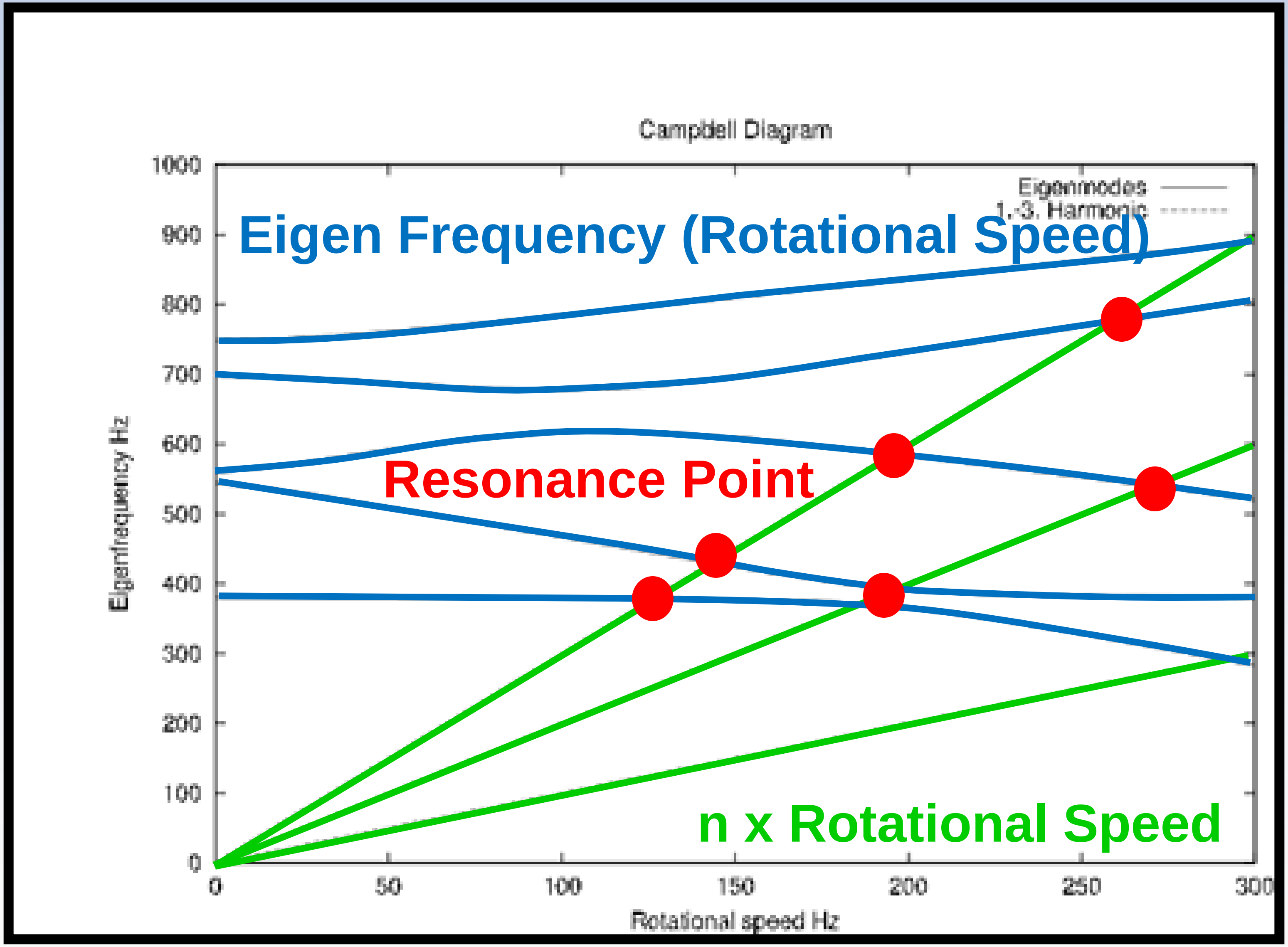
	Vista TF	CFX
m [kg/s]		58.93
Ω [rev/min]		7000
Π_{total} [-]	1.3911	1.3654
η [%]	82.34	78.30



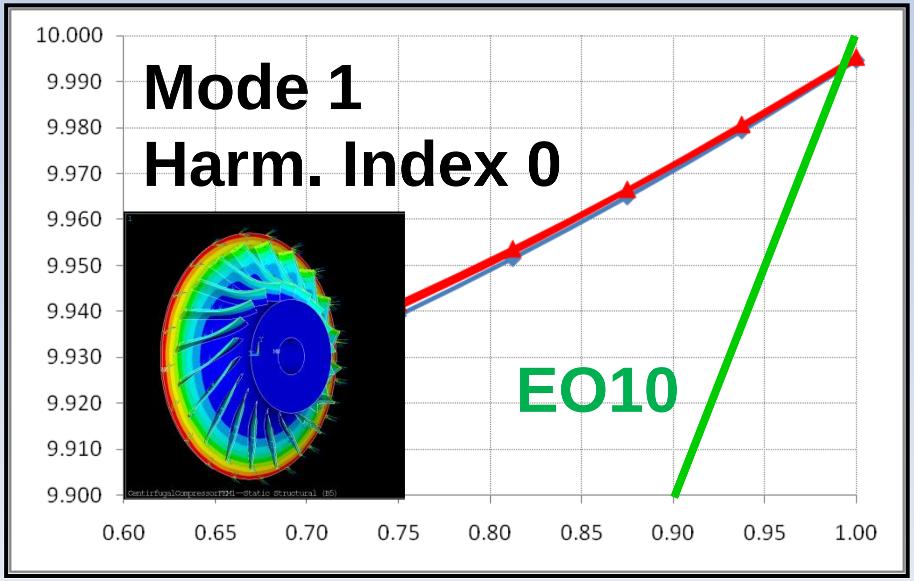


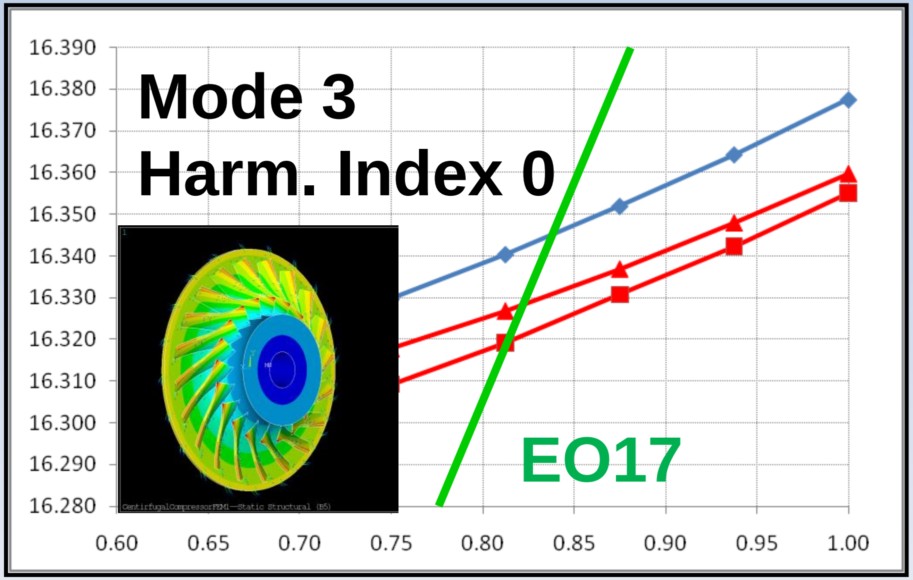


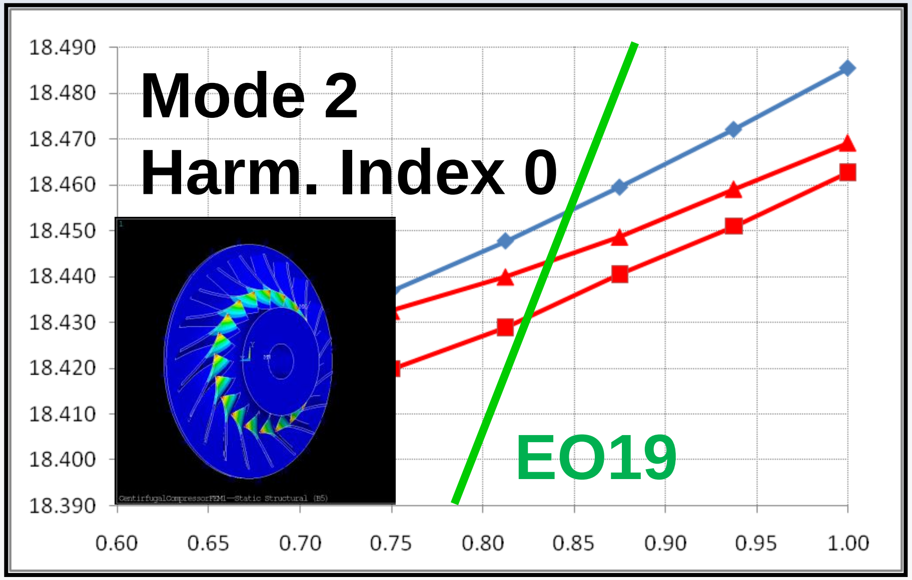
Campbell Diagram

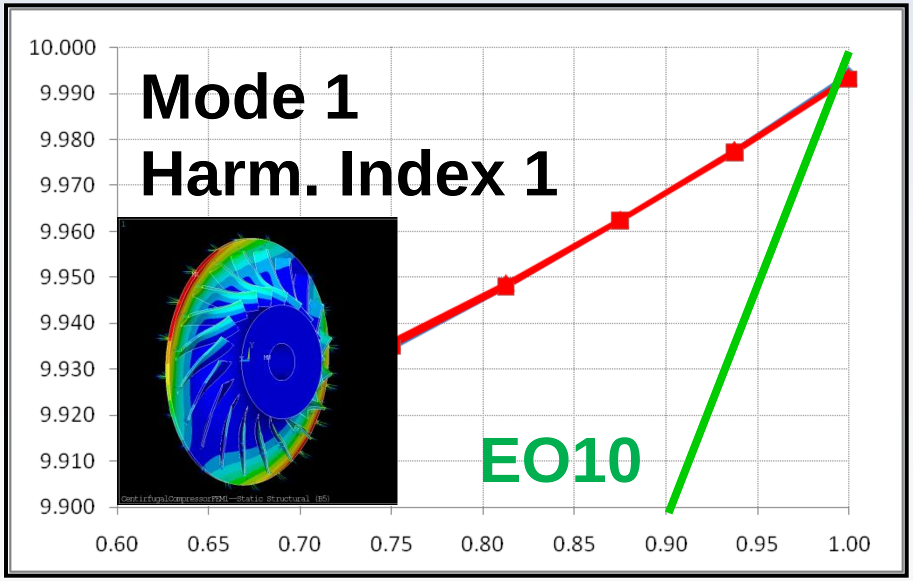


Campbell Diagram

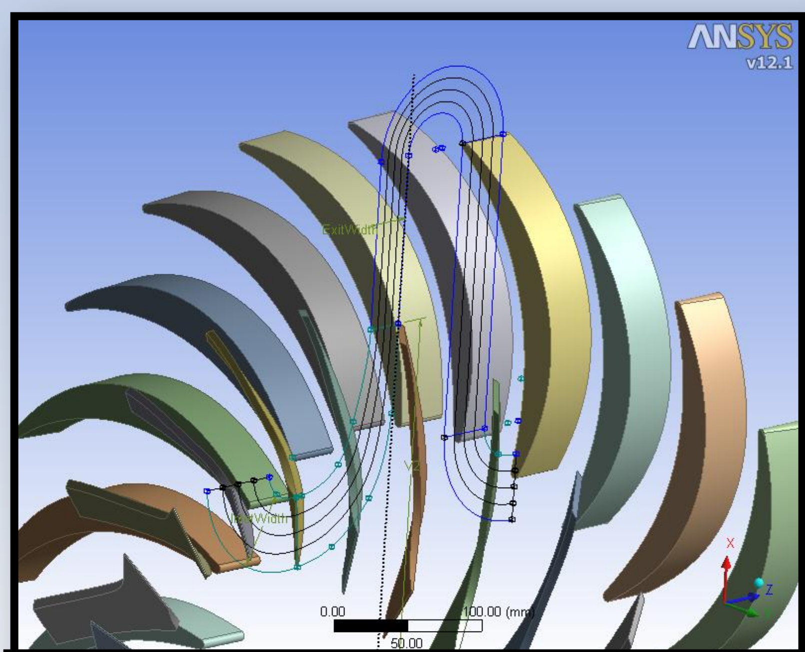


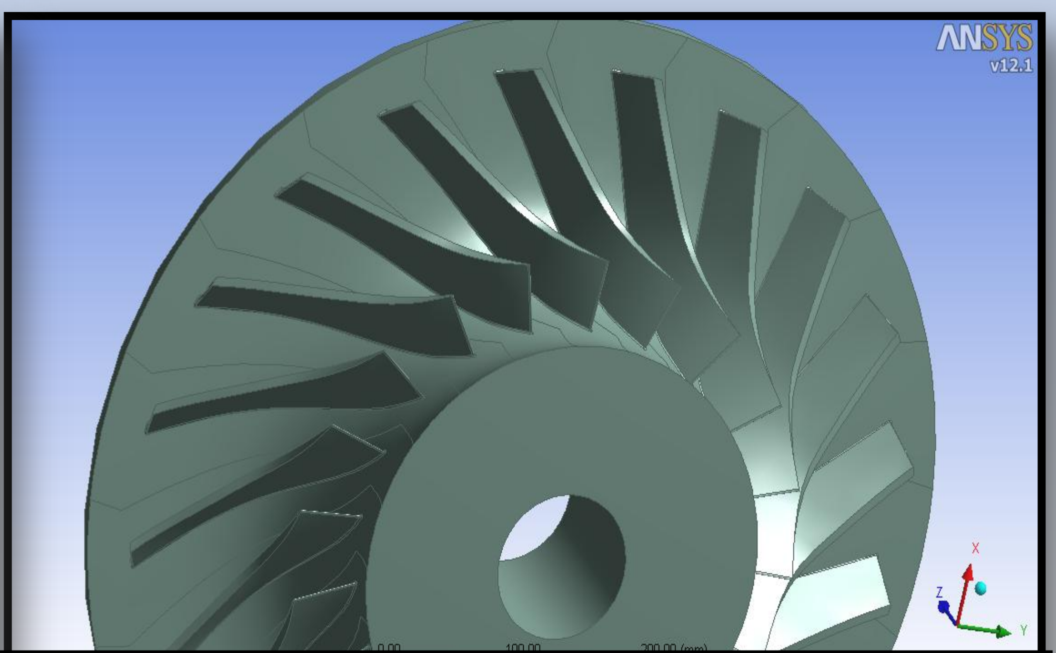






Optimization Objective





Defined Operating Point:

Mass Flow Rate 72.6 kg/s

Rotational Velocity Ω=6644 rev/min

Total Pressure Ratio π=1.35±0.01, Objective

Maximal Efficiency η=max, Objective

No Resonance: $\Omega \neq \omega_i$

Parameter Manager

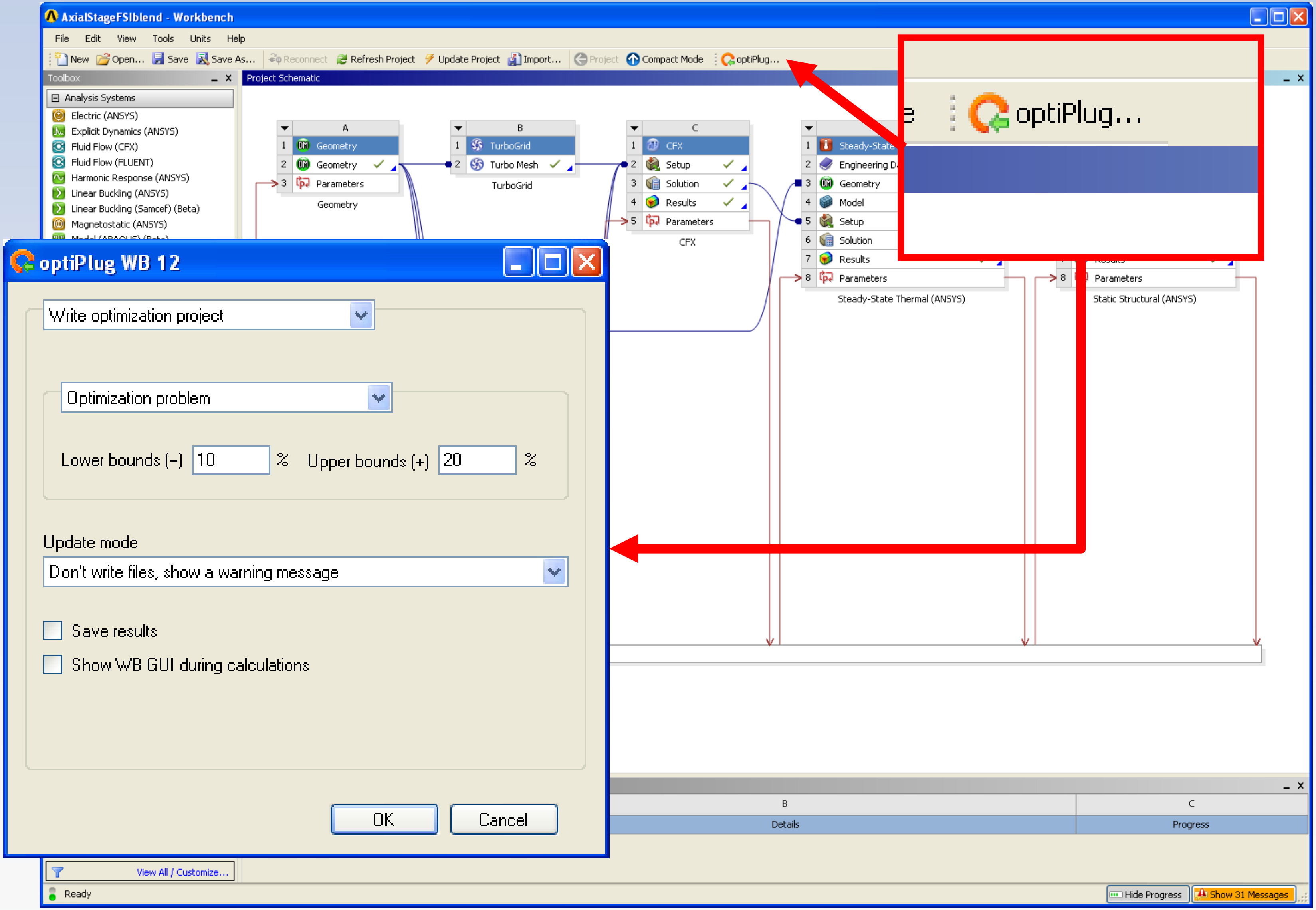
Output Parameters			
P33	nBladesR1	21	
P34	nBladesS1	23	
P36	Ttratio	1.1034	
P37	myMassFlow	72.6	kg s^-1
P38	myeta	0.86738	
P39	ptratio	1.3503	
P40	myPower	-2.2634E+06	W
P41	myTorque	-3234.5	J
P42	myDeltaS	0.012506	
P43	myalphaRS	62.32	degree
P44	myalphaR1	1.4839	degree
P45	myalphaS1	19.598	degree

16	P21	RVHubThk1	45.5	
17	P22	RVHubBeta1	60.5	
18	P23	RVShdBeta1	60.5	
19	P24	RVShdThk1	45.5	
20	P25	ImpellerBlades	21	
21	P26	RVBlades	23	
22	P27	Ttin	313	K
23	P28	myAirCP	1004.4	J kg^-1 K^-1
24	P29	myAirR	287.1	J kg^-1 K^-1
25	P30	mymassin	72.6	kg s^-1
26	P31	myomega	699.76	radian s^-1
27	P35	ptin	1.724E+06	Pa

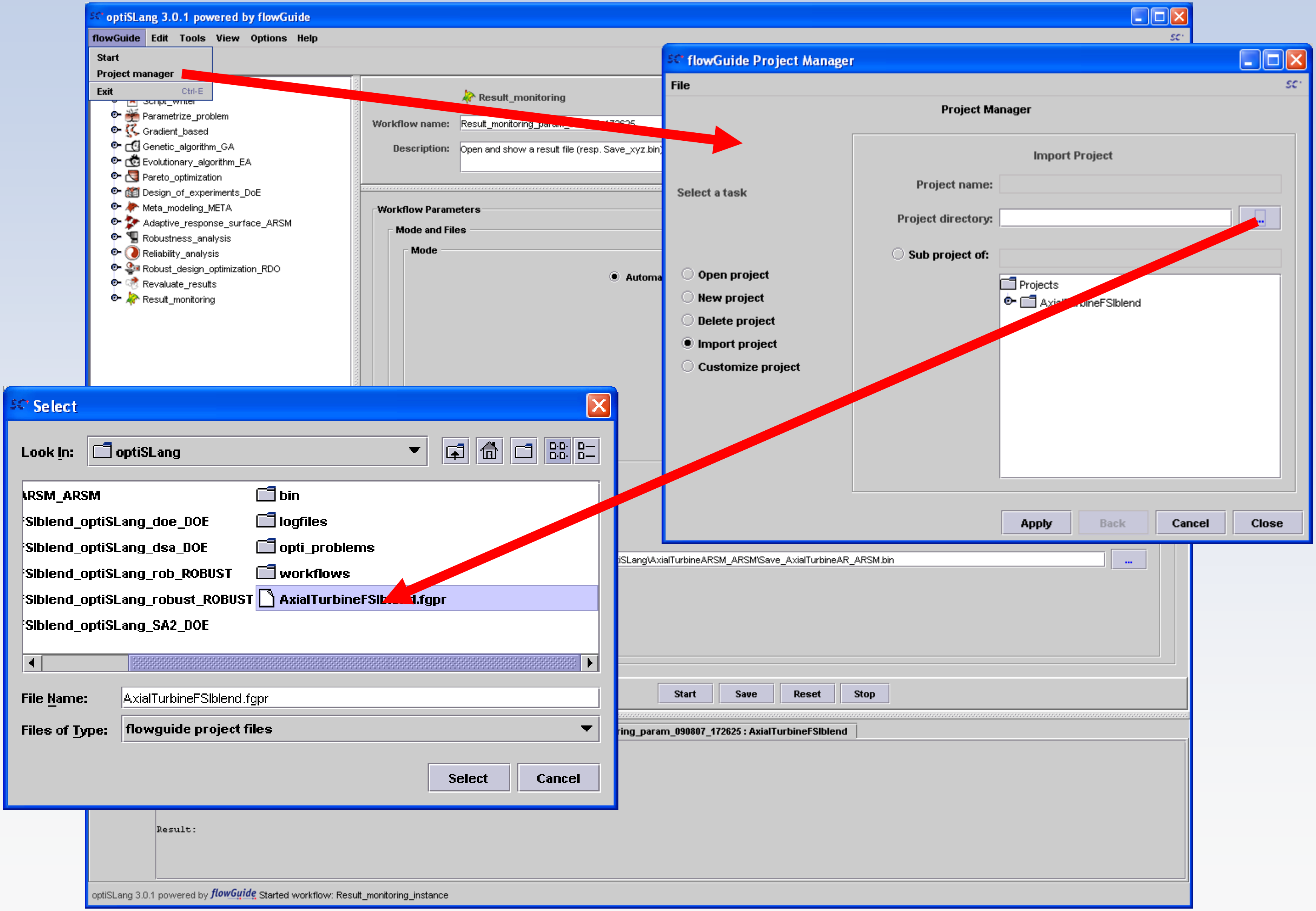
Input Parameter = 26
Output Parameter = 43
Constraints = 68

Input Parameters			
P1	InletWidth	53.1	
P8	ExitWidth	26.2	
P9	RImpeller	305.3	
P10	HubBeta1	-48.4	
P11	HubBeta2	-25.5	
P12	HubBeta3	-25.6	
P13	ShdBeta1	-55.7	
P14	ShdBeta2	-45.7	
P15	ShdBeta3	-30.7	
P16	HubThk1	1.1	
P17	HubThk2	6.2	
P18	ShdThk1	1.1	
P19	ShdThk2	6.1	
P21	RVHubThk1	45.5	
P22	RVHubBeta1	60.5	
P23	RVShdBeta1	60.5	
P24	RVShdThk1	45.5	
P25	ImpellerBlades	21	
P26	RVBlades	23	
P27	Ttin	313	K
P28	myAirCP	1004.4	J kg^-1 K^-1
P29	myAirR	287.1	J kg^-1 K^-1
P30	mymassin	72.6	kg s^-1
P31	myomega	699.76	radian s^-1
P35	ptin	1.724E+06	Pa

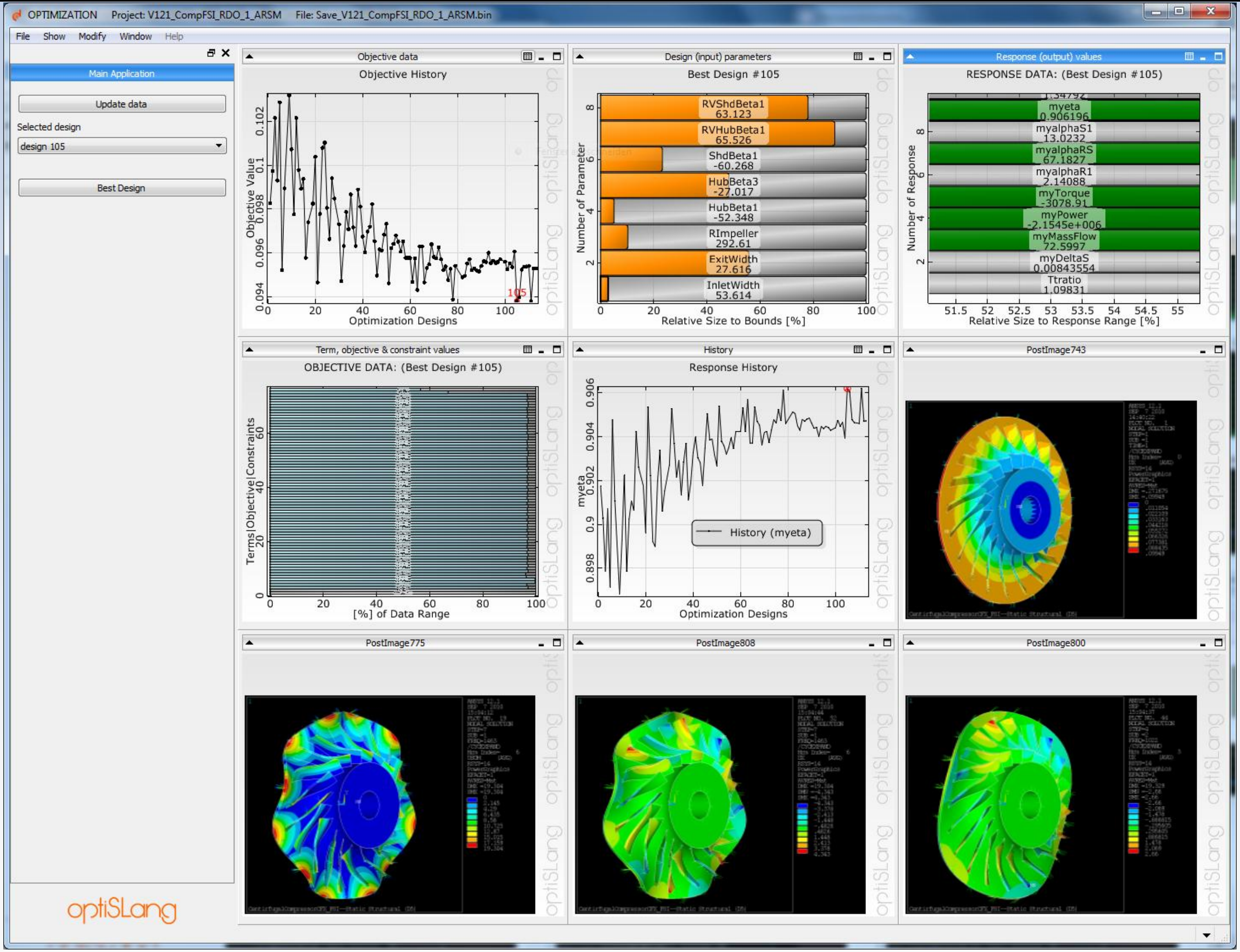
Workbench Interface to optiSLang



Workbench Interface to optiSLang

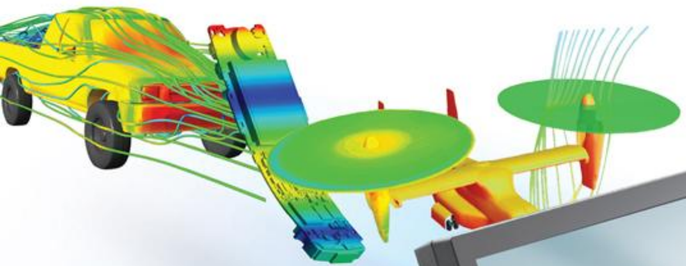


Workbench Interface to optiSLang



Process Integration

Sensitivity Analysis



Design Optimization

Robustness Evaluation

Random Fields

Robust Design Optimization

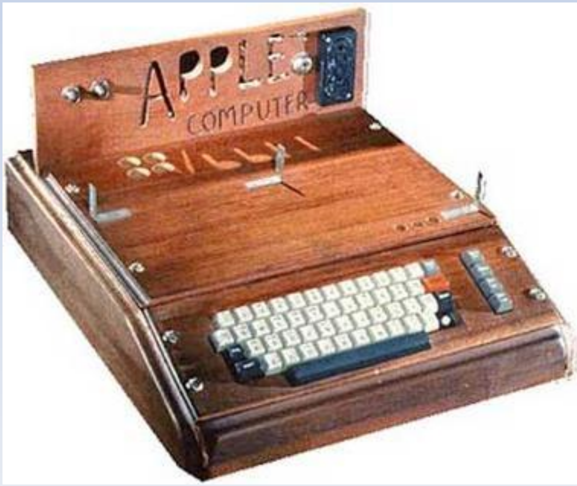
Reliability Analysis

Global variance-based sensitivity analysis

What means sensitivity analysis?

inputs | CAE and experimental | outputs

$$\left.\begin{array}{l} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n_i} \end{array}\right\}$$



$$\left.\begin{array}{l} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_{n_o} \end{array}\right\}$$

$$\left.\begin{array}{l} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n_i} \end{array}\right\}$$

$$\left.\begin{array}{l} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_{n_o} \end{array}\right\}$$

$$\frac{\partial y_j}{\partial x_i}$$ linear sensitivities and importances

$$\left\| \frac{\partial y_1}{\partial x_3} \right\| > \left\| \frac{\partial y_1}{\partial x_1} \right\|$$ nonlinear sensitivities

$$\frac{\partial^n y_j}{\partial x_i^n}$$ multivariate sensitivities

$$\frac{\partial^2 y_j}{\partial x_i \partial x_k}$$ metamodels

$$y_j \approx \tilde{y}_j(x_i)$$ predictability

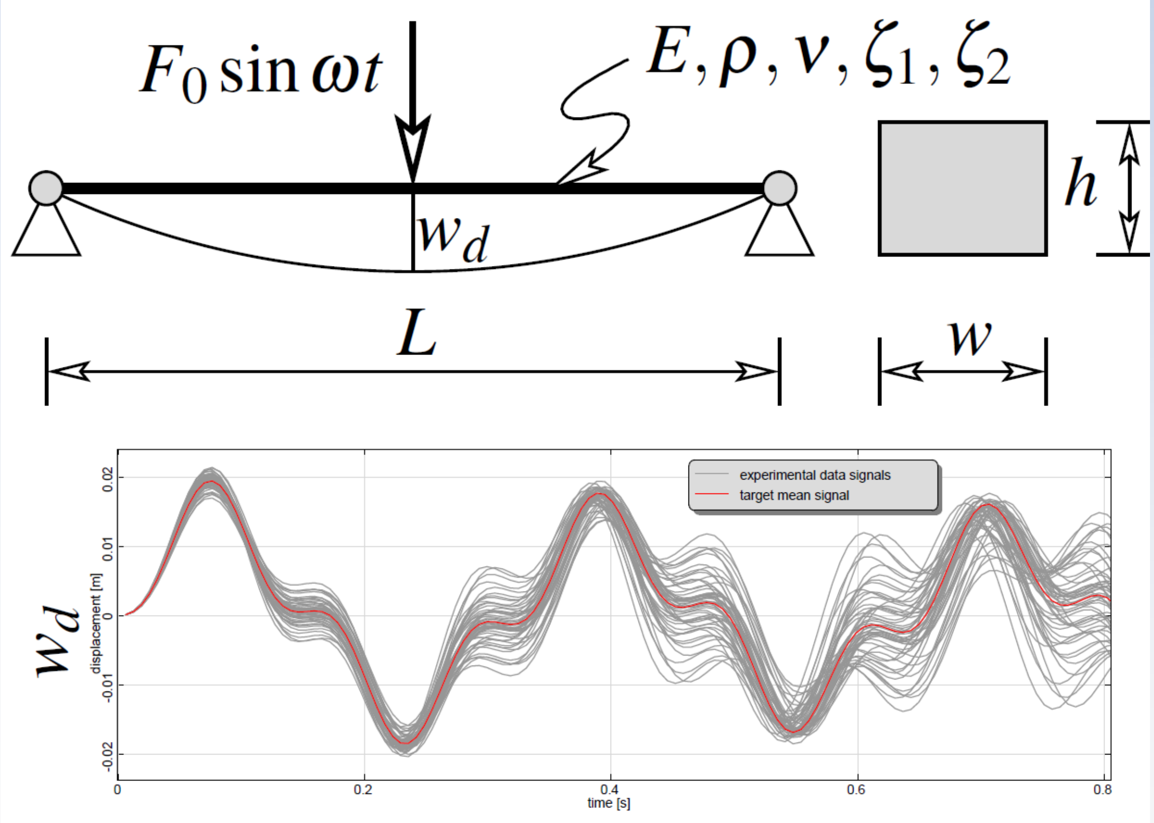
$$R^2$$

$$\mathbf{x}_0$$ start vector

Global variance-based sensitivity analysis

- Analysis of parameter sensitivity means
- investigating the effect of variability (sensitivity) of certain input parameters on the variability of design-relevant response quantities,
- identification of the most important input parameters,
- identification of linear and nonlinear sensitivities,
- identification of multivariate dependencies,
- building metamodels for reducing CAE or experimental costs and gain of knowledge,
- quantification of the model or experimental predictability and noise fraction and
- searching best input parameters vector as starting point for design optimization.

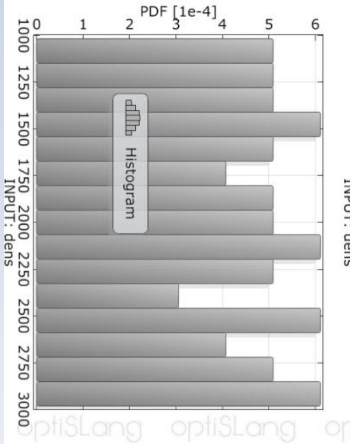
Global variance-based sensitivity analysis

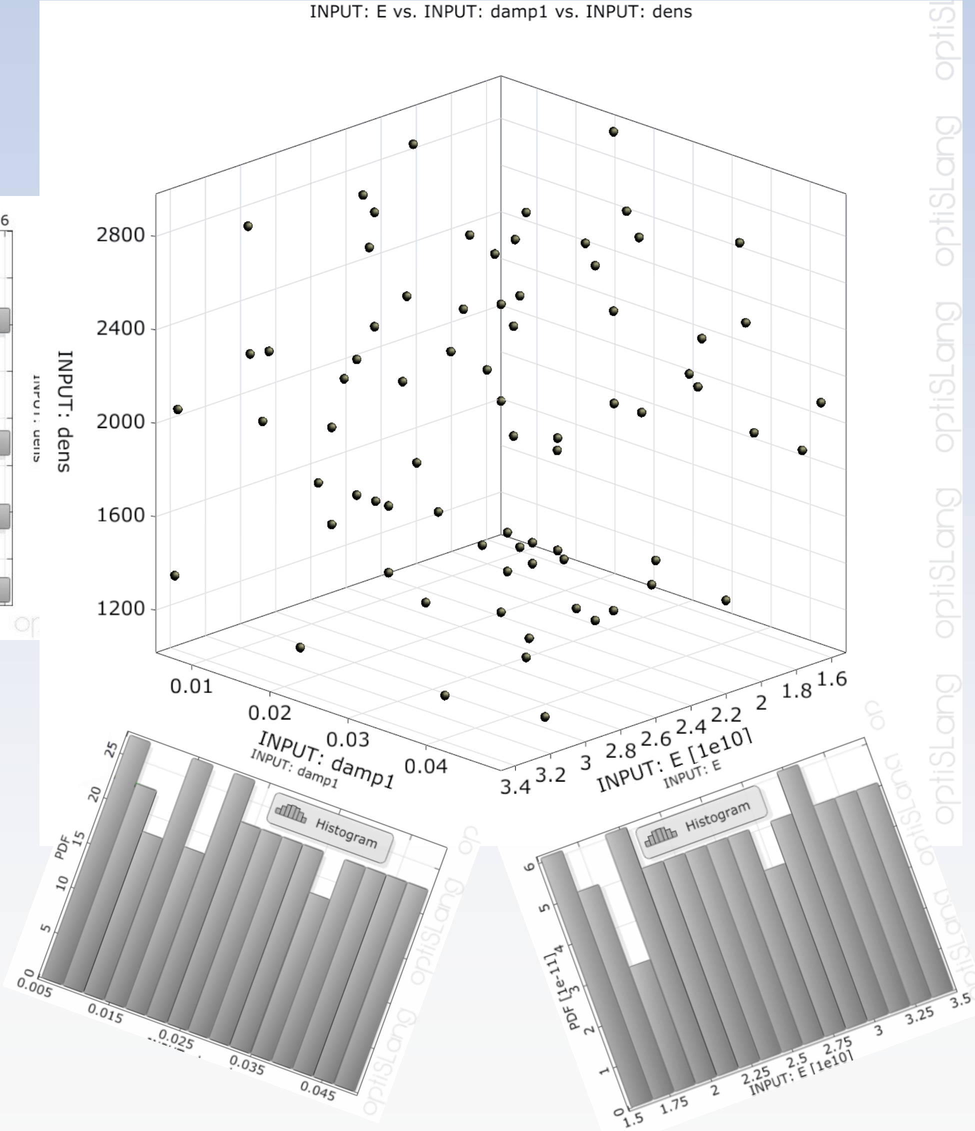


- investigating the input parameter variability effect on the
- variability of the responses
- identification of the most important input parameters
- identification of linear and nonlinear sensitivities
- quantification of the model predictability

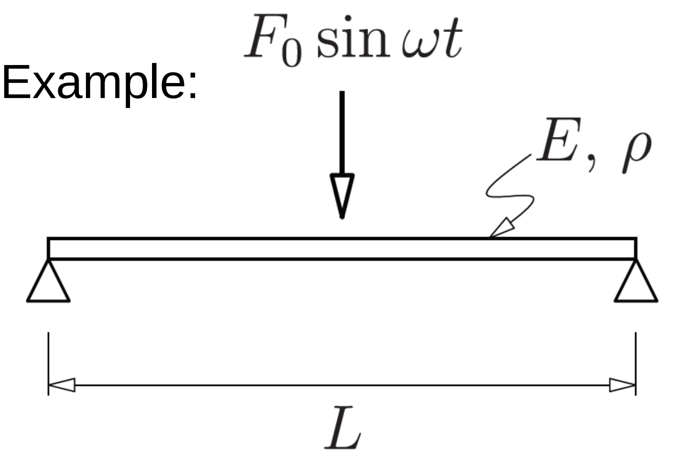
Global variance-based sensitivity analysis

- **Latin hypercube sampling** to
  - Minimize the correlation error of the input variables and to
  - Simulate uniform distributed uncorrelated design variables
  - Required design evaluations N > 50…100



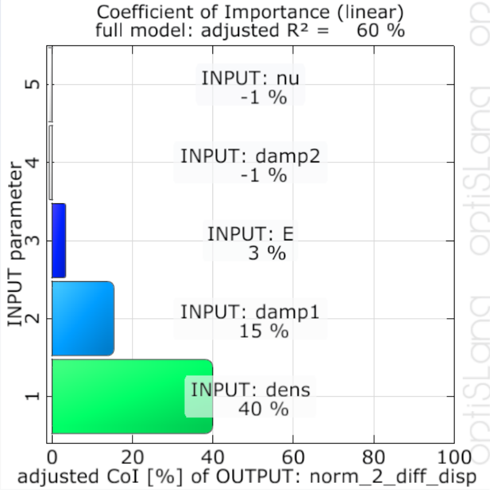


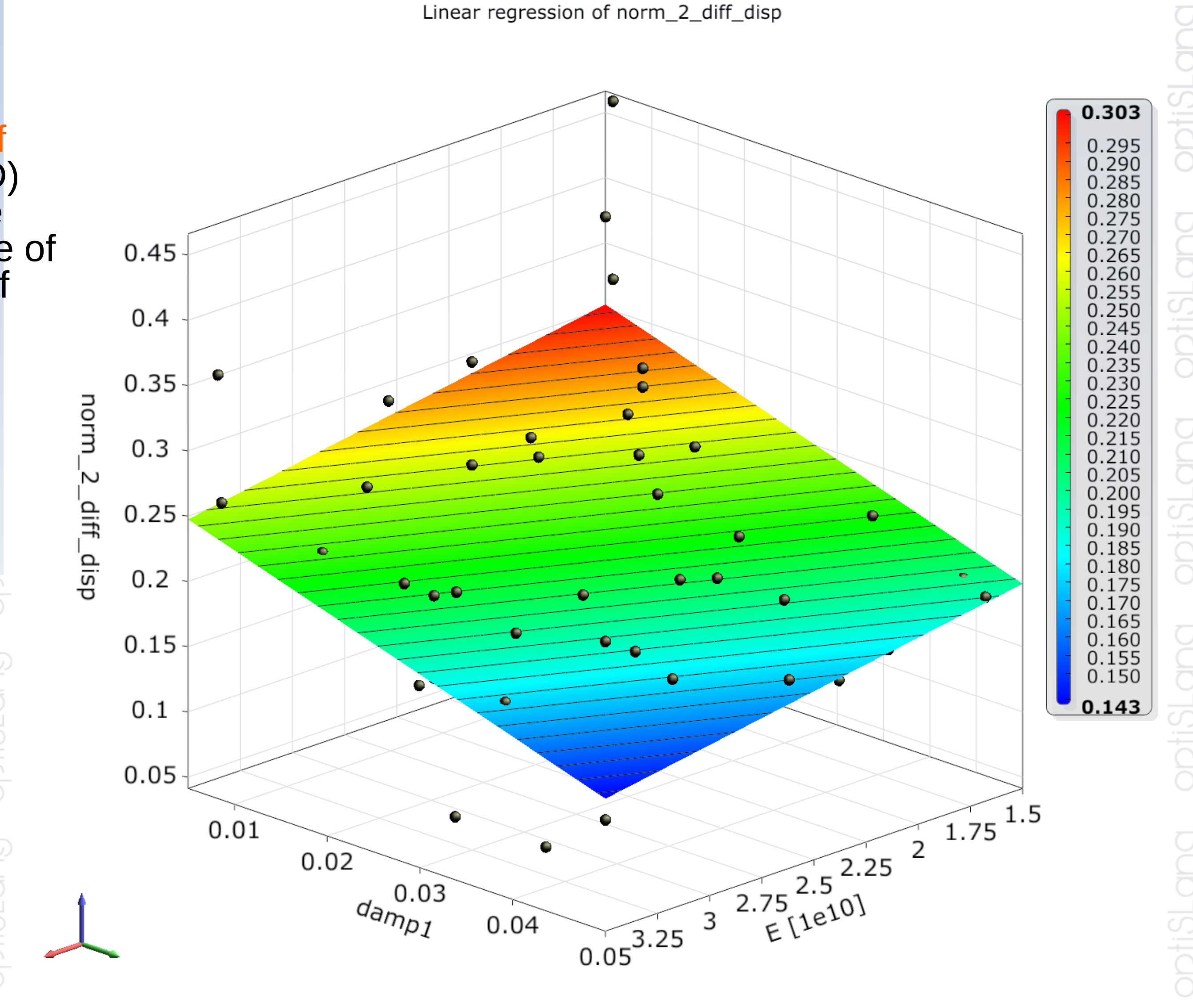
Example:



Global variance-based sensitivity analysis

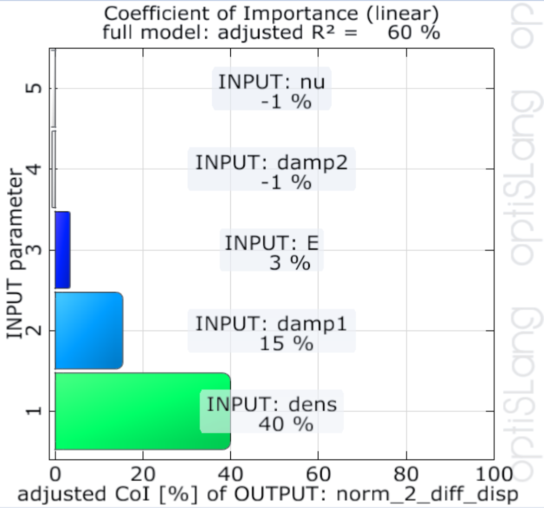
- Linear regression models to calculate
  - Linear coefficient of determination (CoD) (explains the single parameter influence of the total variance of the response performance
  - linear coefficient of importance (CoI) (detects multiple parameter correlations)



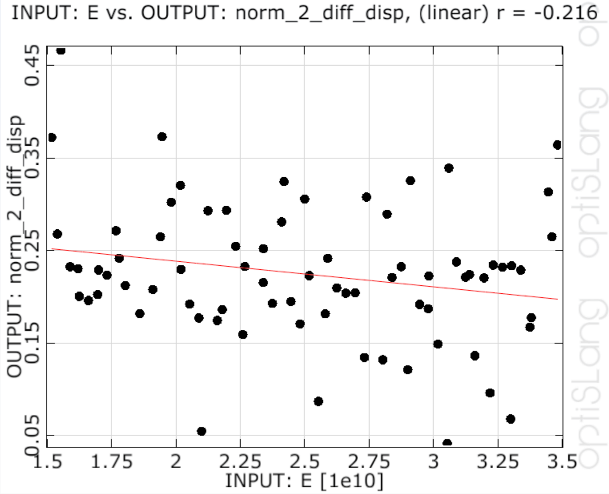


Global variance-based sensitivity analysis

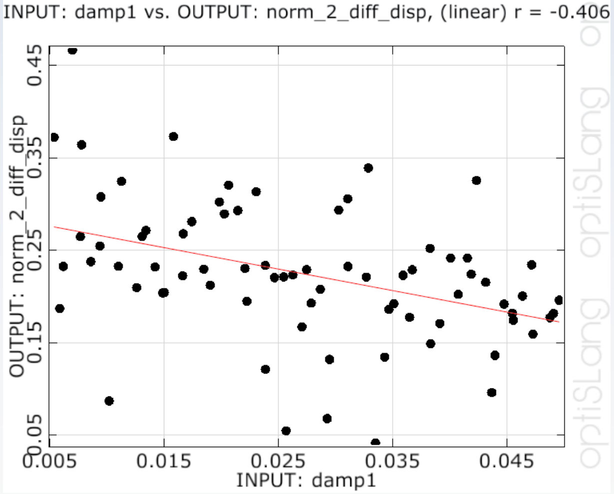
- Linear single parameter correlations



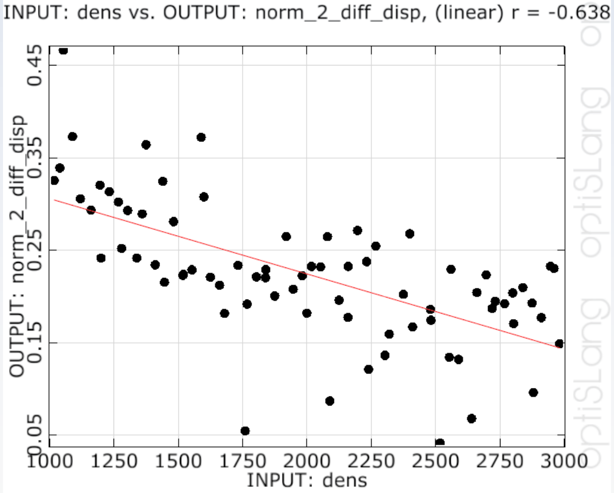
Without linear correlation < 0.3



Weak linear correlation < 0.5

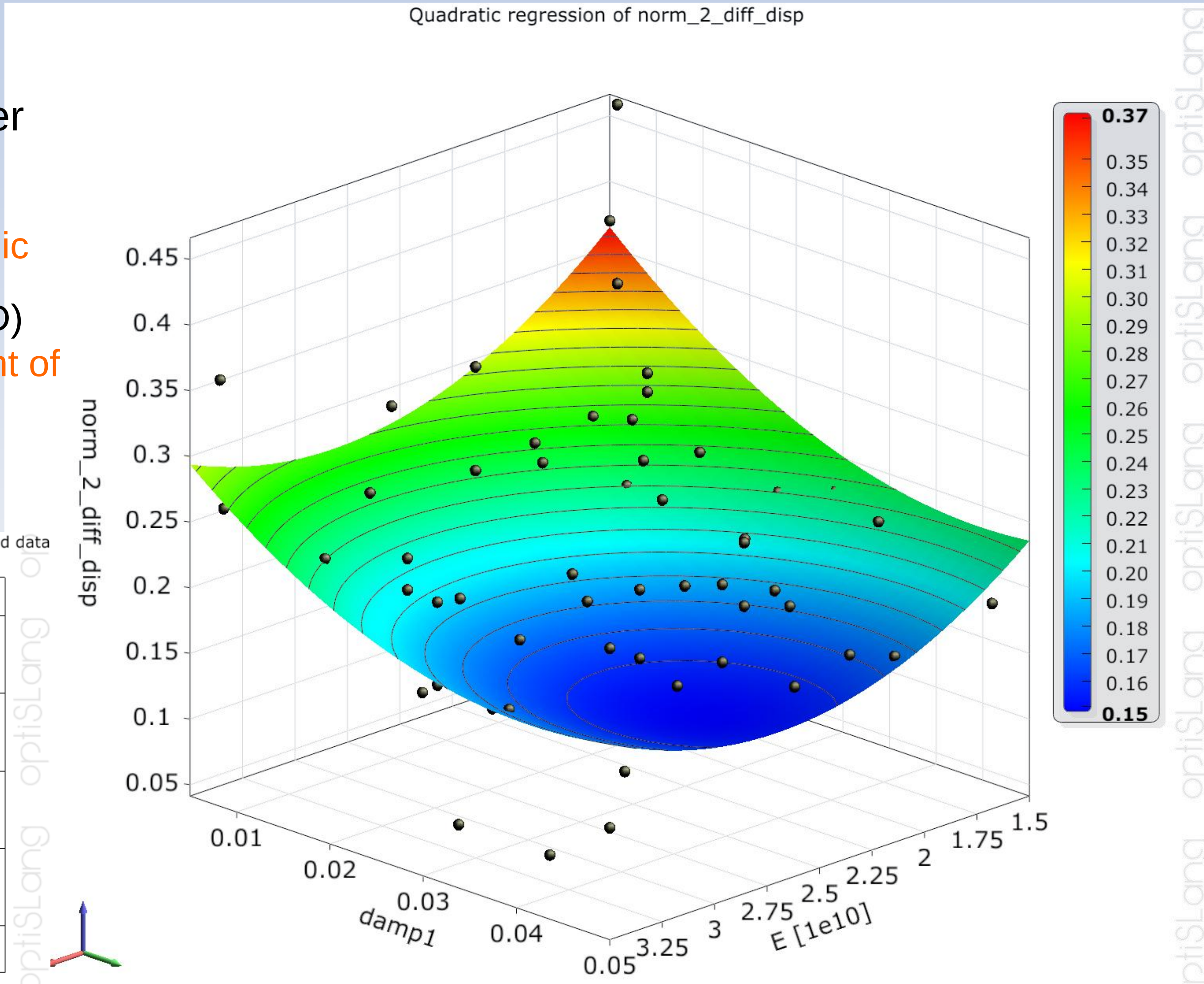


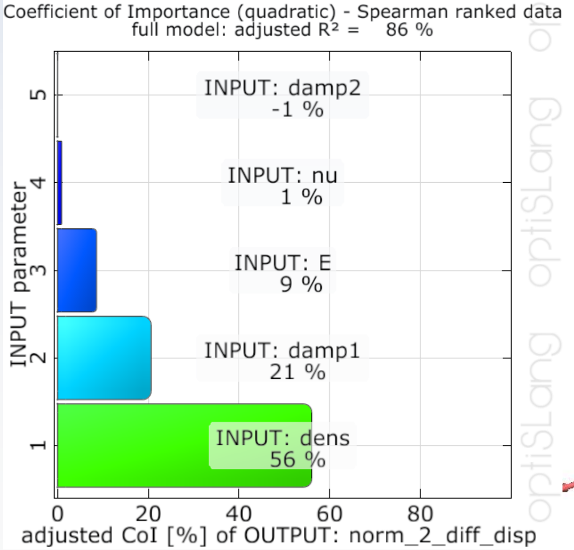
Strong linear correlation > 0.7



Global variance-based sensitivity analysis

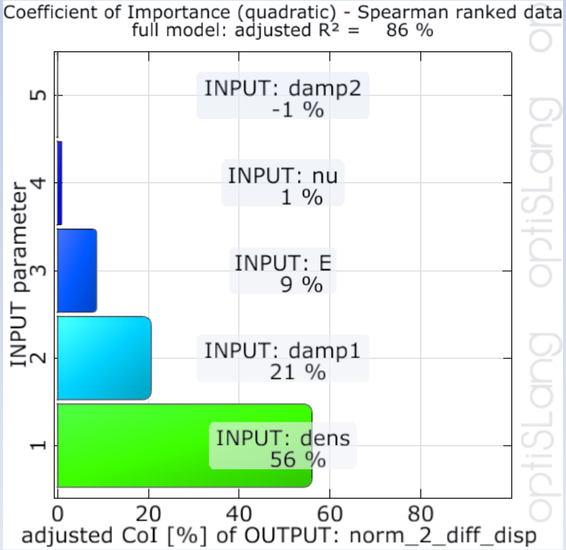
- Nonlinear monotonic regression models (Spearman rank order transformation) to calculate
  - Nonlinear monotonic coefficient of determination (CoD)
  - nonlinear coefficient of importance (CoI)





Global variance-based sensitivity analysis

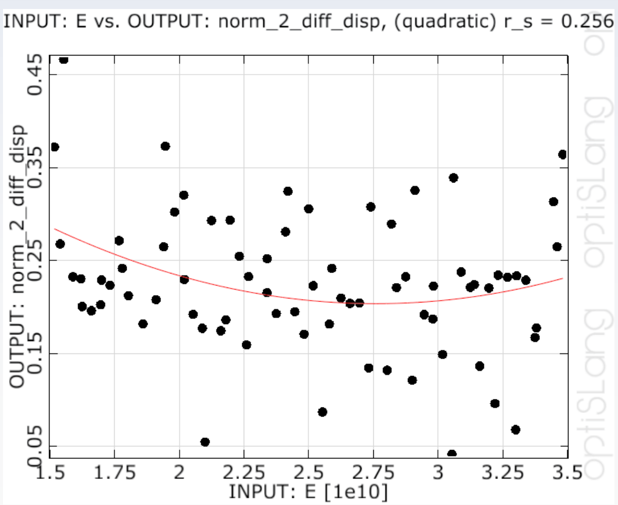
- Nonlinear monotonic single parameter correlations (Spearman correlation)

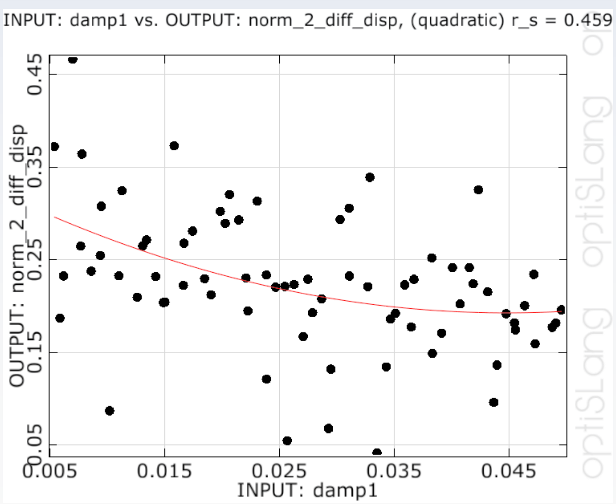


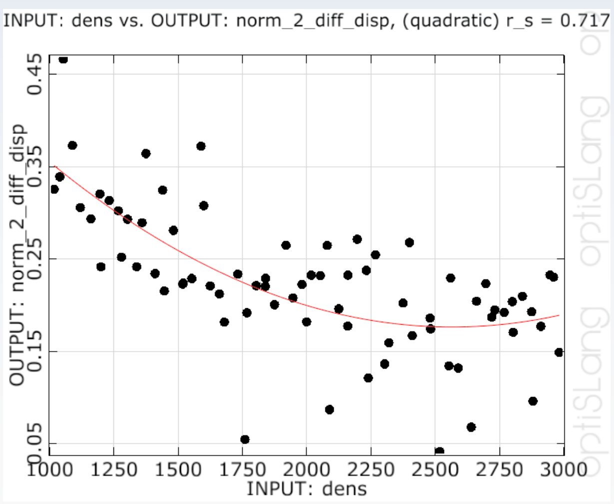
Without linear correlation < 0.3

Weak linear correlation < 0.5

Strong linear correlation > 0.7

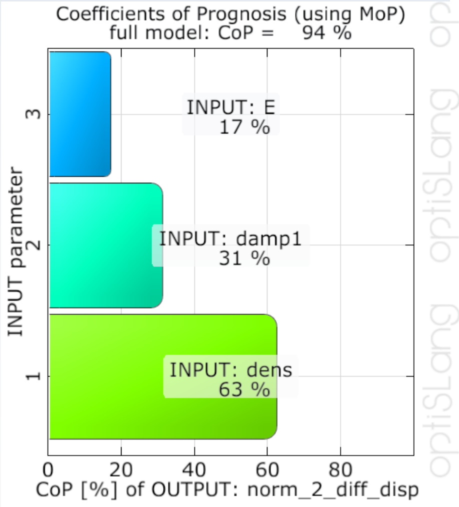


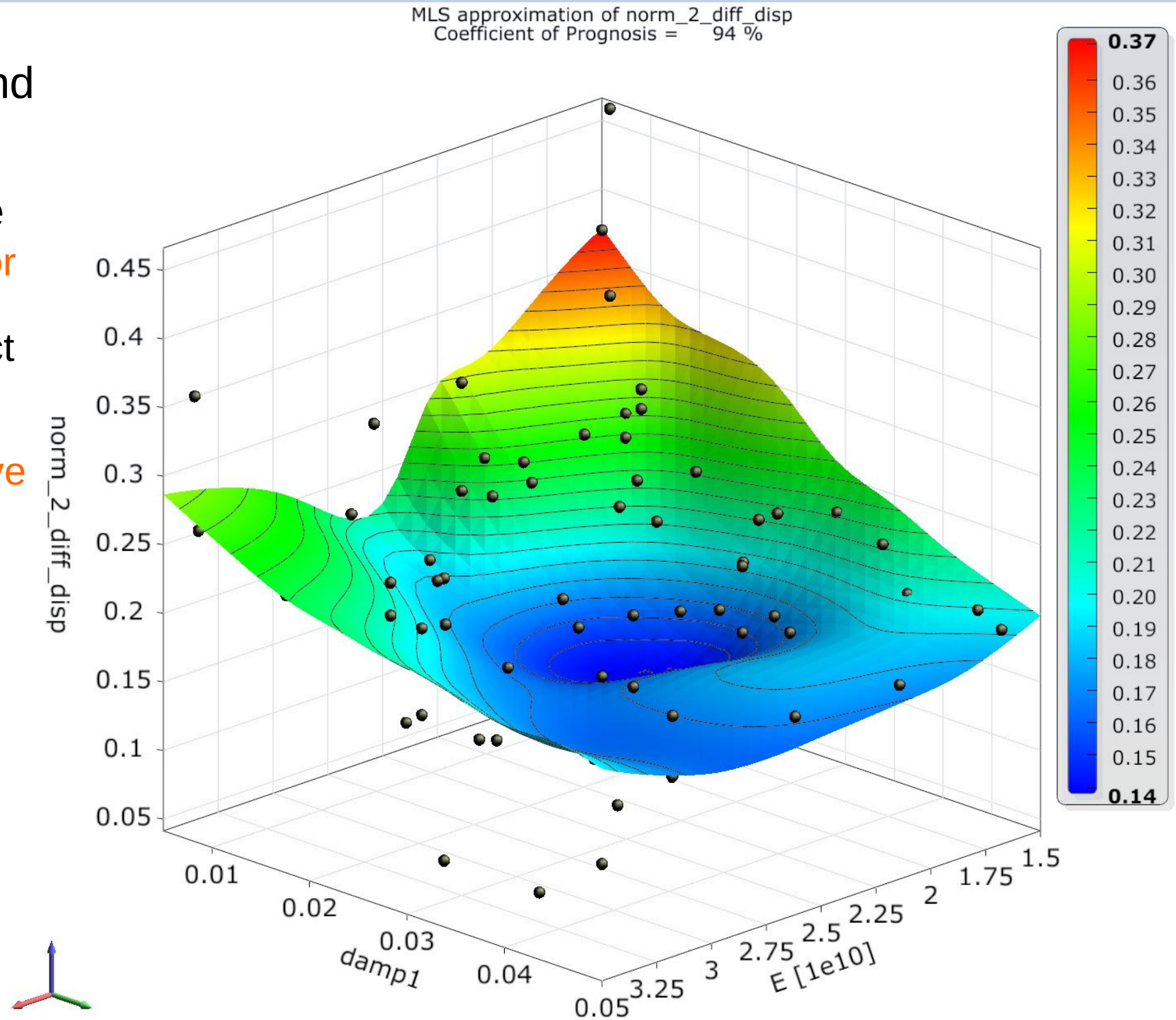




Global variance-based sensitivity analysis

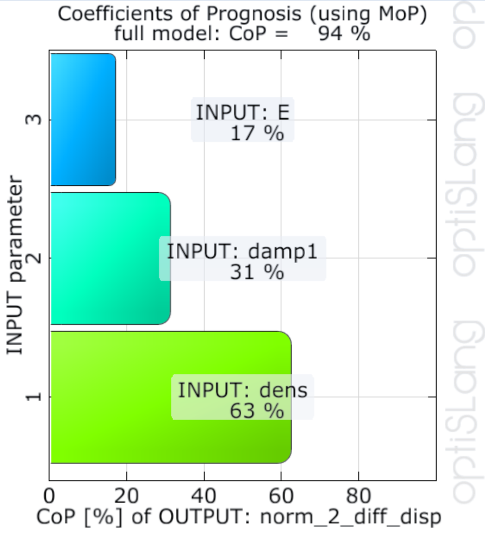
- Metamodels of optimal prognosis (Polynomials and moving least square approximations and interpolations) to calculate
  - Nonlinear model behavior
  - nonlinear coefficient of prognosis (CoP) to detect multivariate parameter sensitivities
  - Estimation of the sensitive design subspace



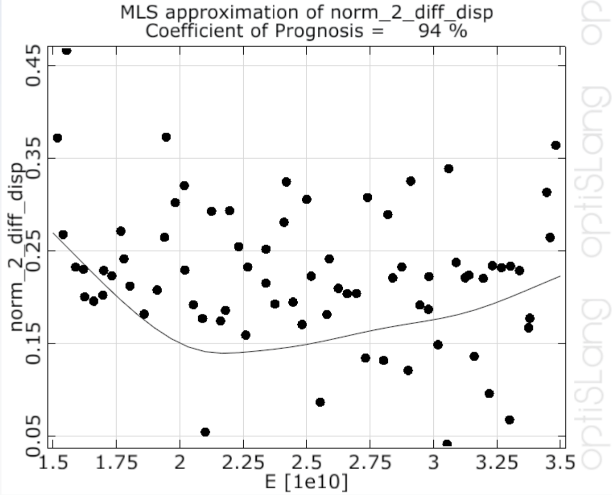


Global variance-based sensitivity analysis

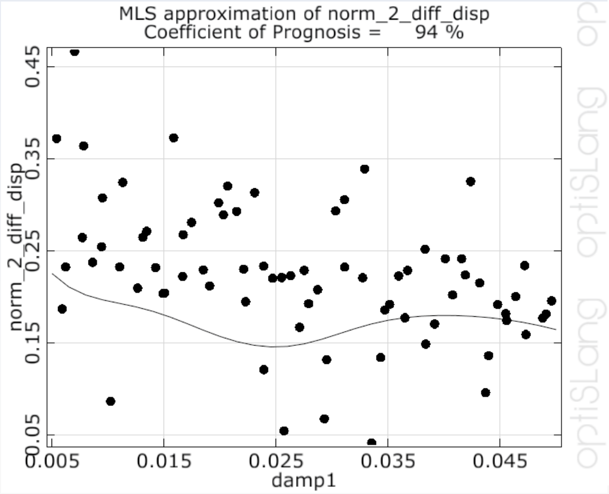
- General nonlinear multivariate parameter correlations (CoP)
- Visualization of the physical parameter dependencies and relationships
- Metamodel of optimal prognosis can be used for optimization and stochastic analysis



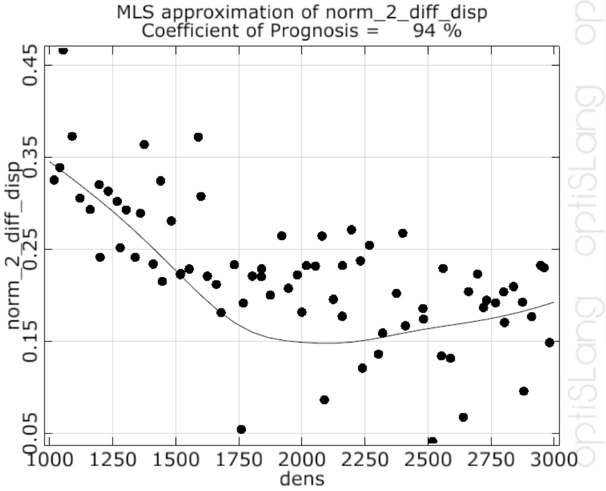
Small influence CoP <10%



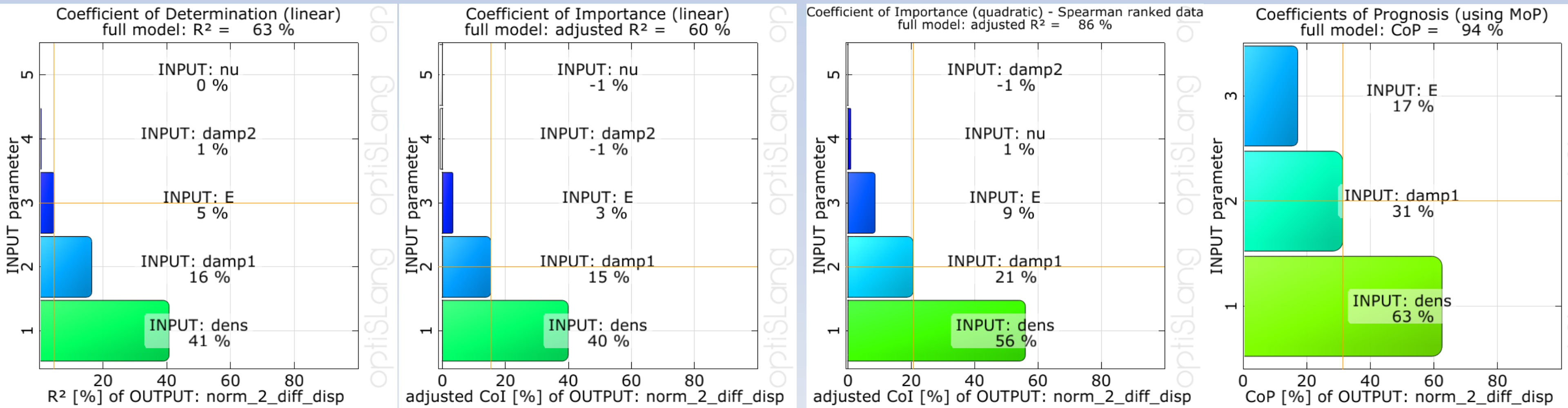
Weak influence CoP <15%



Strong influence CoP >15%

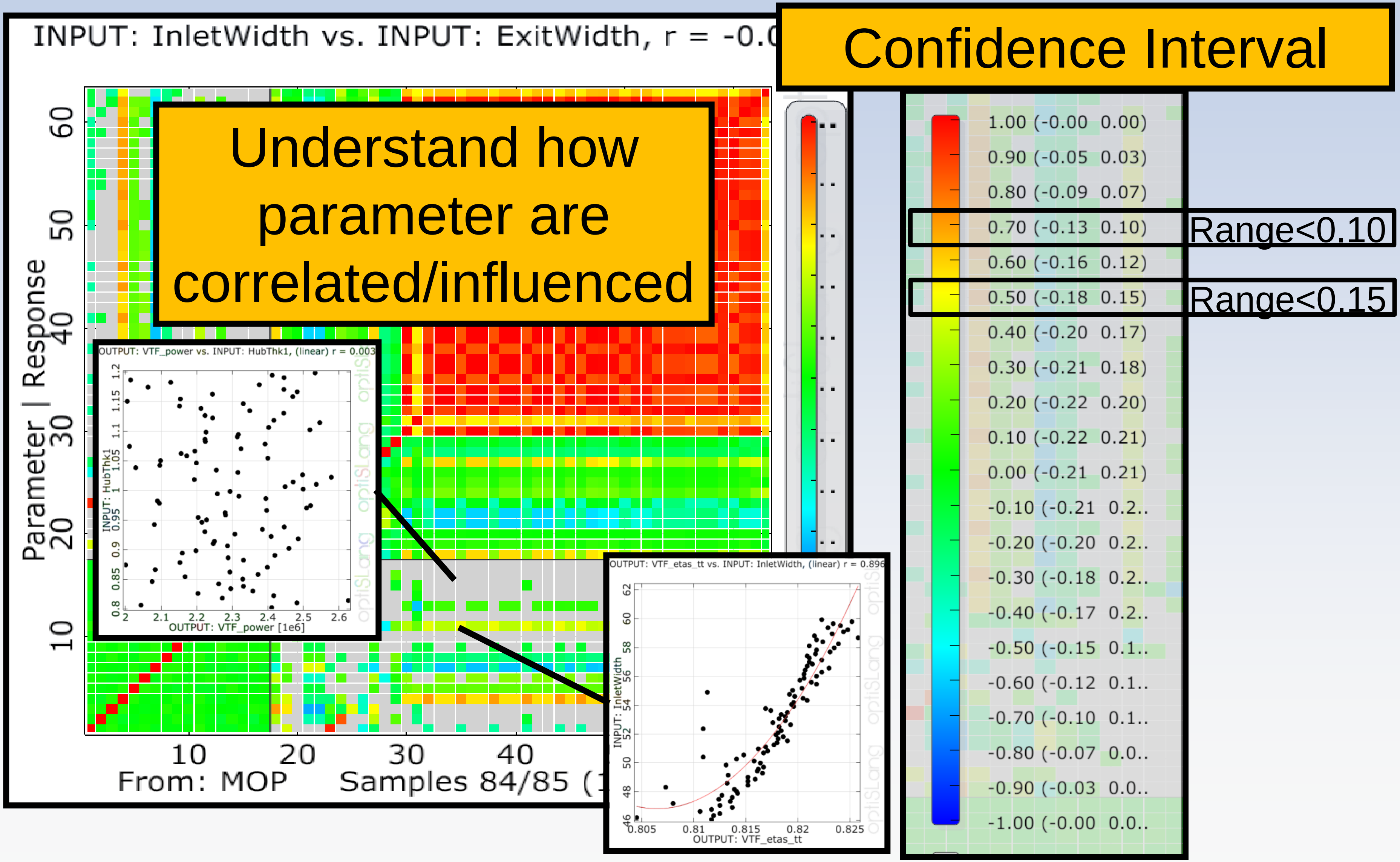


Global variance-based sensitivity analysis



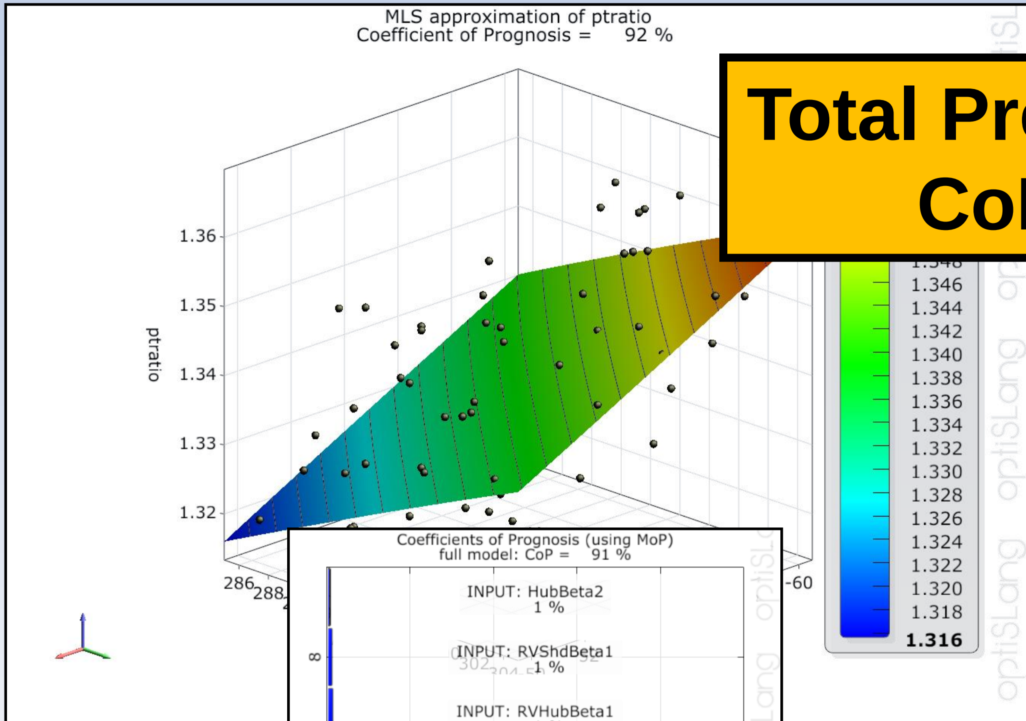
Coefficient of Determination, **CoD** Linear regression	Coefficient of Importance, **CoI** Linear regression	Coefficient of Importance, **CoI** Spearman rankordering	Coefficient of Prognosis, **CoP**, Metamodel of optimal prognosis
Linear relationships	Linear relationships	Nonlinear, monotonic	Nonlinear, general
Single parameter correlations	Single parameter correlations	Single parameter correlations	Multiple parameter correlations
Single parameter dependencies	Qualification of multivariate dependencies	Qualification of multivariate dependencies	Quantification of multivariate dependencies
Identification of input correlations	Identification of multiple parameter correlations	Quantification of predic-table response variance	Identification of the most sensitive design subspace
Examination of the accumulatable single parameter CoDs		CoD total model > 80%	Multivariate sensitivities: single CoP > total CoP

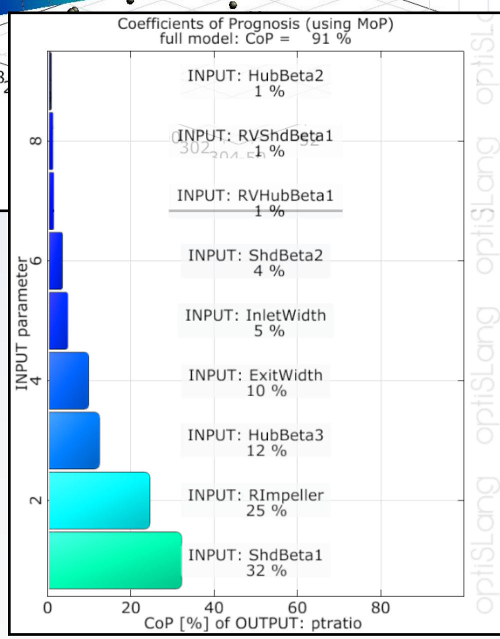
Correlation Matrix

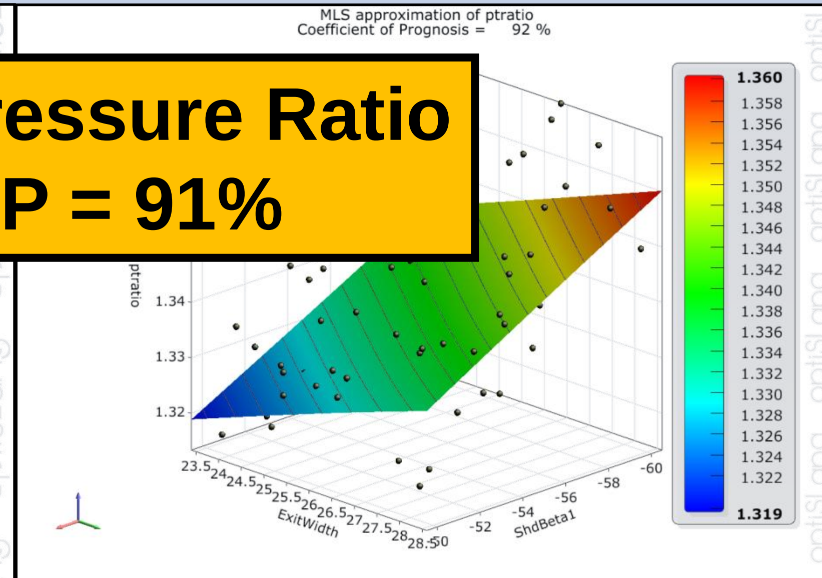


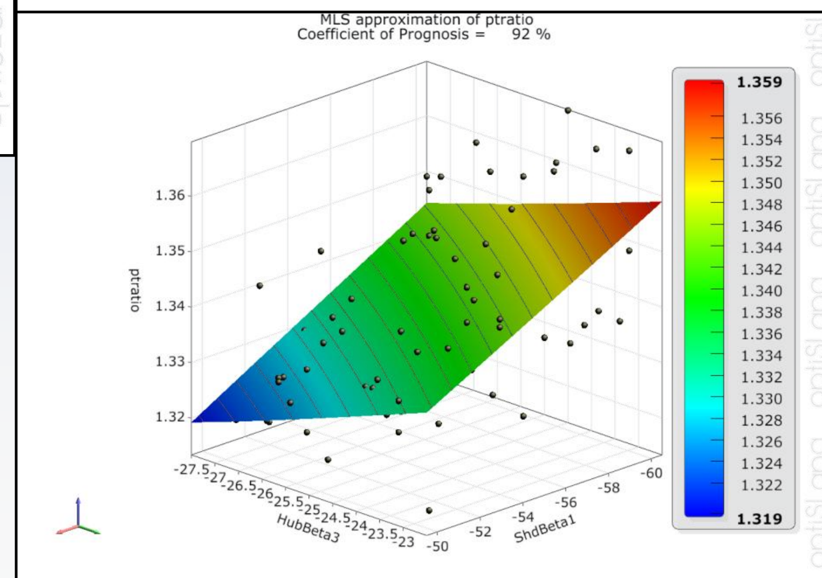
Meta-Model of Best Prognosis

Total Pressure Ratio
CoP = 91%



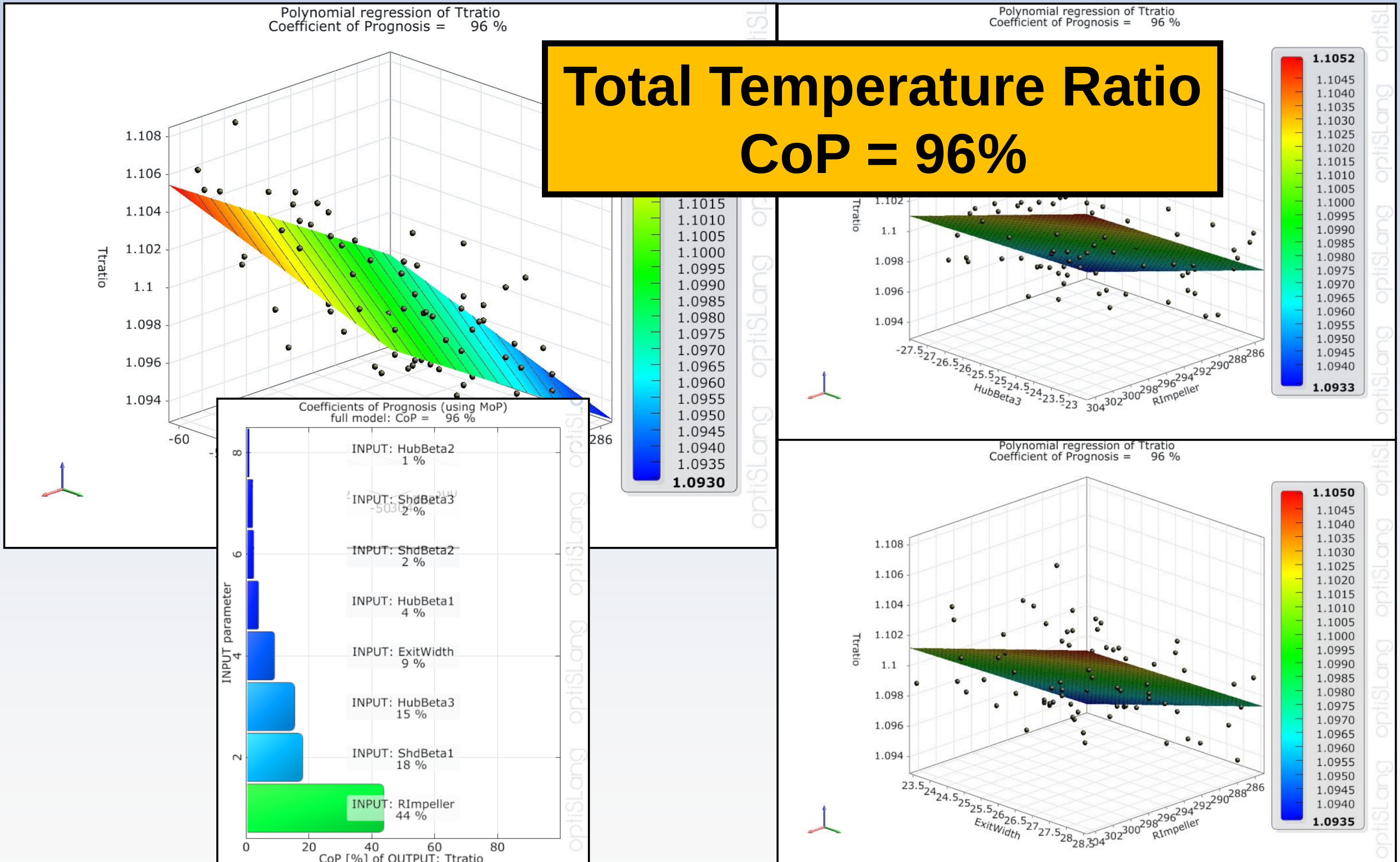




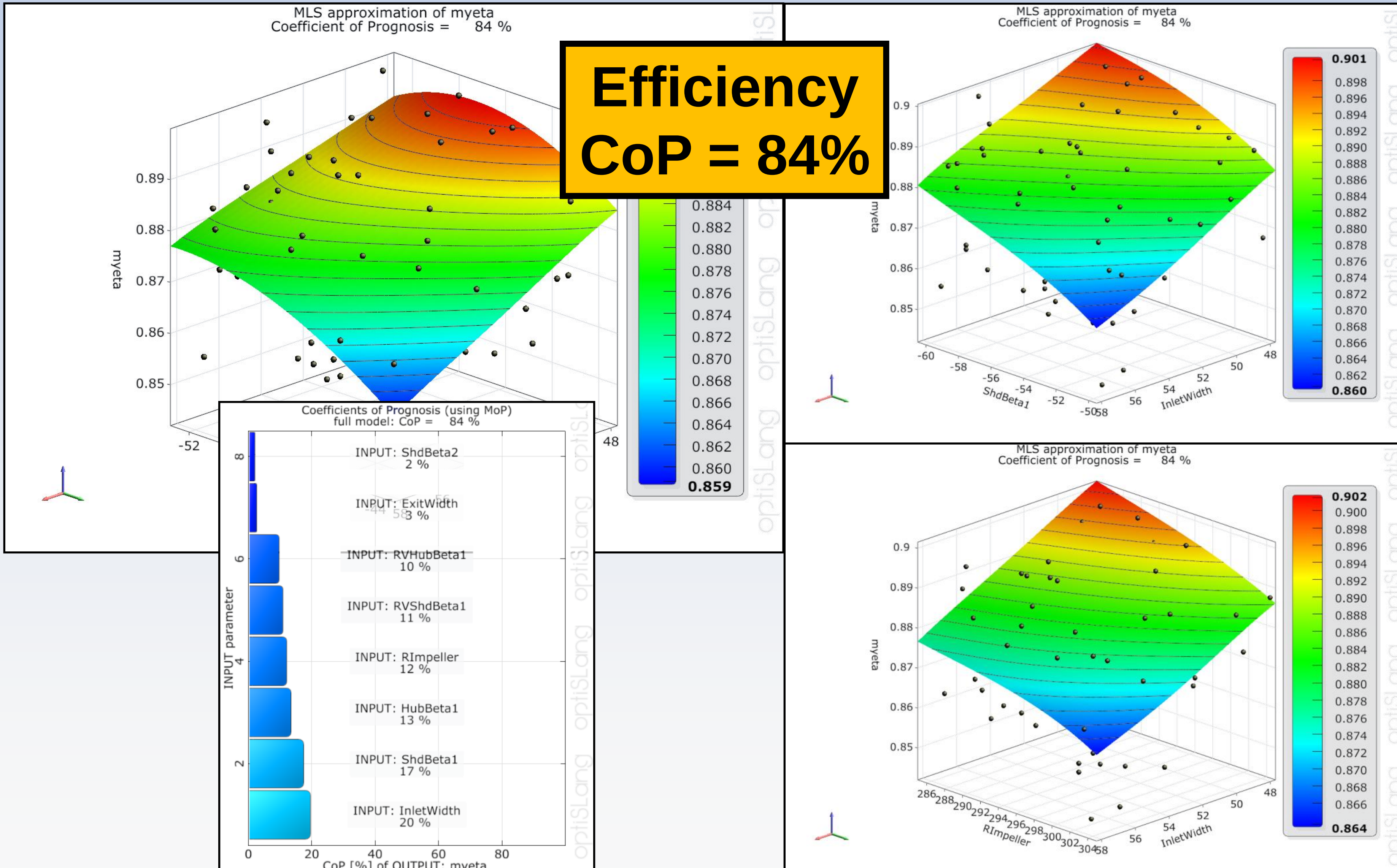


Meta-Model of Best Prognosis

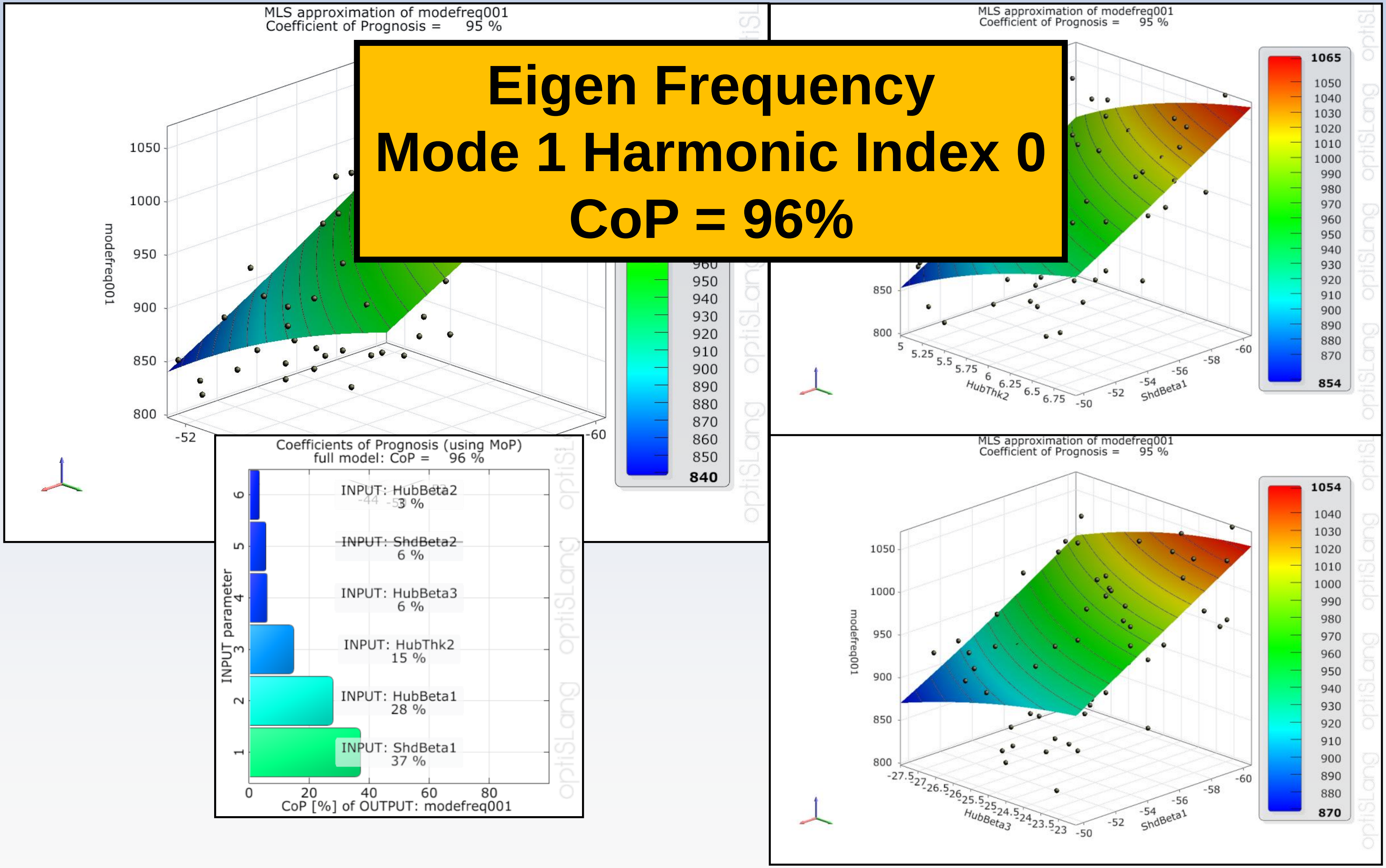
Total Temperature Ratio
CoP = 96%



Meta-Model of Best Prognosis

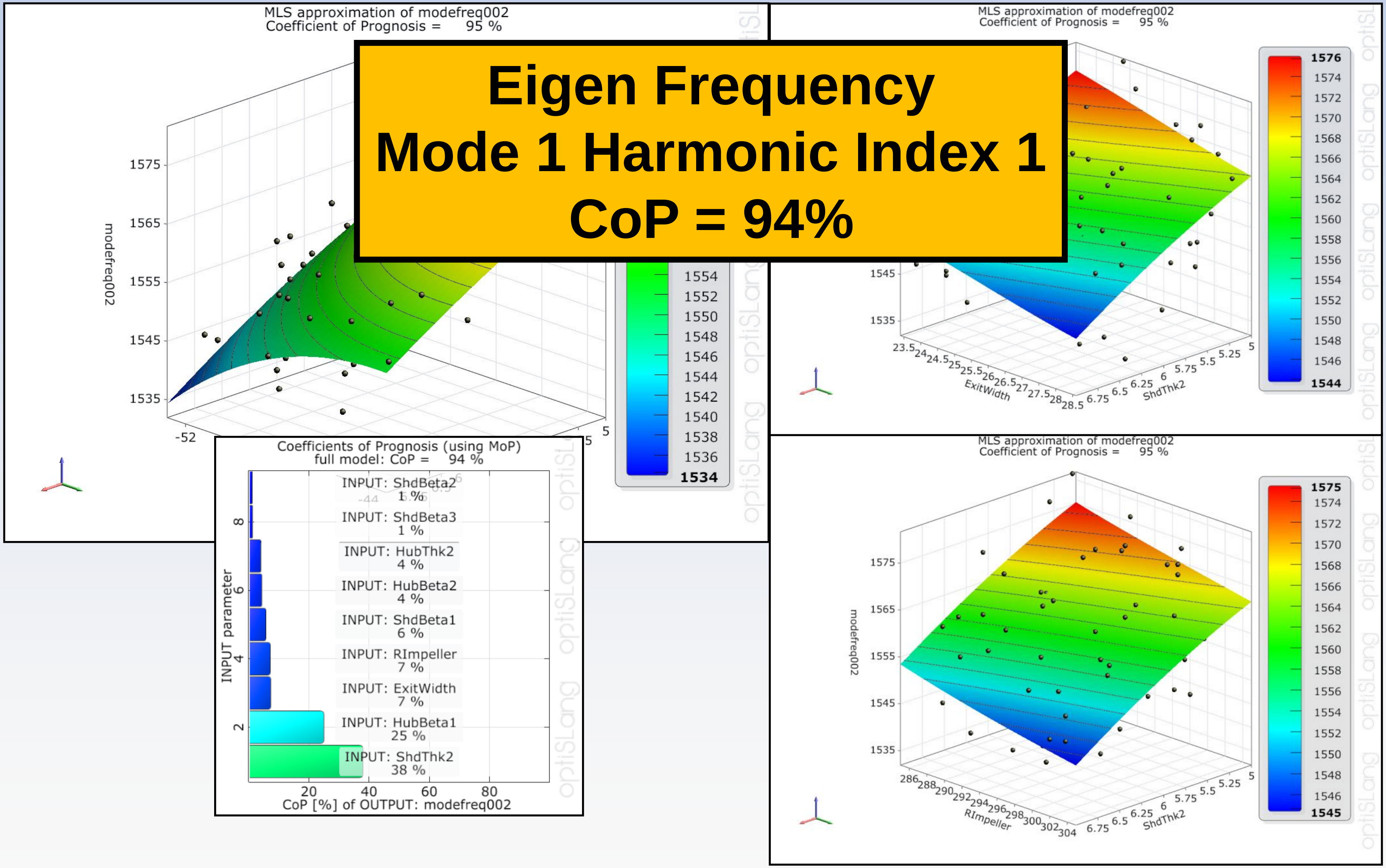


Meta-Model of Best Prognosis

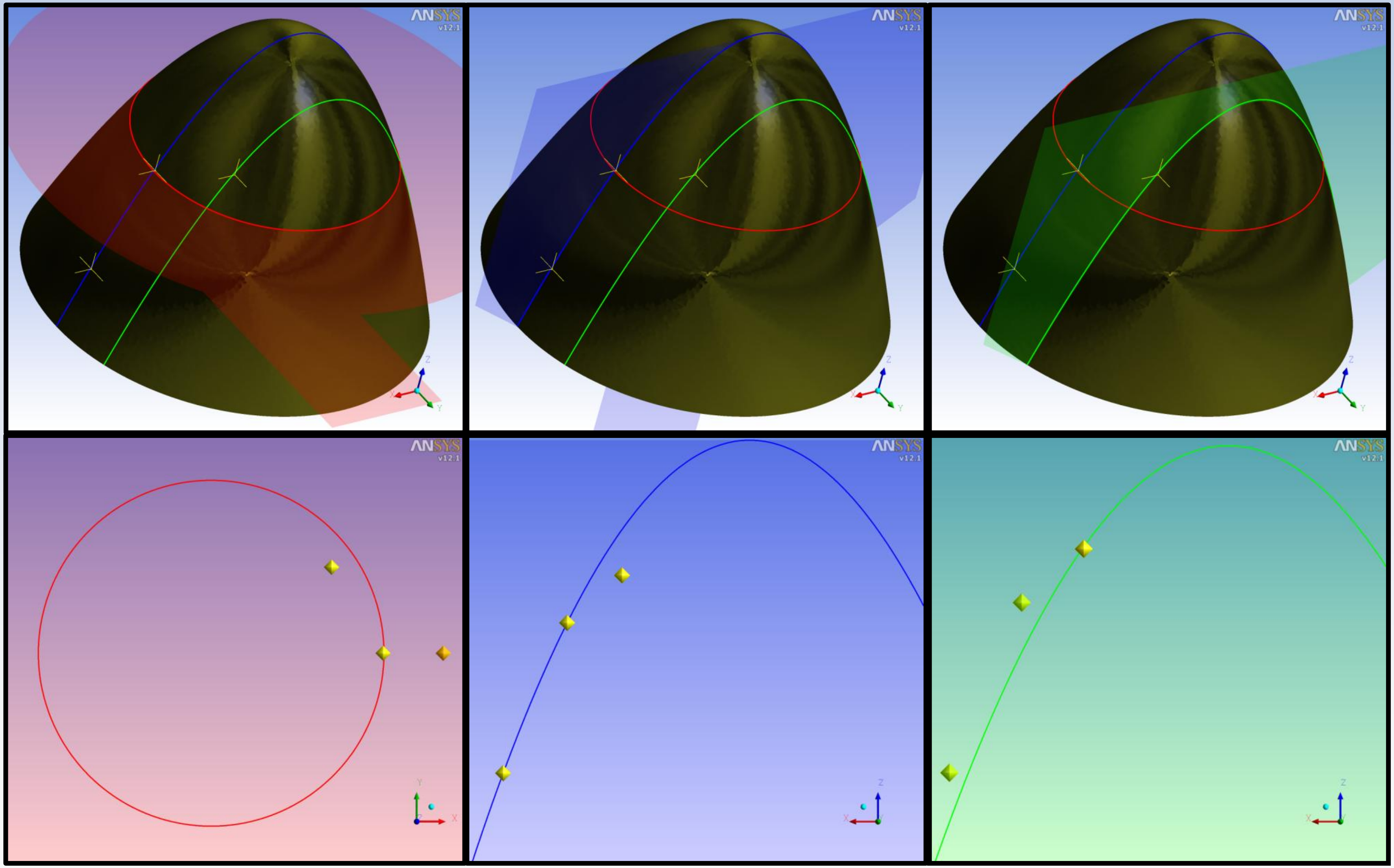


Meta-Model of Best Prognosis

Eigen Frequency
Mode 1 Harmonic Index 1
CoP = 94%



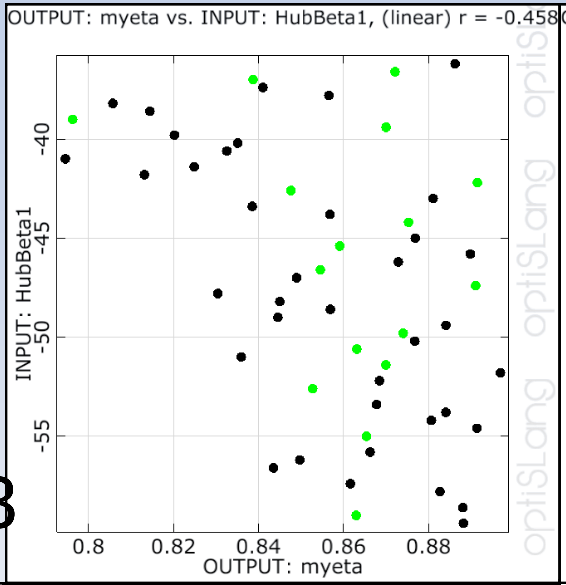
Visualization of MoP

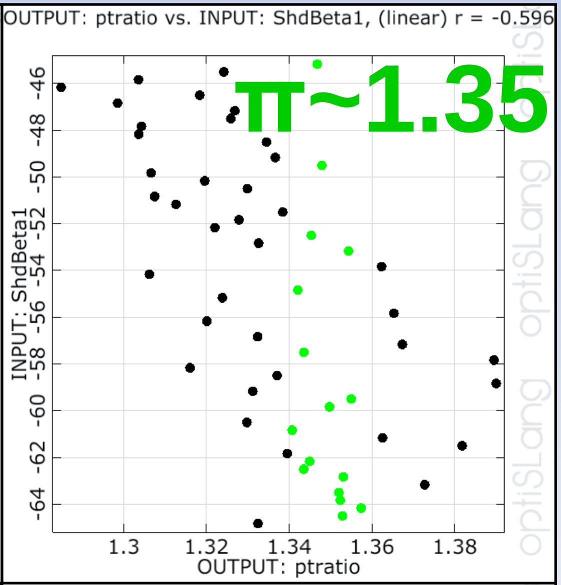


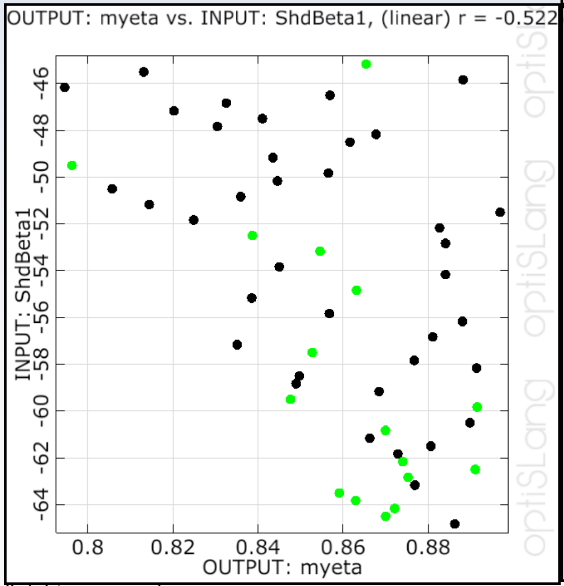
Parameter Reduction

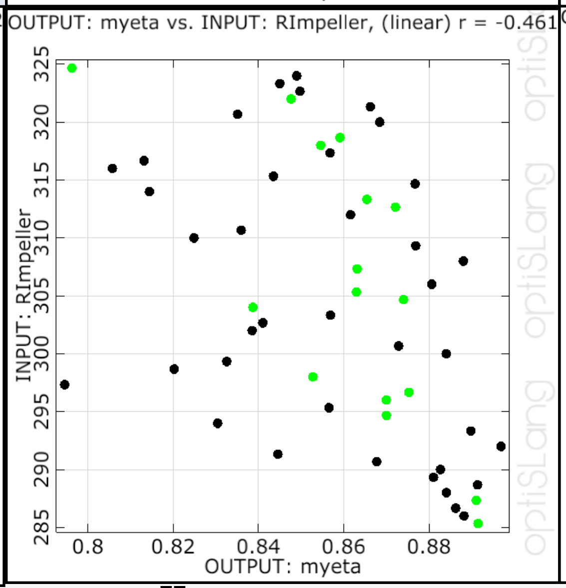
Important Parameter:

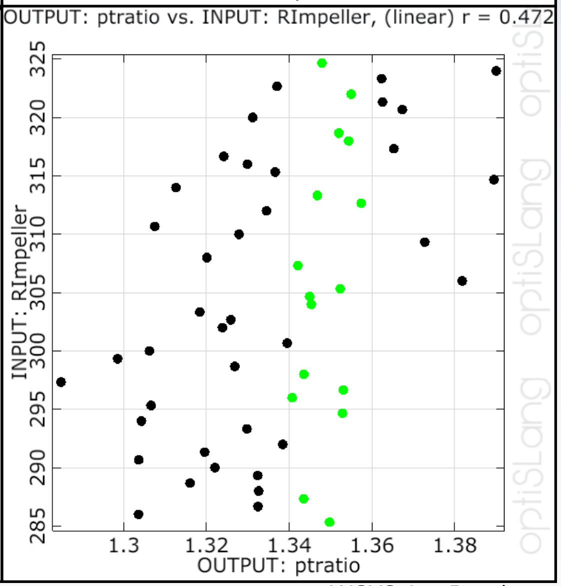
- InletWidth
- ExitWidth
- RImpeller
- RV/Hub/Shd Beta 1,3





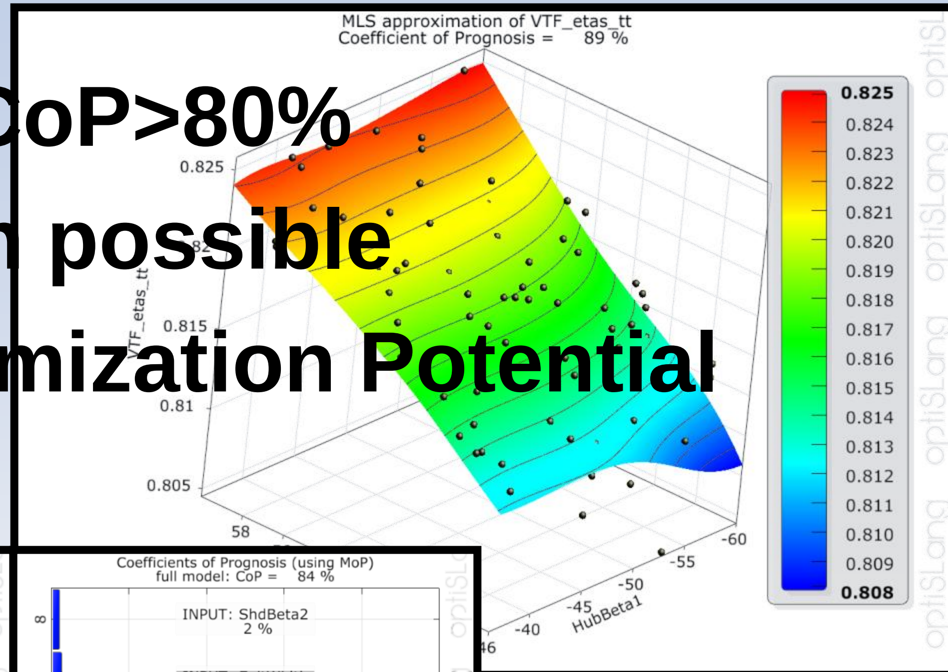


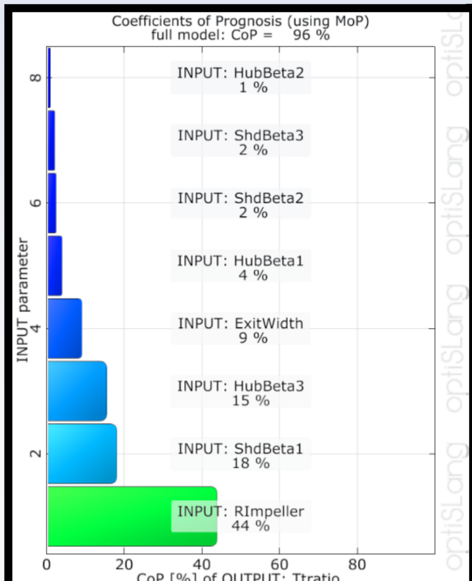


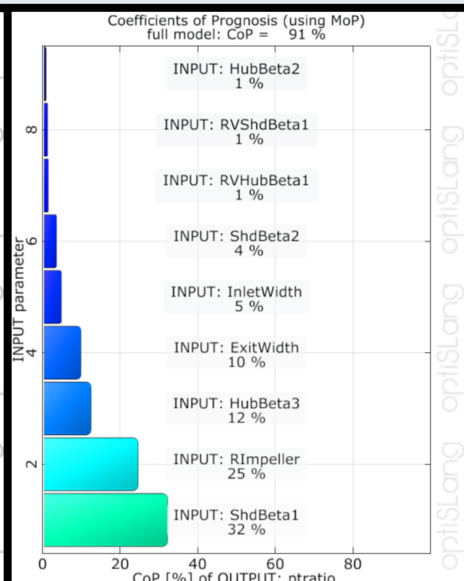


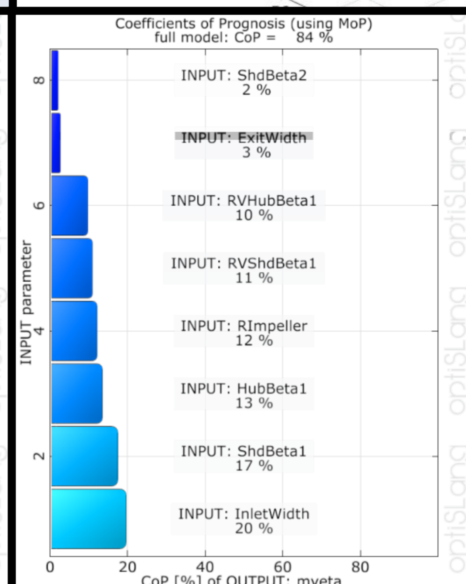
Conclusion Sensitivity Analysis

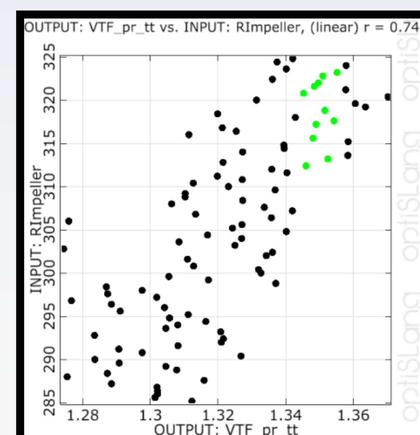
- **Statistic is reliable, CoP>80%**
- **Parameter Reduction possible**
- **Statistic Shows Optimization Potential**











Conclusion Sensitivity Analysis

- Efficiency: **myeta**
  - InletWidth : as smallest as possible
  - ShdBeta1 : as smallest as possible
  - HubBeta1 : as smallest as possible
  - RImpeller : as smallest as possible
  - RVShdBeta1 : as largest as possible
  - RVHubBeta1 : as largest as possible

Opti | Robust | Output | Strings | Constraints | Objectives

#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Type	Format	Active	Con...
1	InletWidth	53	49.237	53.5	57.5	continuous	%20.14f	☑	☐
2	ExitWidth	26	26.91	26.5	28.5	continuous	%20.14f	☑	☐
3	RImpeller	305	288.3	292.0	298.0	continuous	%20.14f	☑	☐
4	HubBeta1	-48	-51.888	-52.5	-49.5	continuous	%20.14f	☑	☐
5	HubBeta3	-25	-25.075	-27.5	-26.5	continuous	%20.14f	☑	☐
6	ShdBeta1	-55	-58.355	-60.5	-59.5	continuous	%20.14f	☑	☐
7	RVHubBeta1	60	64.02	62.0	66.0	continuous	%20.14f	☑	☐
8	RVShdBeta1	60	69.5	60.0	64.0	continuous	%20.14f	☑	☐
9	mymassin	72.6	72.6	65.34	87.119999999...	continuous	%20.14f	☐	☐
10	myomega	699.76	699.76	629.784	839.712	continuous	%20.14f	☐	☐
11	HubBeta2	-25	-26.525	-27.5	-22.5	continuous	%20.14f	☐	☐
12	ShdBeta2	-45	-42.885	-49.5	-40.5	continuous	%20.14f	☐	☐
13	ShdBeta3	-30	-30.69	-33.0	-27.0	continuous	%20.14f	☐	☐
14	HubThk1	1	0.818	0.8	1.2	continuous	%20.14f	☐	☐
15	HubThk2	6	6.01	5.0	7.0	continuous	%20.14f	☐	☐
16	ShdThk1	1	0.93	0.8	1.2	continuous	%20.14f	☐	☐
17	ShdThk2	6	6.85	5.0	7.0	continuous	%20.14f	☐	☐
18	RVHubThk1	45	61.065	45.0	66.0	continuous	%20.14f	☐	☐
19	RVShdThk1	45	38.3	35.0	55.0	continuous	%20.14f	☐	☐
20	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f	☐	☐
21	RVBlades	24	24	21.6	28.799999999...	continuous	%20.14f	☐	☐

Cancel | OK

- Total pressure: **ptratio**
  - ShdBeta1 : as smallest as possible
  - RImpeller : as largest as possible
  - HubBeta3 : as smallest as possible
  - ExitWidth : as largest as possible
  - InletWidth : as smallest as possible

Process Integration

Sensitivity Analysis

Design Optimization

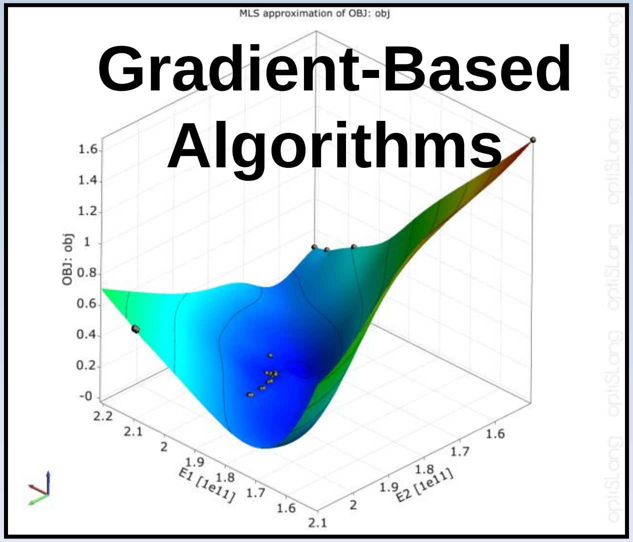
Robustness Evaluation

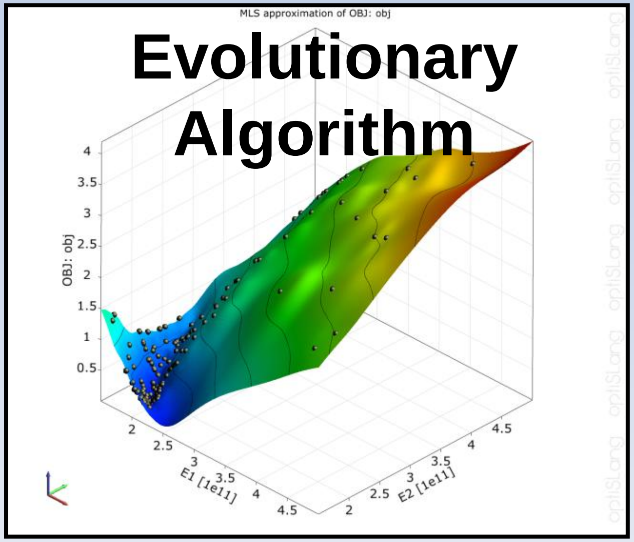
Random Fields

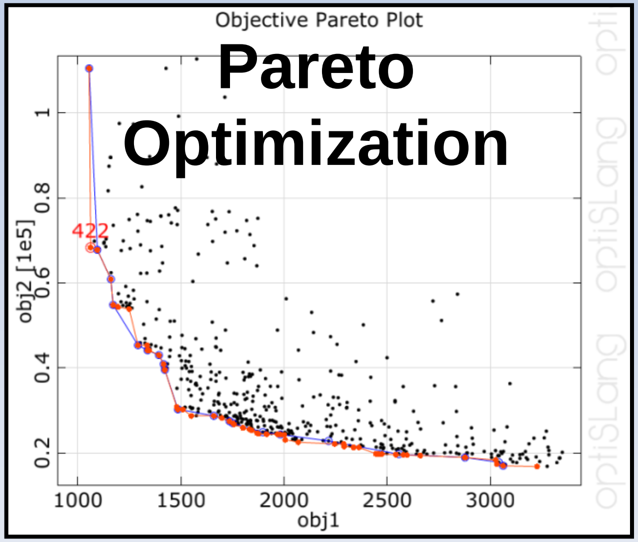
Robust Design Optimization

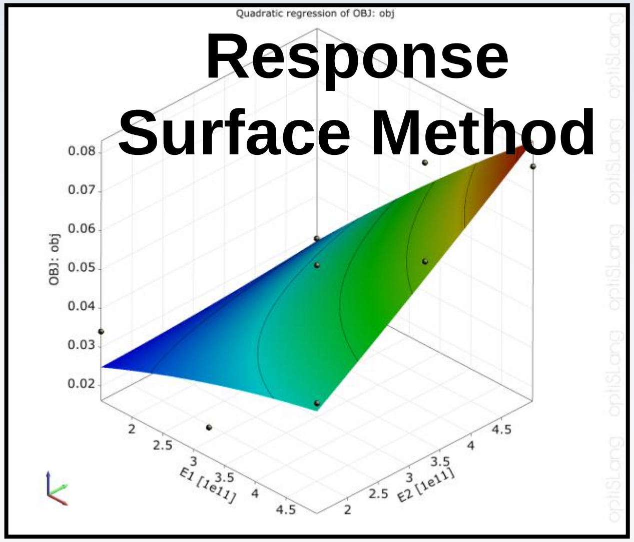
Reliability Analysis

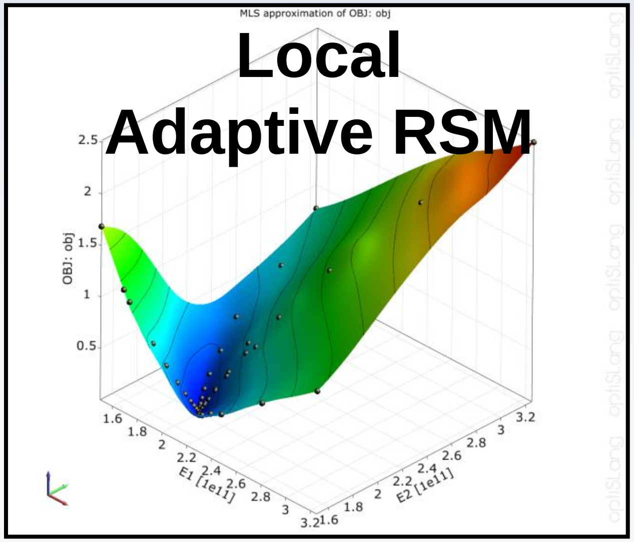
Design Optimization

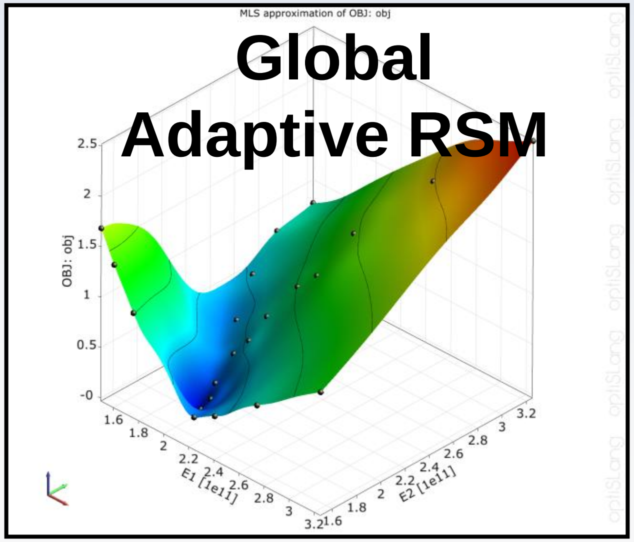








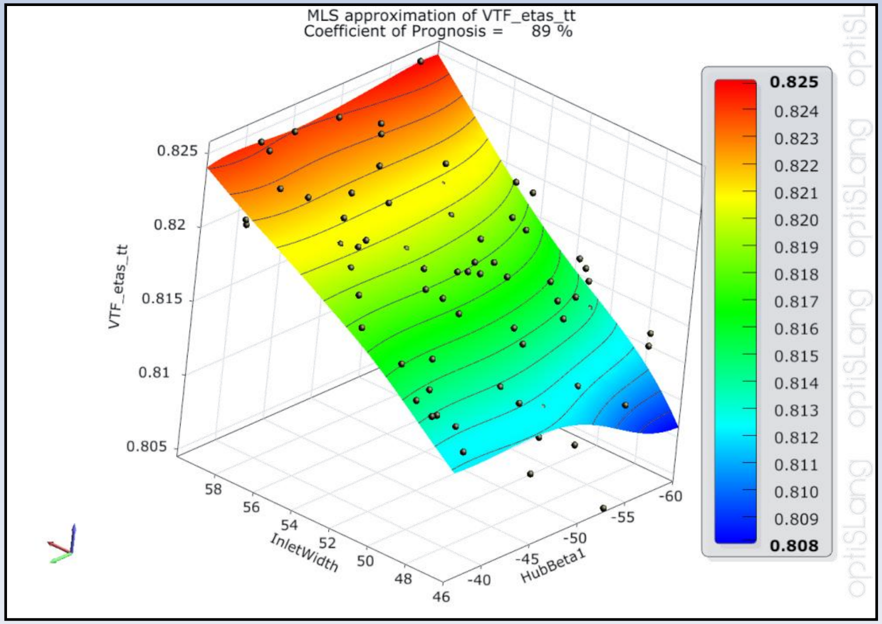




Optimization Strategy

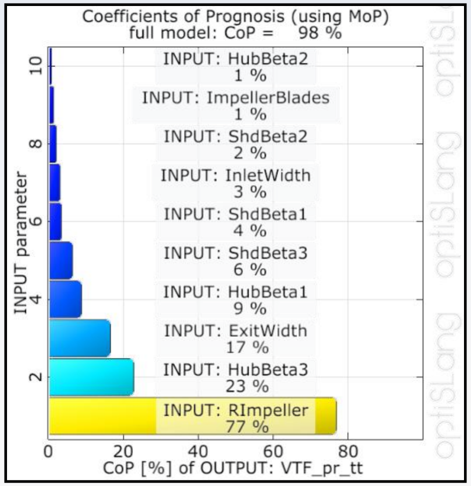
Sensitivity Analysis (SA)

- Shows optimization potential
- Indicates start solution
- Parameter reduction
- Modify parameter space

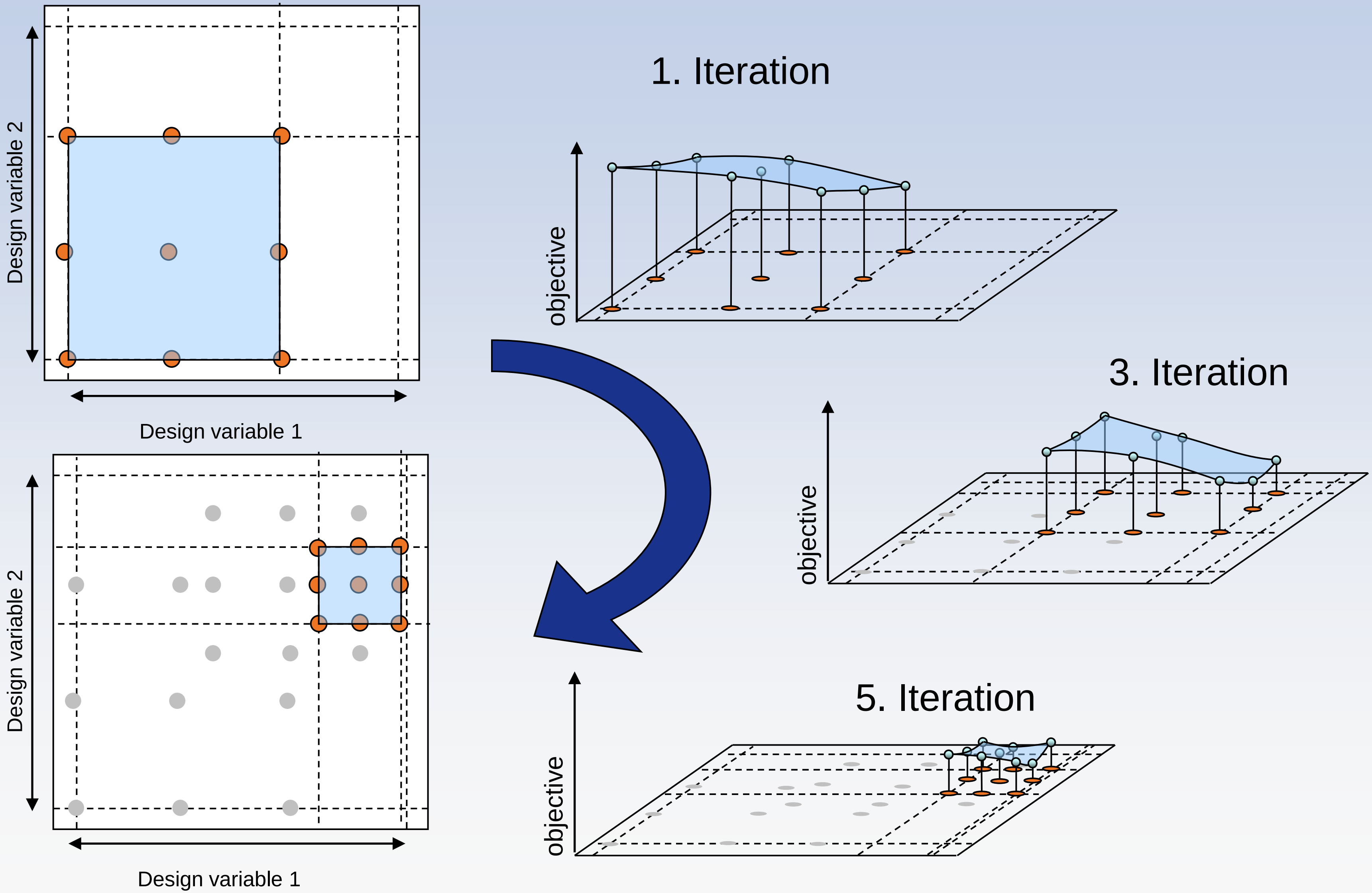


Strategy:

- Start design(s) from SA
- Pre-optimization in sub space, ARSM
- Start design(s) from ARSM
- Local improvement, EA (full space)



Adaptive Response Surface Methods (Local)



Adaptive Response Surface

ARSM with 8 performance-relevant parameters

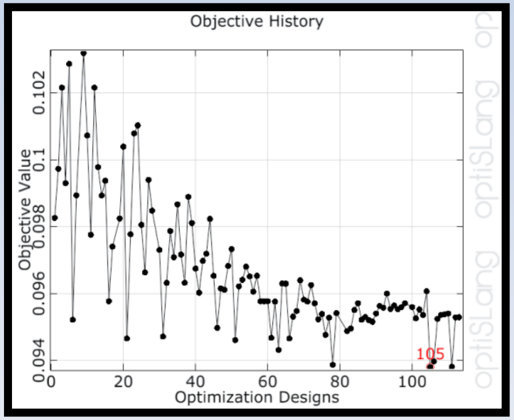
Opti | Robust | Output | Strings | Constraints | Objectives

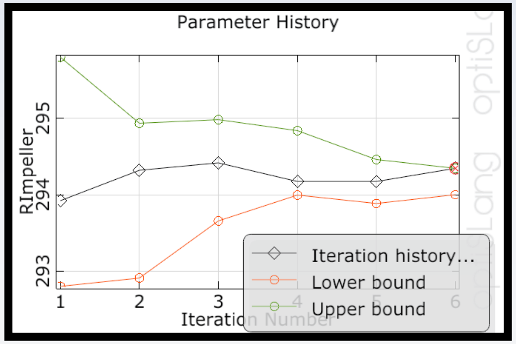
#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Type	Format	Active	Con...
1	InletWidth	53	49.237	53.5	57.5	continuous	%20.14f	[x]	[ ]
2	ExitWidth	26	26.91	26.5	28.5	continuous	%20.14f	[x]	[ ]
3	RImpeller	305	288.3	292.0	298.0	continuous	%20.14f	[x]	[ ]
4	HubBeta1	-48	-51.888	-52.5	-49.5	continuous	%20.14f	[x]	[ ]
5	HubBeta3	-25	-25.075	-27.5	-26.5	continuous	%20.14f	[x]	[ ]
6	ShdBeta1	-55	-58.355	-60.5	-59.5	continuous	%20.14f	[x]	[ ]
7	RVHubBeta1	60	64.02	62.0	66.0	continuous	%20.14f	[x]	[ ]
8	RVShdBeta1	60	69.5	60.0	64.0	continuous	%20.14f	[x]	[ ]
9	mymassin	72.6	72.6	65.34	87.119999999...	continuous	%20.14f	[ ]	[ ]
10	myomega	699.76	699.76	629.784	839.712	continuous	%20.14f	[ ]	[ ]
11	HubBeta2	-25	-26.525	-27.5	-22.5	continuous	%20.14f	[ ]	[ ]
12	ShdBeta2	-45	-42.885	-49.5	-40.5	continuous	%20.14f	[ ]	[ ]
13	ShdBeta3	-30	-30.69	-33.0	-27.0	continuous	%20.14f	[ ]	[ ]
14	HubThk1	1	0.818	0.8	1.2	continuous	%20.14f	[ ]	[ ]
15	HubThk2	6	6.01	5.0	7.0	continuous	%20.14f	[ ]	[ ]
16	ShdThk1	1	0.93	0.8	1.2	continuous	%20.14f	[ ]	[ ]
17	ShdThk2	6	6.85	5.0	7.0	continuous	%20.14f	[ ]	[ ]
18	RVHubThk1	45	61.065	45.0	66.0	continuous	%20.14f	[ ]	[ ]
19	RVShdThk1	45	38.3	35.0	55.0	continuous	%20.14f	[ ]	[ ]
20	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f	[ ]	[ ]
21	RVBlades	24	24	21.6	28.799999999...	continuous	%20.14f	[ ]	[ ]

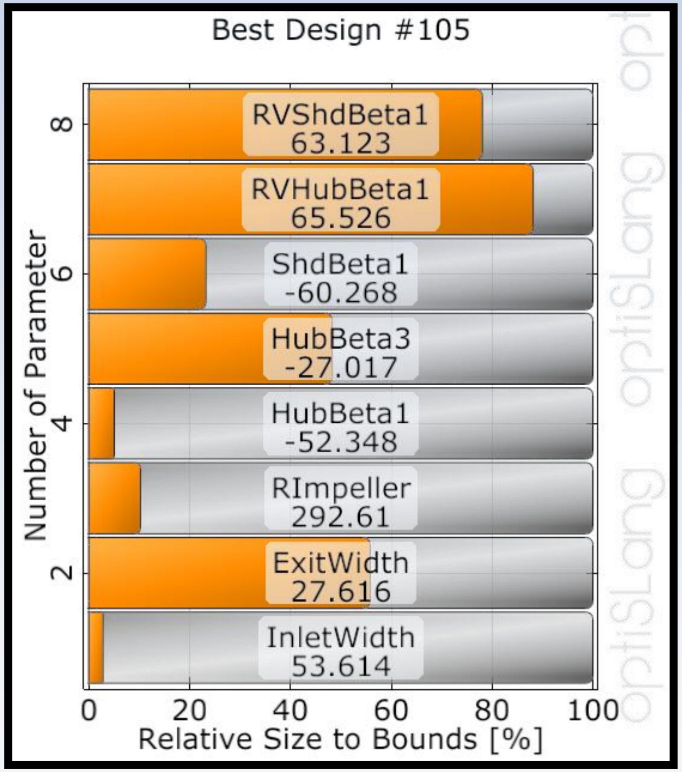
Cancel | OK

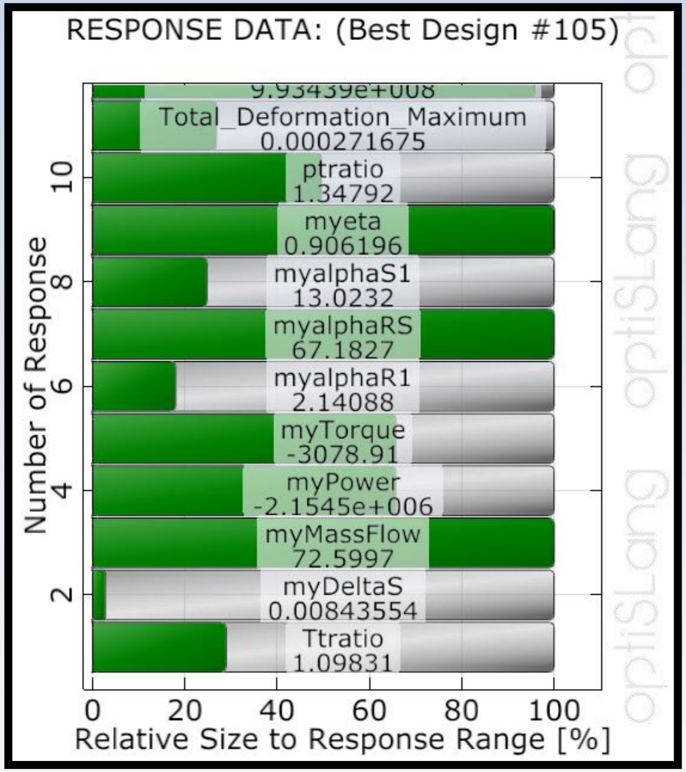
Adaptive Response Surface

ARSM leads to better design:



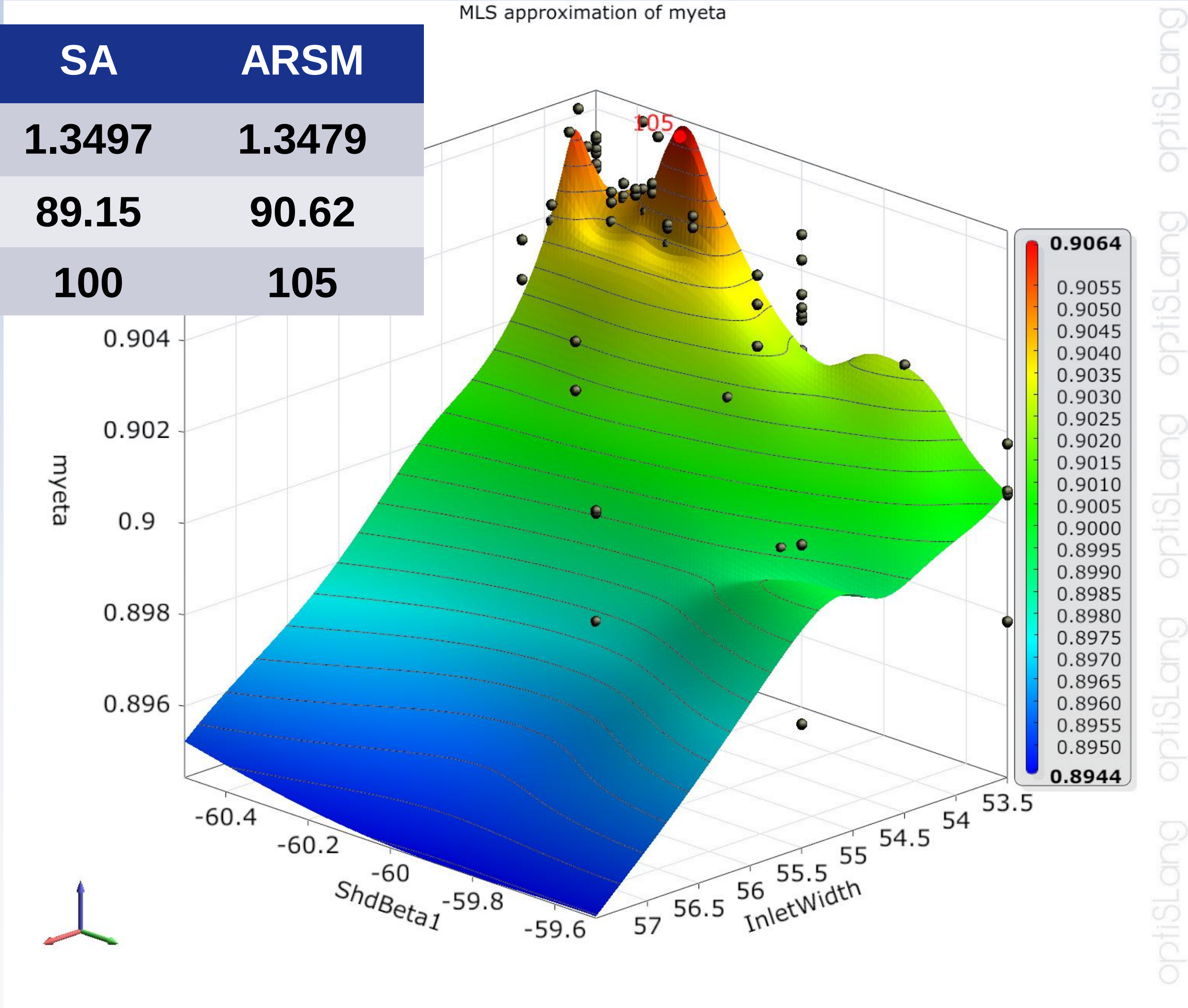






Adaptive Response Surface

	SA	ARSM
Total Pressure Ratio	1.3497	1.3479
Efficiency [%]	89.15	90.62
#Designs	100	105



Evolutionary Algorithm

EA with 17 parameters and 74 constraints leads to further improvement:

Opti | Robust | Output | Strings | Constraints | Objectives

#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Type	Format	Activ
1	InletWidth	53	53.613661065...	53.5	57.5	continuous	%20.14f	✓
2	ExitWidth	26	27.615811531...	26.5	28.5	continuous	%20.14f	✓
3	RImpeller	305	292.61382989...	292.0	298.0	continuous	%20.14f	✓
4	HubBeta1	-48	-52.34845257...	-52.5	-49.5	continuous	%20.14f	✓
5	HubBeta2	-25	-26.525	-27.5	-22.5	continuous	%20.14f	✓
6	HubBeta3	-25	-27.01713251...	-27.5	-26.5	continuous	%20.14f	✓
7	ShdBeta1	-55	-60.26762316...	-60.5	-59.5	continuous	%20.14f	✓
8	ShdBeta2	-45	-42.885	-49.5	-40.5	continuous	%20.14f	✓
9	ShdBeta3	-30	-30.69	-33.0	-27.0	continuous	%20.14f	✓
10	HubThk1	1	0.818	0.8	1.2	continuous	%20.14f	✓
11	HubThk2	6	6.01	5.0	7.0	continuous	%20.14f	✓
12	ShdThk1	1	0.93	0.8	1.2	continuous	%20.14f	✓
13	ShdThk2	6	6.85	5.0	7.0	continuous	%20.14f	✓
14	RVHubThk1	45	61.065	45.0	66.0	continuous	%20.14f	✓
15	RVHubBeta1	60	65.526394007...	62.0	66.0	continuous	%20.14f	✓
16	RVShdBeta1	60	63.123088351...	60.0	64.0	continuous	%20.14f	✓
17	RVShdThk1	45	38.3	35.0	55.0	continuous	%20.14f	✓
18	mymassin	72.6	72.6	65.34	87.119999999...	continuous	%20.14f	
19	myomega	699.76	699.76	629.784	839.712	continuous	%20.14f	
20	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f	
21	RVBlades	24	24	21.6	28.799999999...	continuous	%20.14f	

Cancel

Opti | Robust | Output | Strings | Constraints | Objectives

#	Name	Type	Formula	Active
14	rel2modeh021	ine...	fabs((6.2832*modefreq021-(2*myomega))/(2*myomega))-0.01	✓
15	rel1modeh022	ine...	fabs((6.2832*modefreq022-(1*myomega))/(1*myomega))-0.01	✓
16	rel2modeh022	ine...	fabs((6.2832*modefreq022-(2*myomega))/(2*myomega))-0.01	✓
17	rel1modeh023	ine...	fabs((6.2832*modefreq023-(1*myomega))/(1*myomega))-0.01	✓
18	rel2modeh023	ine...	fabs((6.2832*modefreq023-(2*myomega))/(2*myomega))-0.01	✓
19	rel1modeh031	ine...	fabs((6.2832*modefreq031-(1*myomega))/(1*myomega))-0.01	✓
20	rel2modeh031	ine...	fabs((6.2832*modefreq031-(2*myomega))/(2*myomega))-0.01	✓
21	rel1modeh032	ine...	fabs((6.2832*modefreq032-(1*myomega))/(1*myomega))-0.01	✓
22	rel2modeh032	ine...	fabs((6.2832*modefreq032-(2*myomega))/(2*myomega))-0.01	✓
23	rel1modeh033	ine...	fabs((6.2832*modefreq033-(1*myomega))/(1*myomega))-0.01	✓
24	rel2modeh033	ine...	fabs((6.2832*modefreq033-(2*myomega))/(2*myomega))-0.01	✓
25	rel1modeh041	ine...	fabs((6.2832*modefreq041-(1*myomega))/(1*myomega))-0.01	✓
26	rel2modeh041	ine...	fabs((6.2832*modefreq041-(2*myomega))/(2*myomega))-0.01	✓
27	rel1modeh042	ine...	fabs((6.2832*modefreq042-(1*myomega))/(1*myomega))-0.01	✓
28	rel2modeh042	ine...	fabs((6.2832*modefreq042-(2*myomega))/(2*myomega))-0.01	✓
29	rel1modeh043	ine...	fabs((6.2832*modefreq043-(1*myomega))/(1*myomega))-0.01	✓
30	rel2modeh043	ine...	fabs((6.2832*modefreq043-(2*myomega))/(2*myomega))-0.01	✓
31	rel1modeh051	ine...	fabs((6.2832*modefreq051-(1*myomega))/(1*myomega))-0.01	✓
32	rel2modeh051	ine...	fabs((6.2832*modefreq051-(2*myomega))/(2*myomega))-0.01	✓
33	rel1modeh052	ine...	fabs((6.2832*modefreq052-(1*myomega))/(1*myomega))-0.01	✓
34	rel2modeh052	ine...	fabs((6.2832*modefreq052-(2*myomega))/(2*myomega))-0.01	✓
35	rel1modeh053	ine...	fabs((6.2832*modefreq053-(1*myomega))/(1*myomega))-0.01	✓
36	rel2modeh053	ine...	fabs((6.2832*modefreq053-(2*myomega))/(2*myomega))-0.01	✓
37	rel1modeh061	ine...	fabs((6.2832*modefreq061-(1*myomega))/(1*myomega))-0.01	✓
38	rel2modeh061	ine...	fabs((6.2832*modefreq061-(2*myomega))/(2*myomega))-0.01	✓
39	rel1modeh062	ine...	fabs((6.2832*modefreq062-(1*myomega))/(1*myomega))-0.01	✓
40	rel2modeh062	ine...	fabs((6.2832*modefreq062-(2*myomega))/(2*myomega))-0.01	✓
41	rel1modeh063	ine...	fabs((6.2832*modefreq063-(1*myomega))/(1*myomega))-0.01	✓
42	rel2modeh063	ine...	fabs((6.2832*modefreq063-(2*myomega))/(2*myomega))-0.01	✓
43	rel1modeh071	ine...	fabs((6.2832*modefreq071-(1*myomega))/(1*myomega))-0.01	✓
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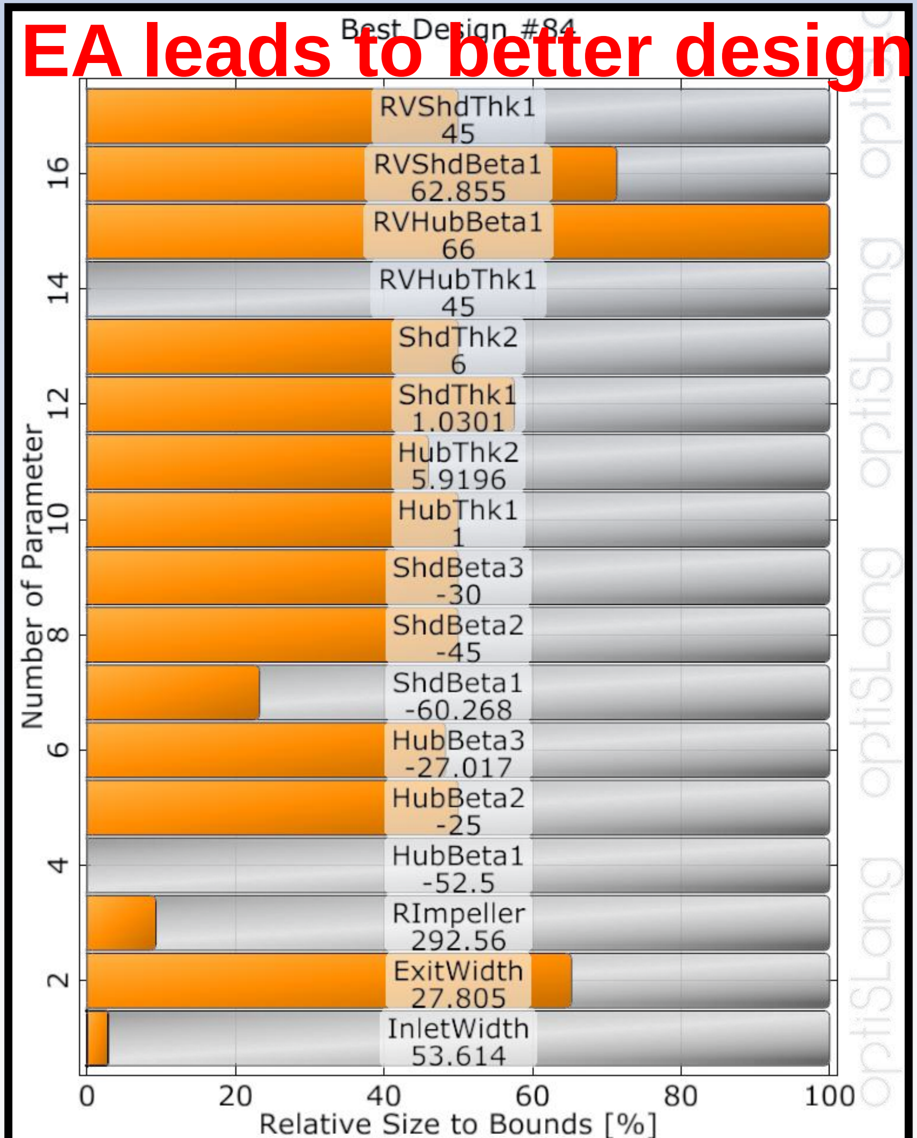
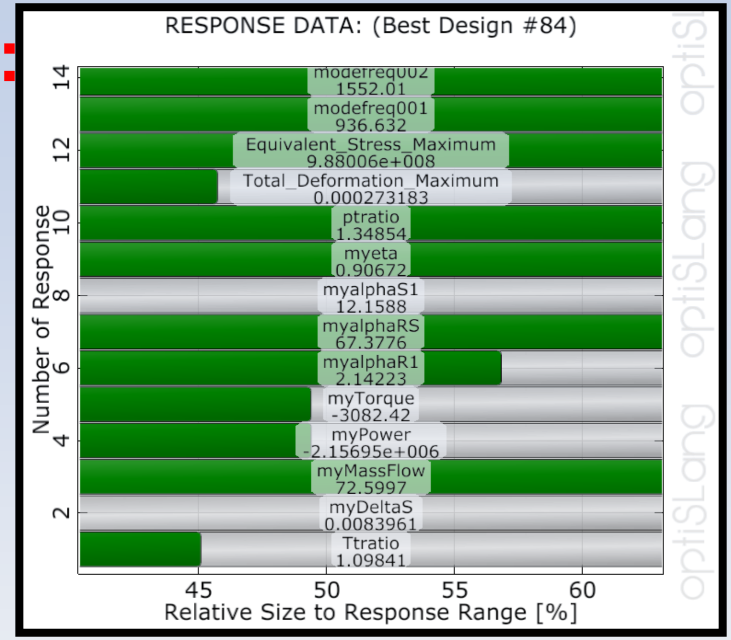
49	rel2modeh081	ine...	fabs((6.2832*modefreq081-(2*myomega))/(2*myomega))-0.01	✓
50	rel1modeh082	ine...	fabs((6.2832*modefreq082-(1*myomega))/(1*myomega))-0.01	✓
51	rel1modeh081	ine...	fabs((6.2832*modefreq081-(1*myomega))/(1*myomega))-0.01	✓
52	rel2modeh082	ine...	fabs((6.2832*modefreq082-(2*myomega))/(2*myomega))-0.01	✓
53	rel1modeh083	ine...	fabs((6.2832*modefreq083-(1*myomega))/(1*myomega))-0.01	✓
54	rel2modeh083	ine...	fabs((6.2832*modefreq083-(2*myomega))/(2*myomega))-0.01	✓
55	rel2modeh091	ine...	fabs((6.2832*modefreq091-(2*myomega))/(2*myomega))-0.01	✓
56	rel1modeh092	ine...	fabs((6.2832*modefreq092-(1*myomega))/(1*myomega))-0.01	✓
57	rel1modeh091	ine...	fabs((6.2832*modefreq091-(1*myomega))/(1*myomega))-0.01	✓
58	rel2modeh092	ine...	fabs((6.2832*modefreq092-(2*myomega))/(2*myomega))-0.01	✓
59	rel1modeh093	ine...	fabs((6.2832*modefreq093-(1*myomega))/(1*myomega))-0.01	✓
60	rel2modeh093	ine...	fabs((6.2832*modefreq093-(2*myomega))/(2*myomega))-0.01	✓
61	rel2modeh101	ine...	fabs((6.2832*modefreq101-(2*myomega))/(2*myomega))-0.01	✓
62	rel1modeh102	ine...	fabs((6.2832*modefreq102-(1*myomega))/(1*myomega))-0.01	✓
63	rel1modeh101	ine...	fabs((6.2832*modefreq101-(1*myomega))/(1*myomega))-0.01	✓
64	rel2modeh102	ine...	fabs((6.2832*modefreq102-(2*myomega))/(2*myomega))-0.01	✓
65	rel1modeh103	ine...	fabs((6.2832*modefreq103-(1*myomega))/(1*myomega))-0.01	✓
66	rel2modeh103	ine...	fabs((6.2832*modefreq103-(2*myomega))/(2*myomega))-0.01	✓
67	rel2modeh111	ine...	fabs((6.2832*modefreq111-(2*myomega))/(2*myomega))-0.01	✓
68	rel1modeh112	ine...	fabs((6.2832*modefreq112-(1*myomega))/(1*myomega))-0.01	✓
69	rel1modeh111	ine...	fabs((6.2832*modefreq111-(1*myomega))/(1*myomega))-0.01	✓
70	rel2modeh112	ine...	fabs((6.2832*modefreq112-(2*myomega))/(2*myomega))-0.01	✓
71	rel1modeh113	ine...	fabs((6.2832*modefreq113-(1*myomega))/(1*myomega))-0.01	✓
72	rel2modeh113	ine...	fabs((6.2832*modefreq113-(2*myomega))/(2*myomega))-0.01	✓
73	maxptratio	ine...	1.36-ptratio	✓
74	minptratio	ine...	ptratio-1.34	✓

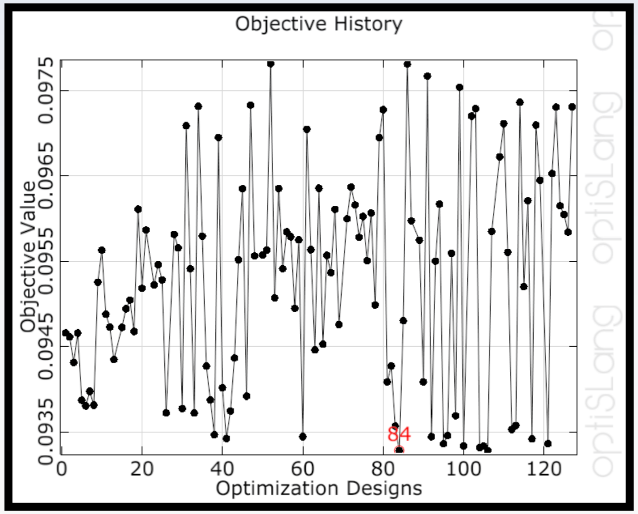
Cancel | OK

Evolutionary Algorithm

EA leads to better design:

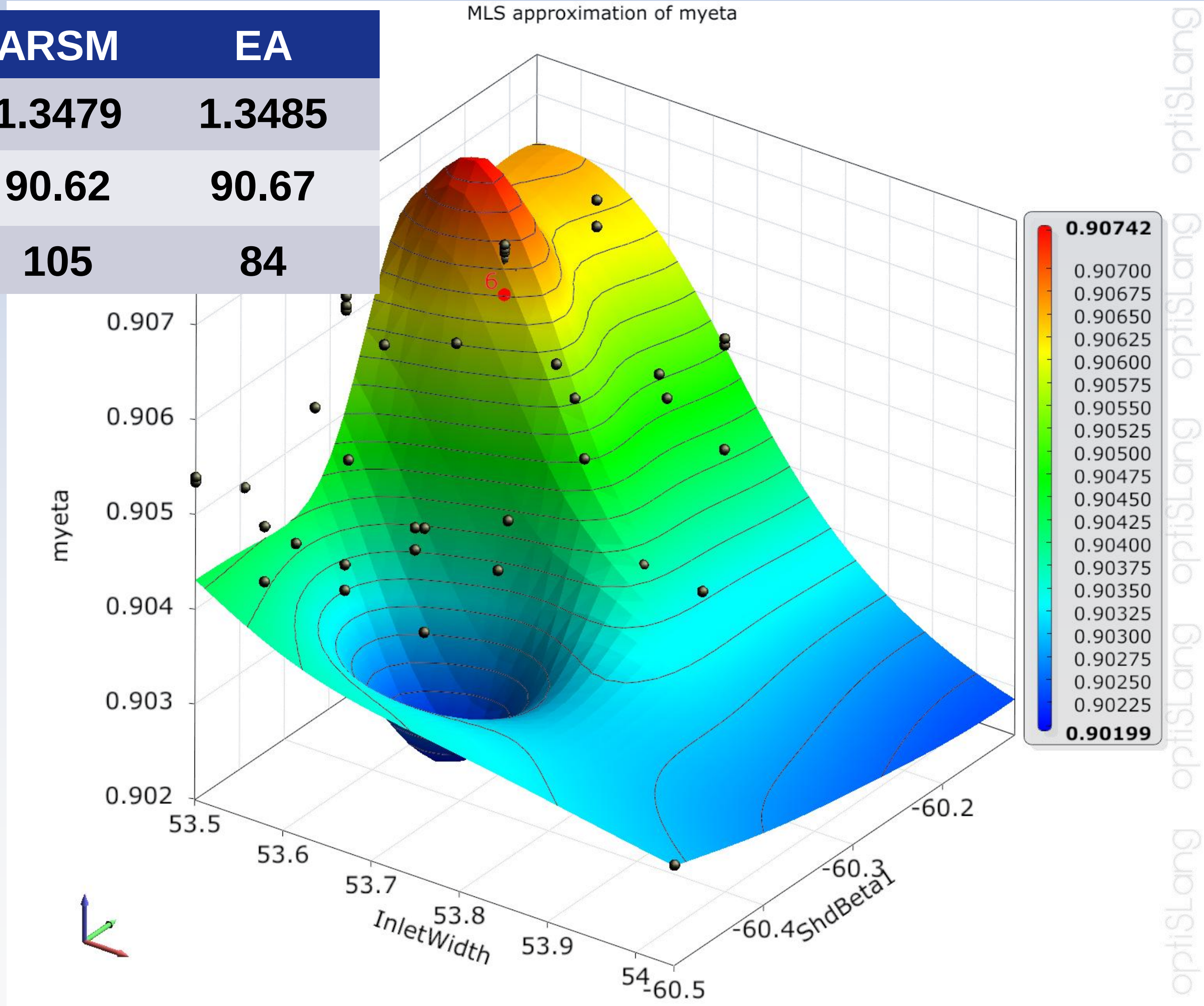






Evolutionary Algorithm

	ARSM	EA
Total Press. Ratio	1.3479	1.3485
Efficiency [%]	90.62	90.67
#Designs	105	84

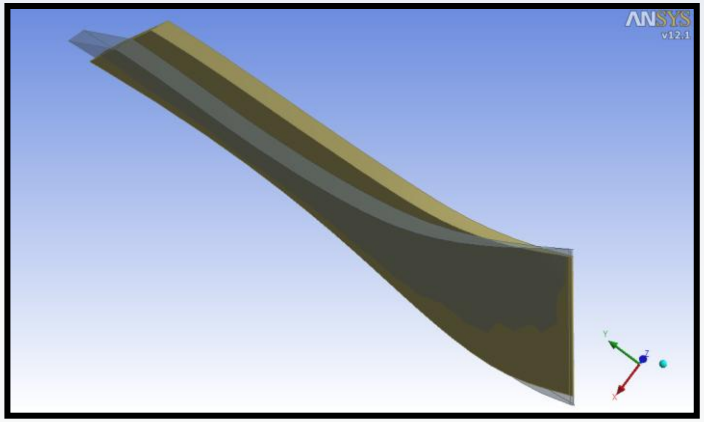


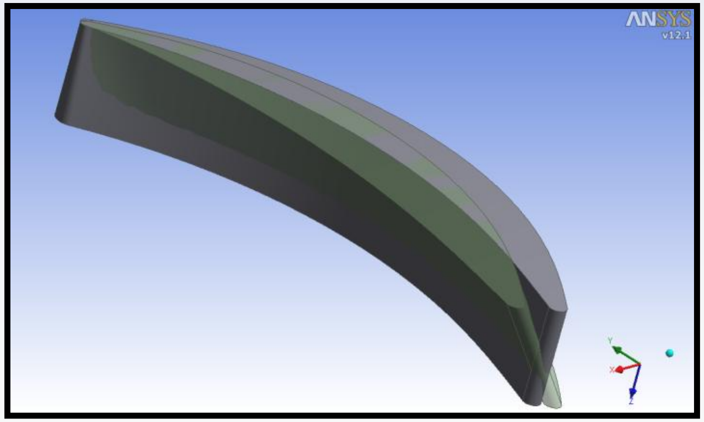
- Additional Results
  - RVHubThk1 ->35
  - RVShdThk1 -> 35
  - RVHubBeta1 -> 68
  - HubBeta1 -> -55
  - InletWidth -> 52.5
  - myomega 699 - 703

Conclusion Optimization

- Sensitivity shows optimization potential
- Pre-Optimization, ARSM, increases quality
- EA leads to further improvement

	Initial	SA	ARSM	EA
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485
Efficiency [%]	86.72	89.15	90.62	90.67
#Designs	-	100	105	84





Process Integration

Sensitivity Analysis

Design Optimization

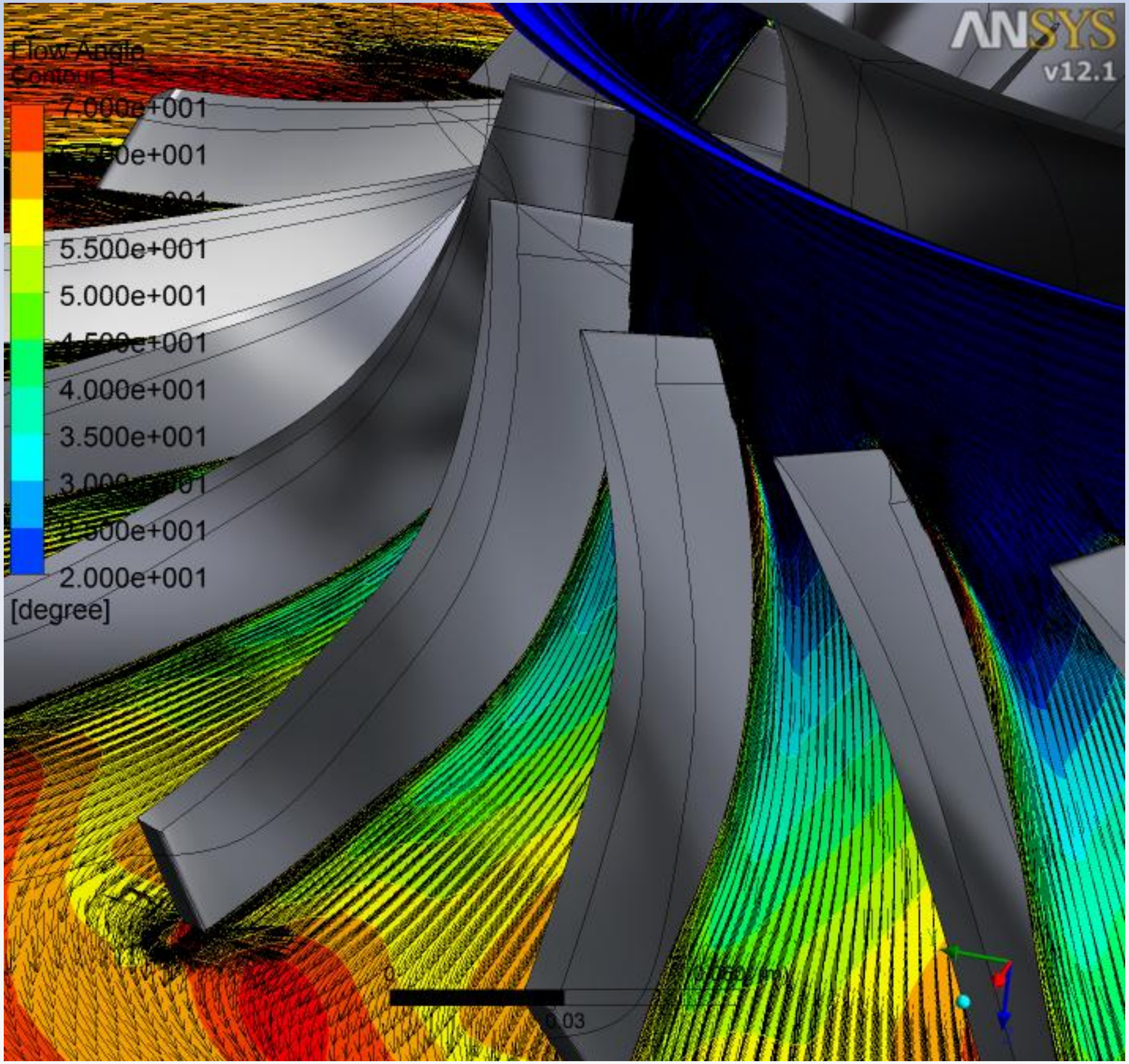
Robustness Evaluation

Random Fields

Robust Design Optimization

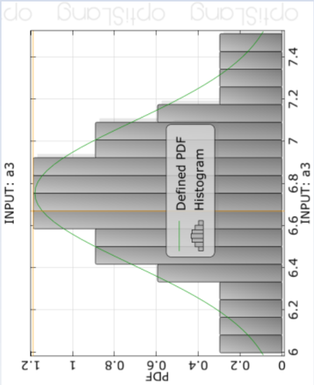
Reliability Analysis

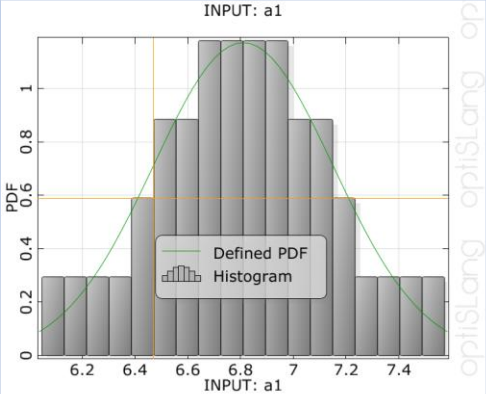
Application of stochastic and robustness evaluation

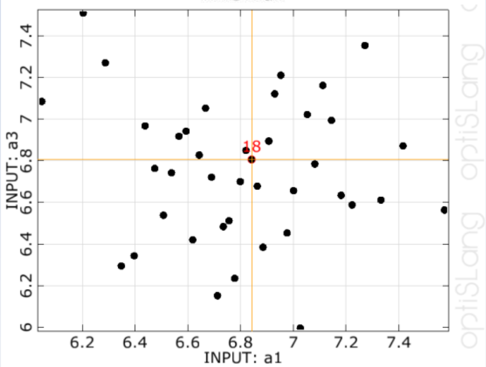


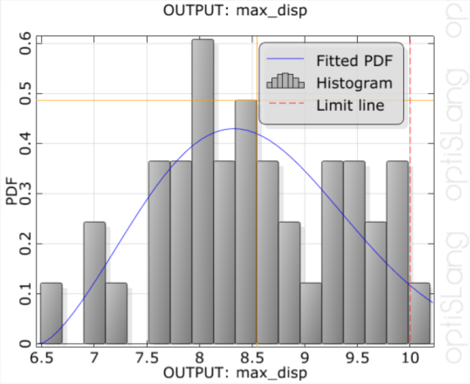
- Analysis models become increasingly detailed
- Numerical procedures become more and more complex
- Substantially more precise data are required for the analysis
- Deterministic optimum design is frequently pushed to the design space boundaries
- Optimized designs lead to high imperfection sensitivities
- Optimized designs tend to loose robustness
- But assessment of design robustness, reliability and safety will be more and more important
- Because of that integration of optimization and stochastic analysis methods will be necessary

What does robustness evaluation mean?





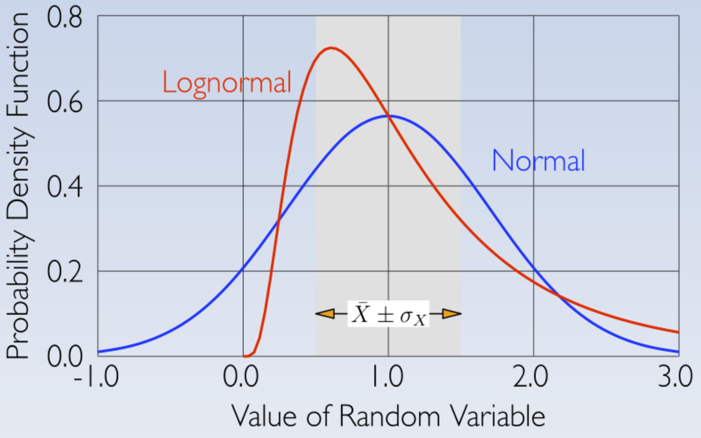




- **Actual, robustness evaluation means**
  - global variance-based sensitivity analysis
  - of computational design models
  - according to random parameters with
  - input correlation and
  - specification of random parameter sensitivities for stochastic analysis and
  - identification of non-robust behavior.
- **But in many cases the non-robustness of**
  - computational methods
  - computational software
  - computational hardware
  - ...
- **is involved.**

Statistical data

- Where to get statistical data**?**
  - From lab testing (always best)
  - From measurements of suppliers/manufacturers
  - From technical references (mostly only mean values provides)
  - Estimation - mostly Gaussian or lognormal distribution standard deviation of about ±10%
- Typical scatter for metallic materials
  - Young's modulus - Gaussian with a standard deviation of about ±3-5%
  - Shear modulus - Gaussian with a standard deviation of about ±4-8%
  - Poisson's ratio - Lognormal with a standard deviation of about ±10-20%
  - Density - Gaussian with a standard deviation of about ±1-6%
  - Thermal expansion coeff. - Gaussian with a standard deviation of about ±4-5%
  - Heat conductivity - Gaussian with a standard deviation of about ±3-5%
  - Heat capacity - Gaussian with a standard deviation of about ±5-7%
  - Yield strength Rp0.2 - Lognormal with a standard deviation of ±5%



Uncertainties and Tolerances

Property	SD/Mean %
Metallic materiales, yield	15
Carbon fiber composites, rupture	17
Metallic shells, buckling strength	14
Junction by screws, rivet, welding	8
Bond insert, axial load	12
Honeycomb, tension	16
Honeycomb, shear, compression	10
Honeycomb, face wrinkling	8
Launch vehicle , thrust	5
Transient loads	50
Thermal loads	7.5
Deployment shock	10
Acoustic loads	40
Vibration loads	20

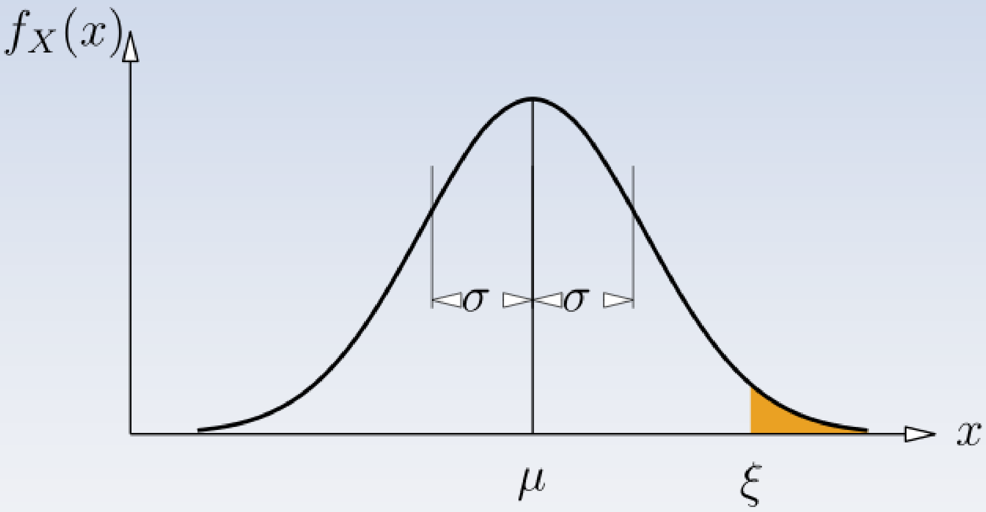
- **Design**
- **Material, geometry, loads, constrains,…**
- **Manufacturing**
- **Operating processes (misuse)**
- **Resulting from Deterioration**

…

[Klein, Schueller et.al. Probabilistic Approach to Structural Factors of Safety in Aerospace. Proc. CNES Spacecraft Structures and Mechanical Testing Conf., Paris 1994]

Exceedance Probability

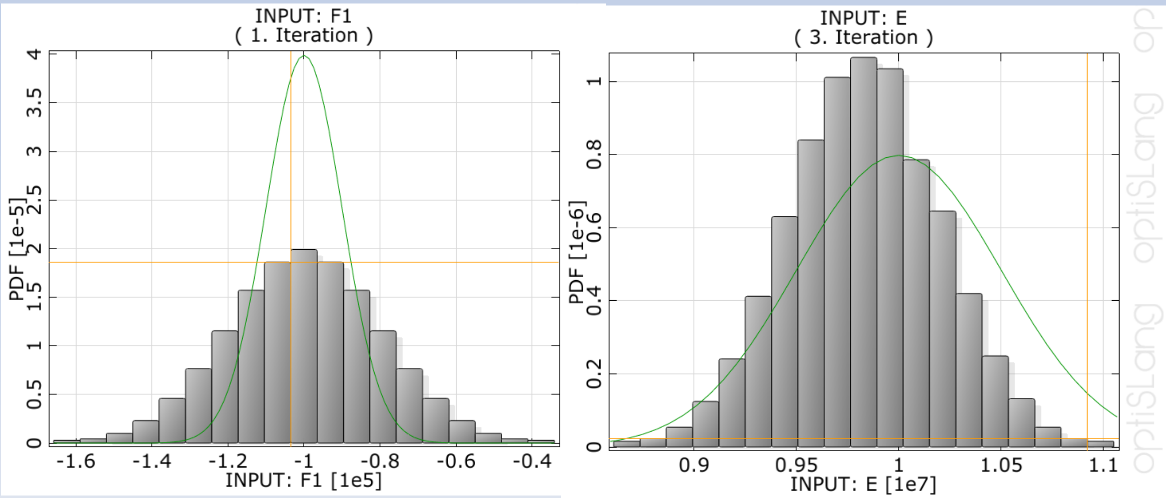
- Probability of reaching values above a threshold for Gaussian distribution

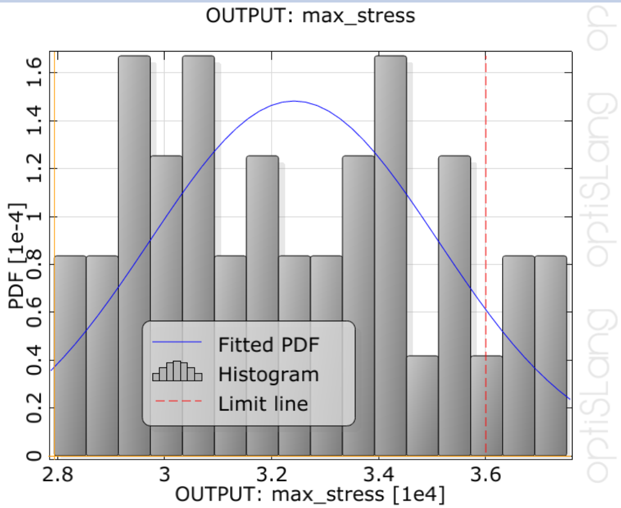


$$P_\xi = P[X \geq \xi]$$

ξ	μ	$\mu + \sigma$	$\mu + 2\sigma$	$\mu + 3\sigma$	$\mu + 4\sigma$
P_ξ	$5 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	$2.3 \cdot 10^{-2}$	$1.4 \cdot 10^{-3}$	$3.2 \cdot 10^{-5}$

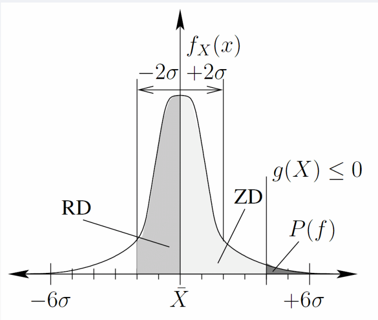
Benefit of robustness evaluation





Statistic data			
Min:	2.794e+04	Max:	3.751e+04
Mean:	3.242e+04	Sigma:	2691
CV:	0.08302		
Skewness:	0.2302	Kurtosis:	1.967
Fitted PDF: Normal			
Mean:	3.242e+04	Sigma:	2691
Limit x = 3.6e+04			
P_rel =	0.9	P_fit =	0.908393

- Identification of non-robustness
  - variance of the responses increases
  - moving of the mean values
  - responses exceed the limit states
  - undesired outliers
  - system failures
    (structural buckling, system resonances, etc.)
- Identification the most-relevant random parameters to reduce the stochastic problem
- Estimation of the Sigma Level



Random variables of robustness evaluation

Opti | Robust | Output | Strings | Constraints | Objectives

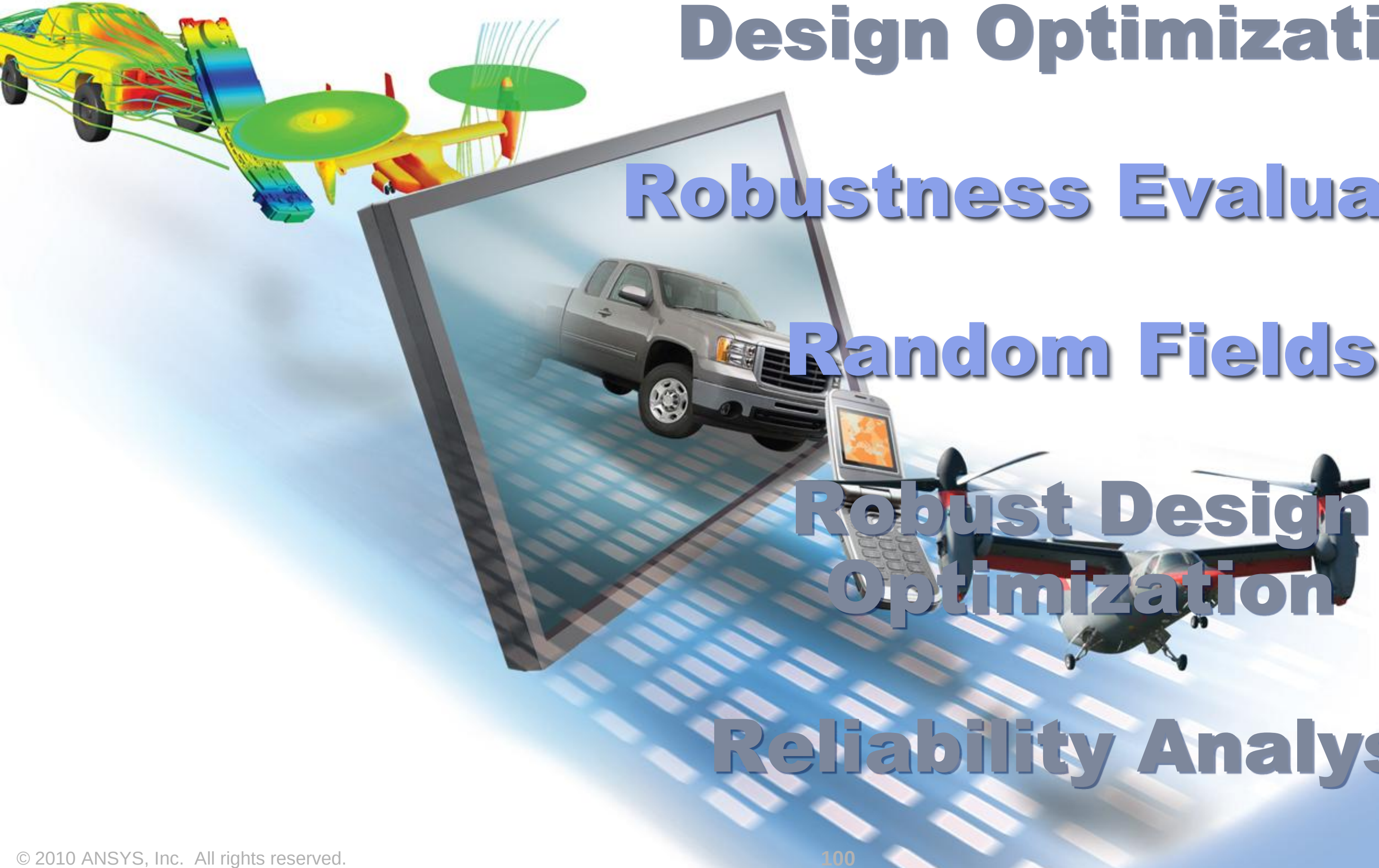
#	Name	Distribution	Mean	CoV	Stddev	Lower Cut	Upper...	For...	A...	C...
1	Ttin	Normal	313.0	0.0050	1.565	-	-	%...	☑	☐
2	myAirCP	Normal	1004.4	0.0050	5.022	-	-	%...	☑	☐
3	myAirR	Normal	287.1	0.0050	1.435500000000...	-	-	%...	☑	☐
4	mymassin	Normal	72.6	0.0050	0.363	-	-	%...	☑	☐
5	myomega	Normal	699.76	0.0050	3.4988	-	-	%...	☑	☐
6	ptin	Normal	1724000.0	0.0050	8620.0	-	-	%...	☑	☐
7	InletWidth	Normal	53.61366106578...	0.005000...	0.268068305328...	-	-	%...	☑	☐
8	ExitWidth	Normal	27.8049298398772	0.0050	0.139024649199...	-	-	%...	☑	☐
9	RImpeller	Normal	292.56	0.0050	1.4628	-	-	%...	☑	☐
10	HubBeta1	Normal	-52.5	-0.0050	0.2625	-	-	%...	☑	☐
11	HubBeta2	Normal	-25.0	-0.0050	0.125	-	-	%...	☑	☐
12	HubBeta3	Normal	-27.01713251922...	-0.0050	0.135085662596...	-	-	%...	☑	☐
13	ShdBeta1	Normal	-60.26762316109...	-0.0050	0.301338115805...	-	-	%...	☑	☐
14	ShdBeta2	Normal	-45.0	-0.0050	0.225	-	-	%...	☑	☐
15	ShdBeta3	Normal	-30.0	-0.0050	0.15	-	-	%...	☑	☐
16	HubThk1	Normal	1.0	0.0050	0.0050	-	-	%...	☑	☐
17	HubThk2	Normal	5.9196	0.0050	0.029598	-	-	%...	☑	☐
18	ShdThk1	Normal	1.0301	0.0050	0.005150500000...	-	-	%...	☑	☐
19	ShdThk2	Normal	6.0	0.05	0.300000000000...	-	-	%...	☑	☐
20	RVHubThk1	Normal	45.0	0.05	2.25	-	-	%...	☑	☐
21	RVHubBeta1	Normal	66.0	0.05	3.300000000000...	-	-	%...	☑	☐
22	RVShdBeta1	Normal	62.85486468353...	0.05	3.142743234176...	-	-	%...	☑	☐
23	RVShdThk1	Normal	45.0	0.05	2.25	-	-	%...	☑	☐
24	Density	Normal	7850.0	0.05	392.5	-	-	%...	☑	☐
25	Youngs_Modulus	Normal	2.0E11	0.05	1.0E10	-	-	%...	☑	☐
26	Poissons_Ratio	Log-Normal	0.3	0.1	0.03	-	-	%...	☑	☐

Cancel | OK

Process Integration

Sensitivity Analysis

Design Optimization



Robustness Evaluation

Random Fields

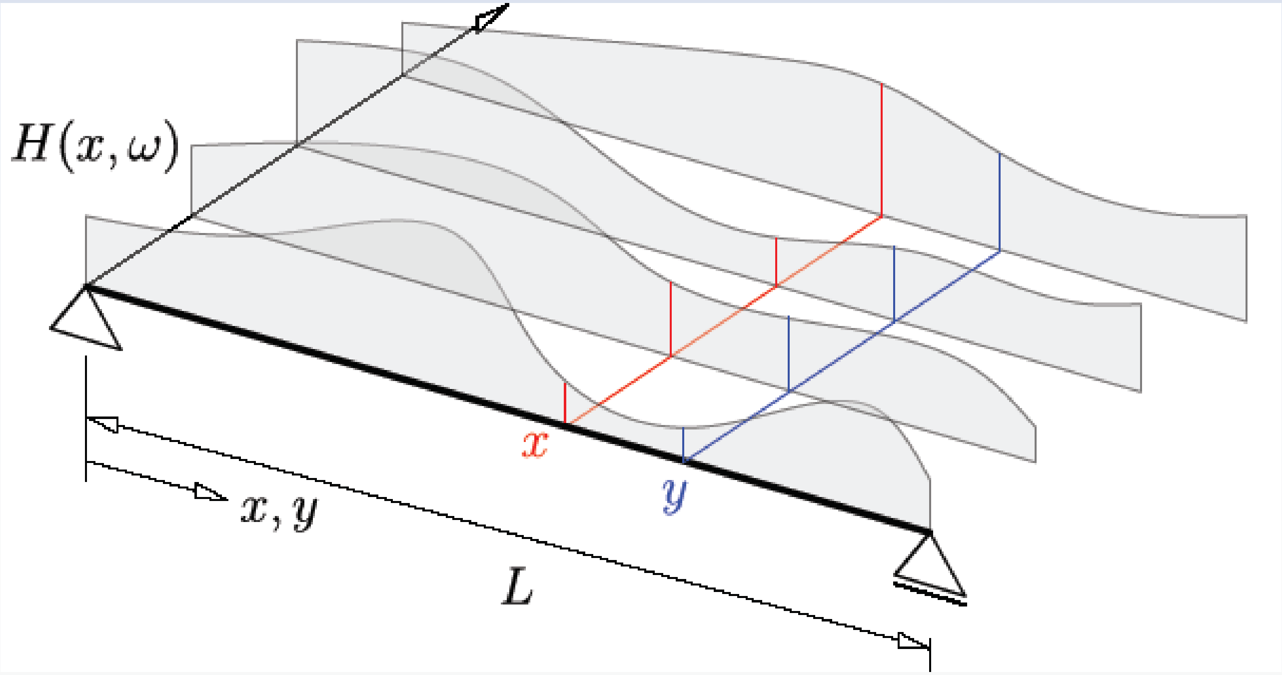
Robust Design Optimization

Reliability Analysis

Random Fields: Background

What is a random field?

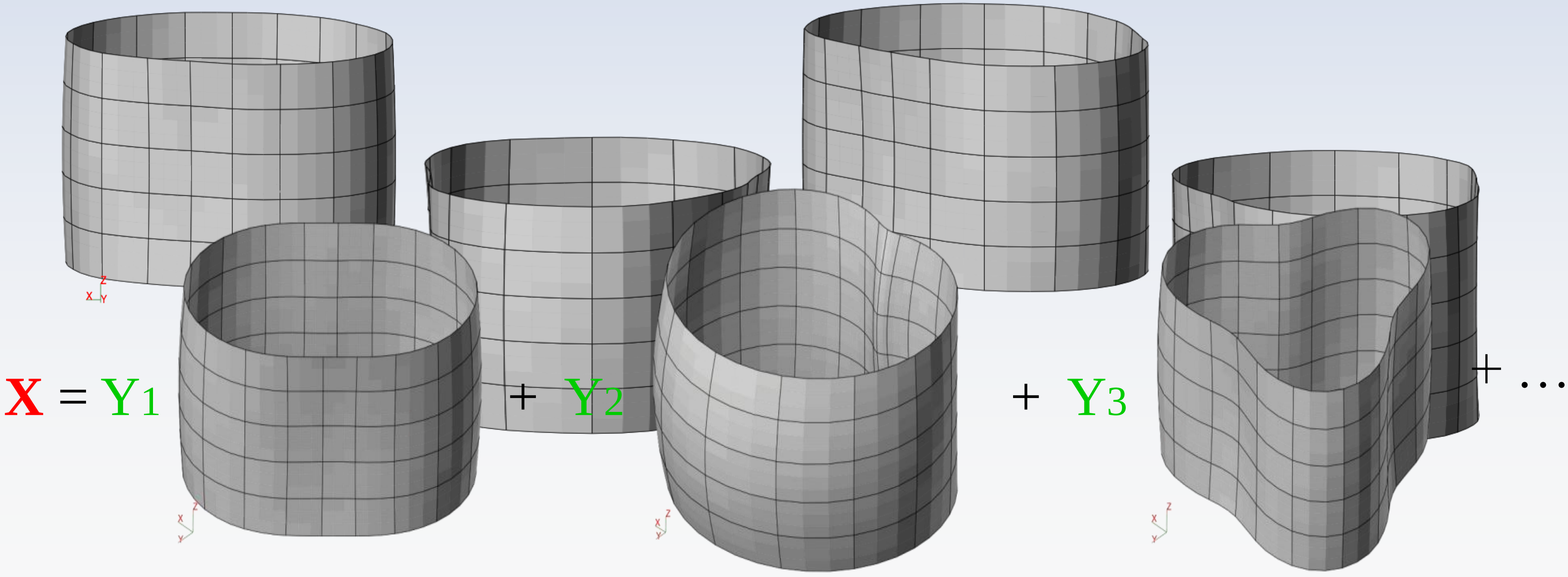
- A random function, defined on a structure, which takes random values at any location. One outcome is called realization, the set of all realizations is called ensemble.
- Stochastic properties at each point are defined by stochastic moments (mean, standard deviation …) and distribution type.
- Dependency between different locations is defined by the correlation function.



Random Fields: Background

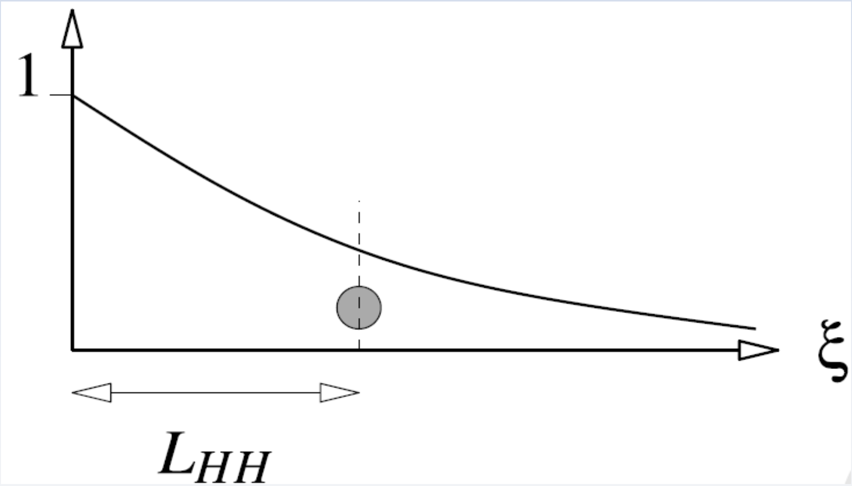
Random Fields Simulation:

- Measured Data
- Decomposition of the Covariance Matrix, the Random Field is expanded as a series of deterministic Shape Functions and Random Amplitudes



Theoretical Basics

The correlation coefficient function is a function of the distance between two points. It is characterized by the correlation length

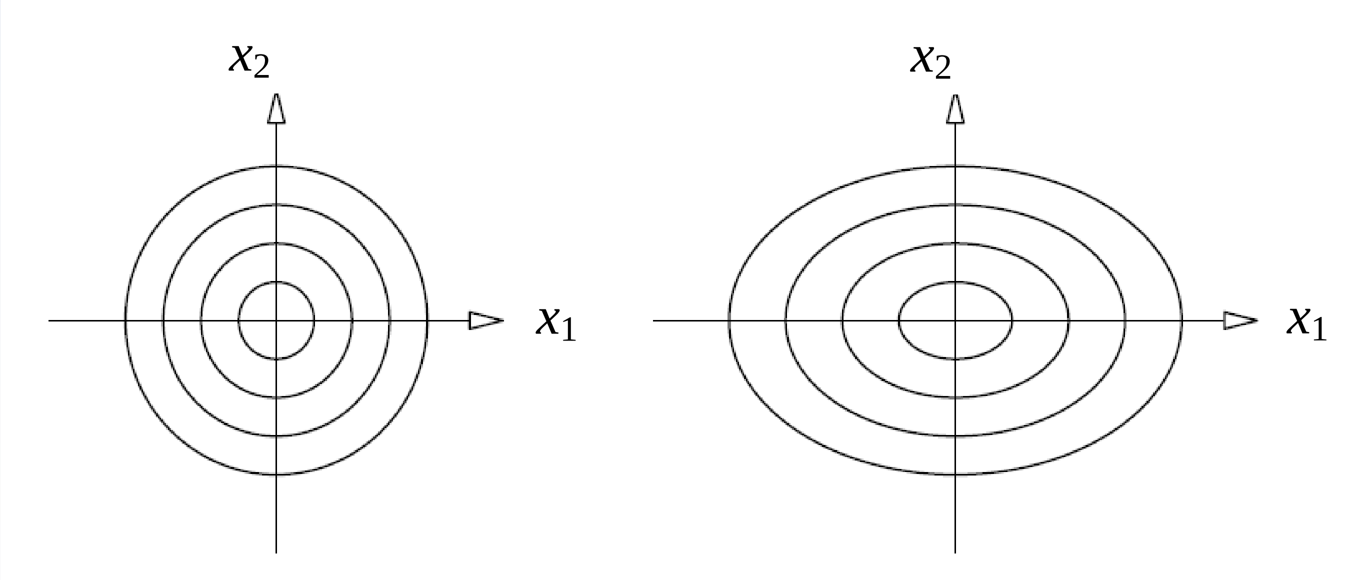


The correlation function must be positive semi-definite. Examples: exponential, triangular.

Theoretical Basics

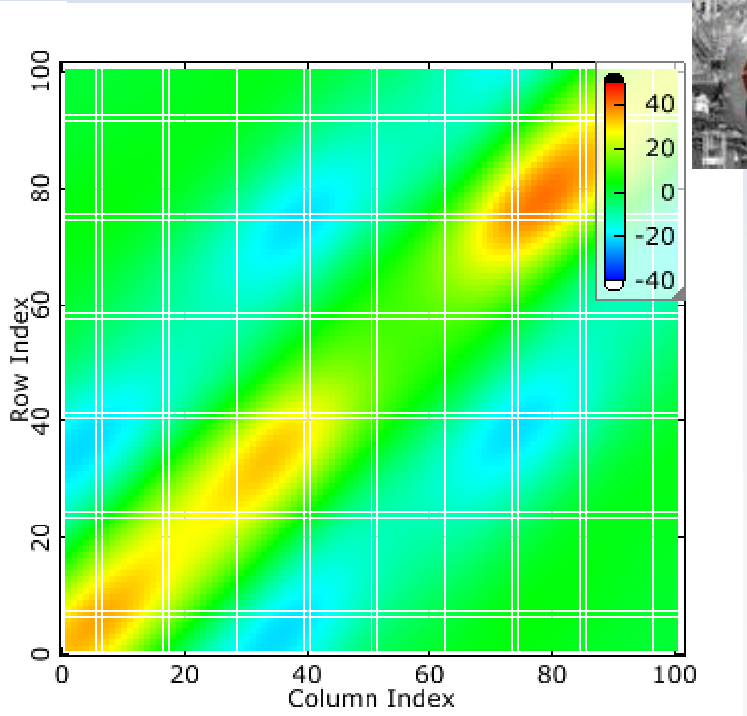
Special cases:

- Normal distribution type: the random field is characterized completely by mean and covariance function
- Homogeneity: same stochastic properties at any point throughout the structure
- Isotropy: correlation depends on the distance between two points, not the direction
- Zero means: correlation function and covariance function are identical

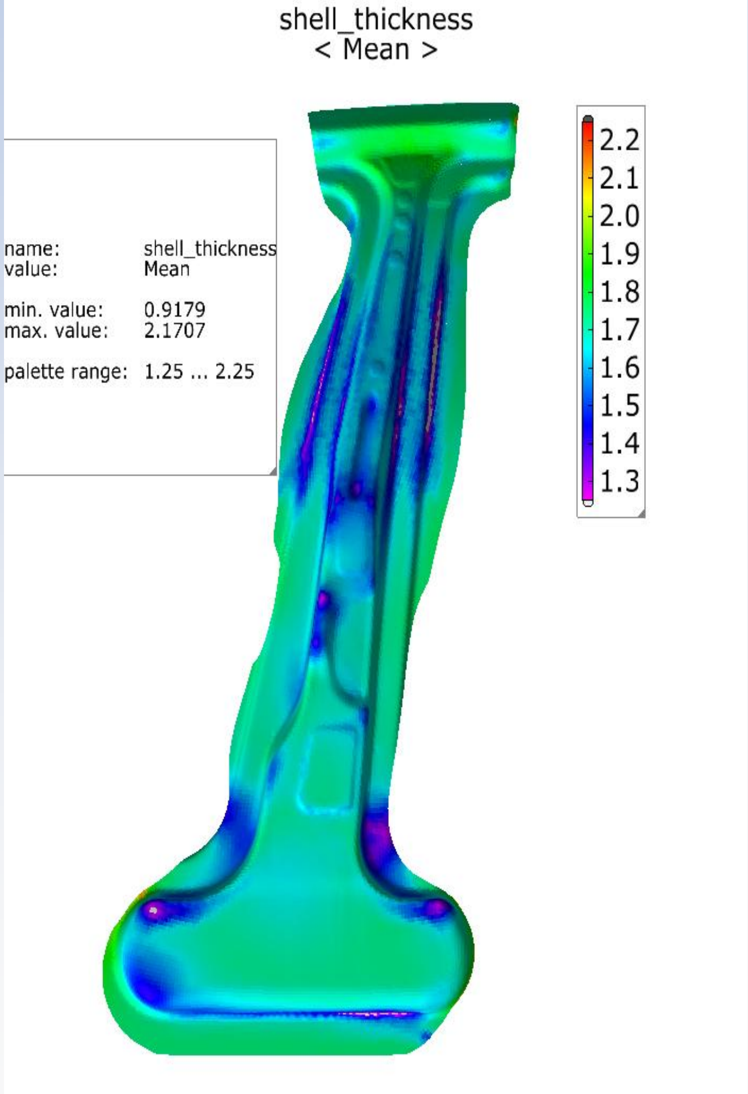


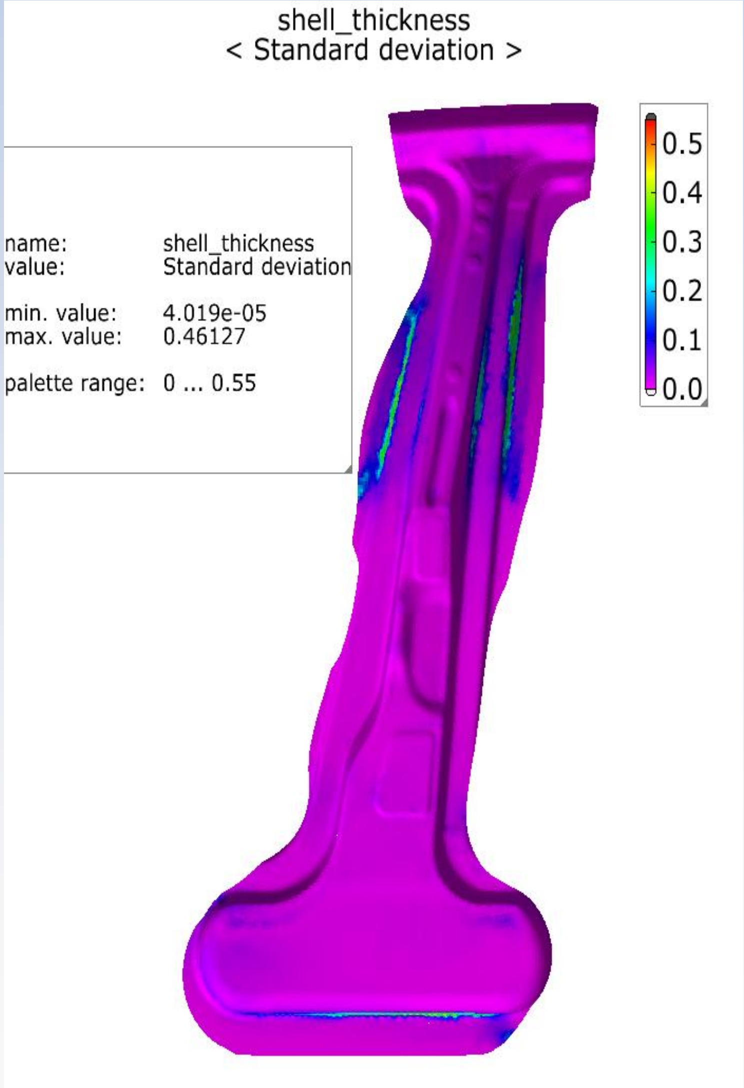
Measure Spatial Deviations

- Using 3D digitizer
- Based on the principle of triangulation, projected fringe patterns are observed with cameras, 3D coordinates for each camera pixel are calculated, a polygon mesh of the object's surface is generated
- Deviations from CAD geometry can be calculated

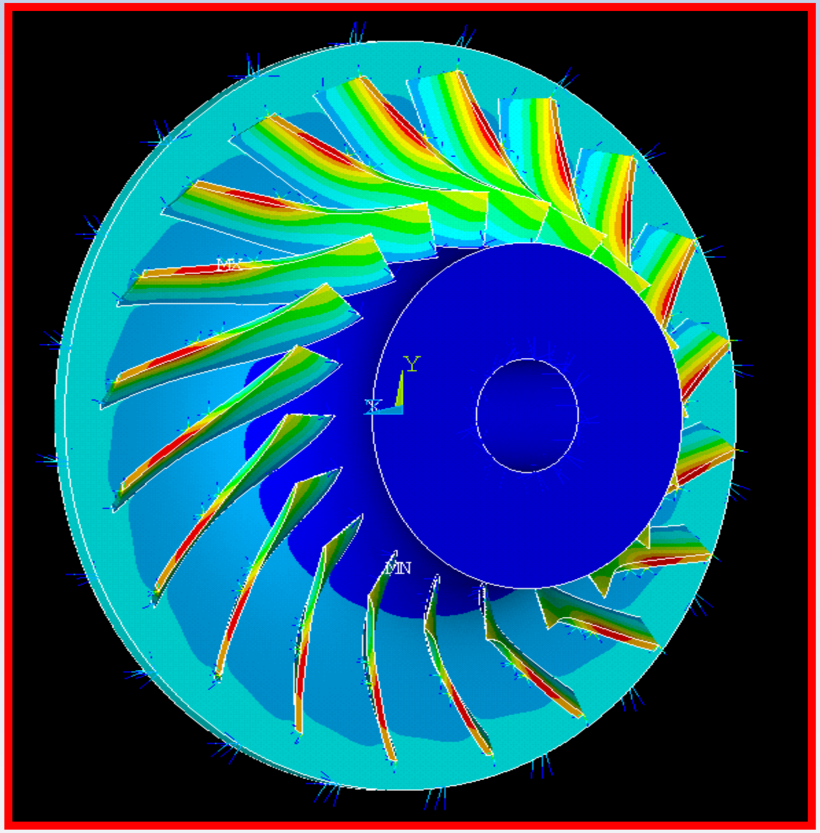


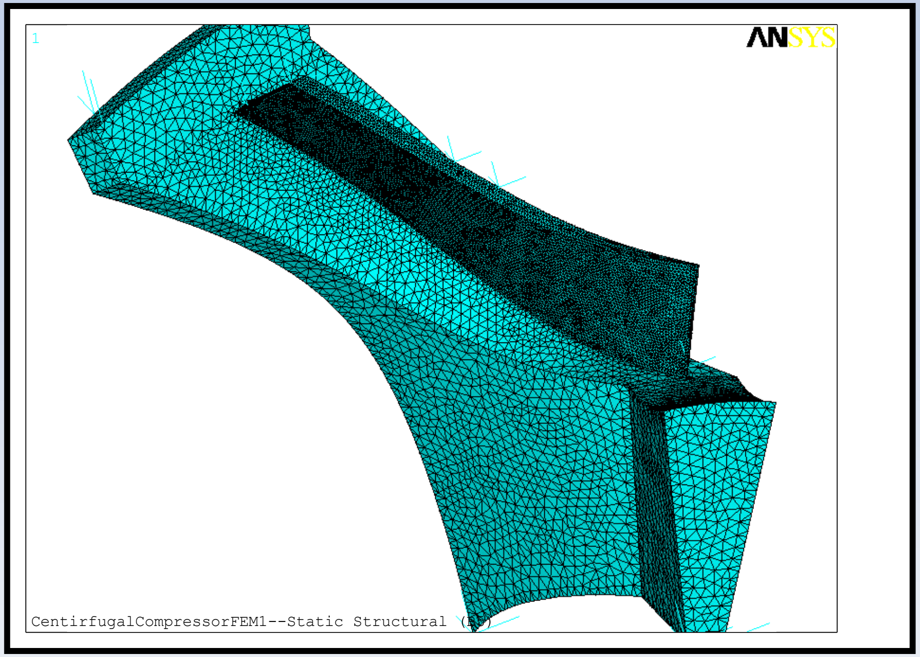
Modeling Spatial Deviations

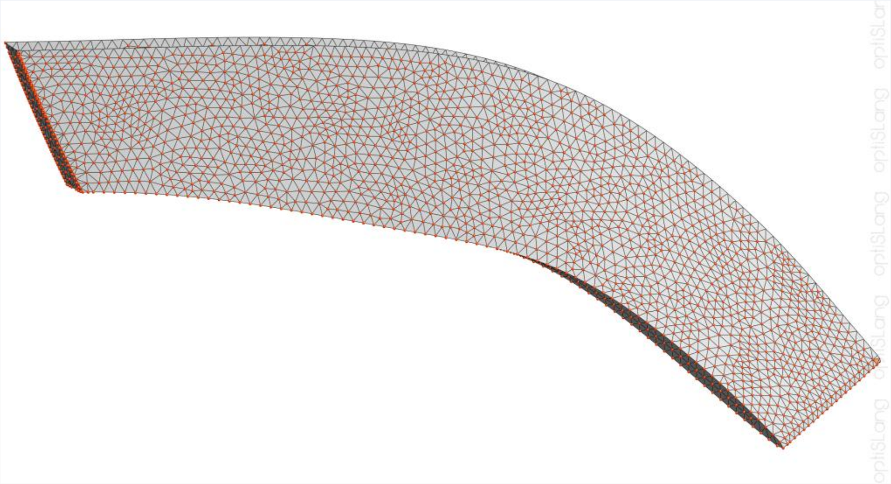




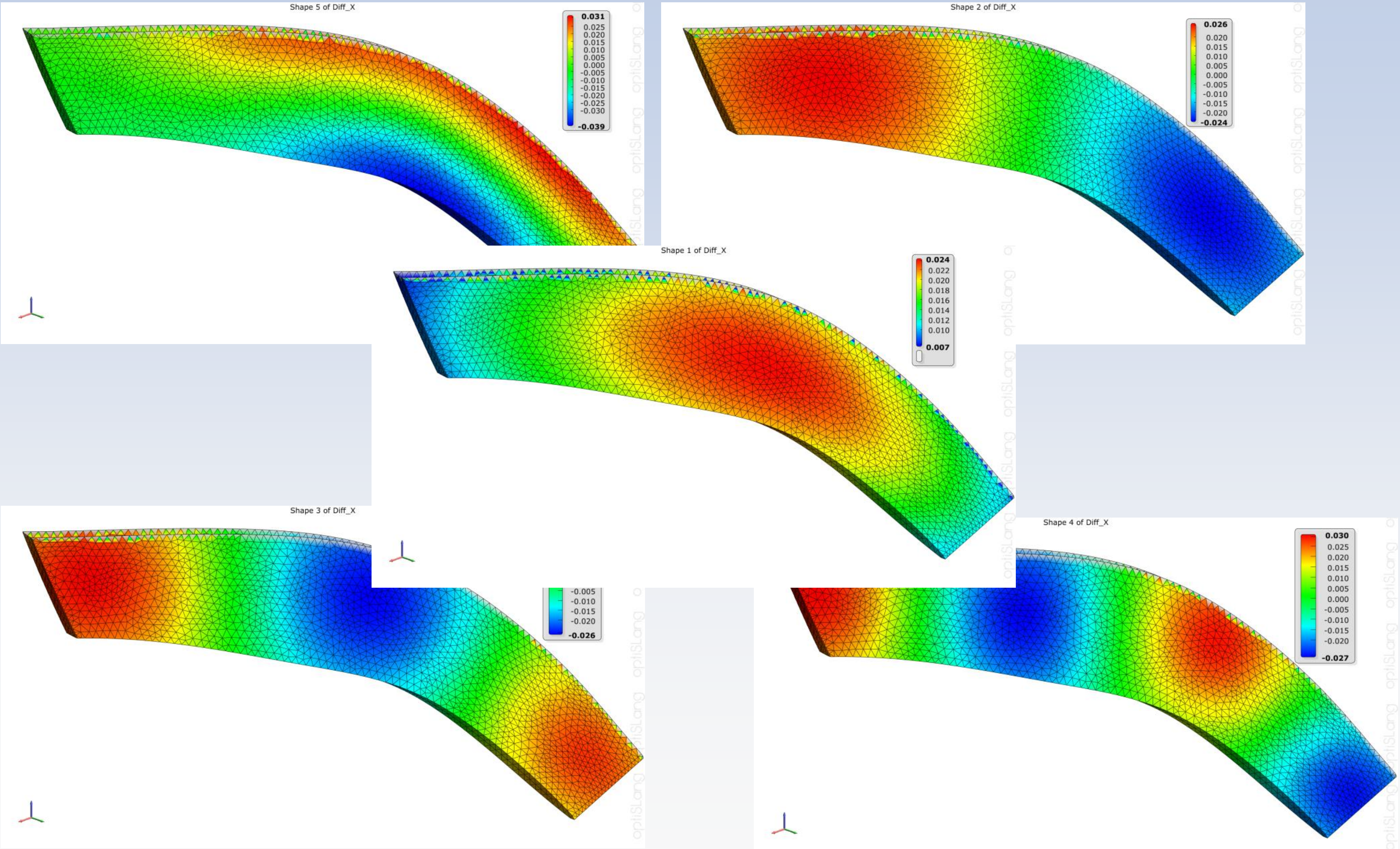
Random field modeling







Imperfection modes of the random field

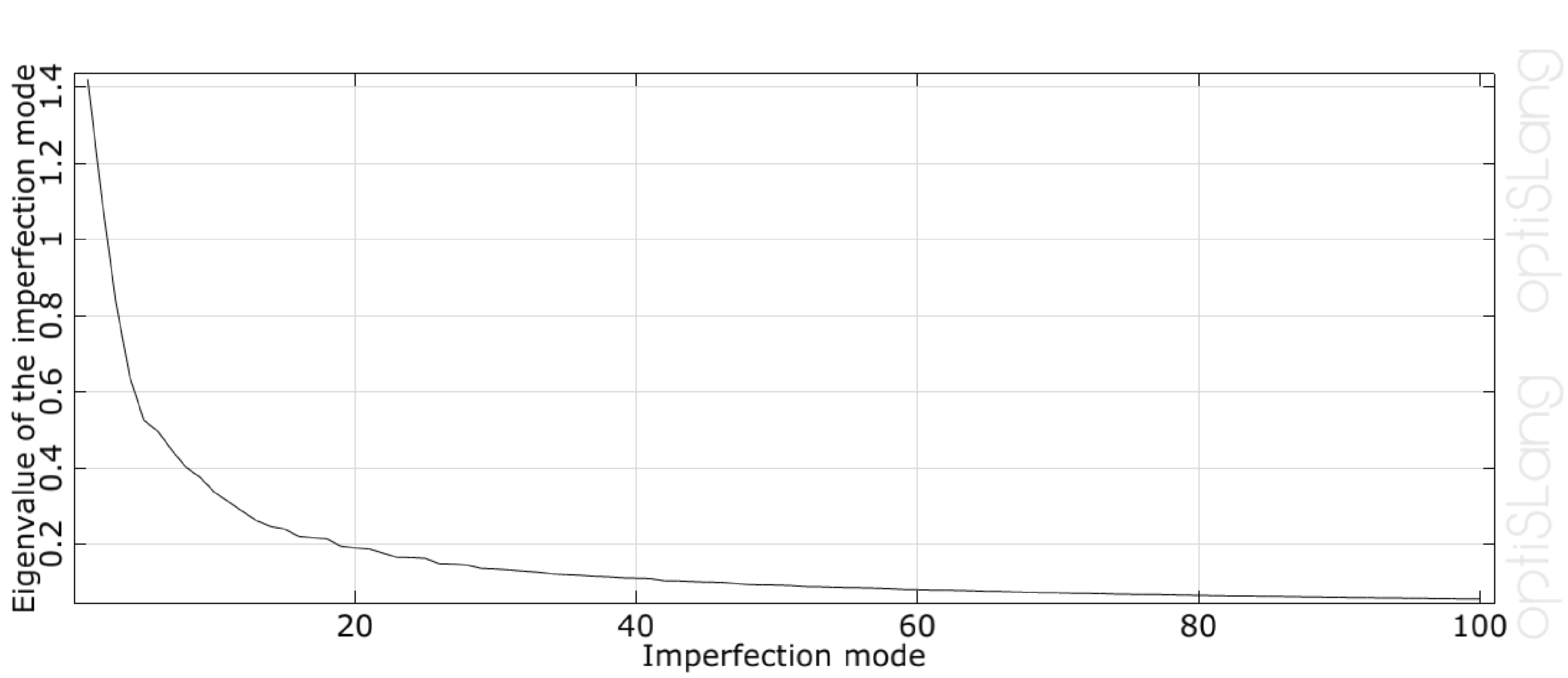


Data Reduction

Truncation of random field expansion

- Eigenvalues are sorted in (usu. strongly) decreasing order
- Highest eigenvalues contribute most to total variance

➔ Neglect variables with minor contribution



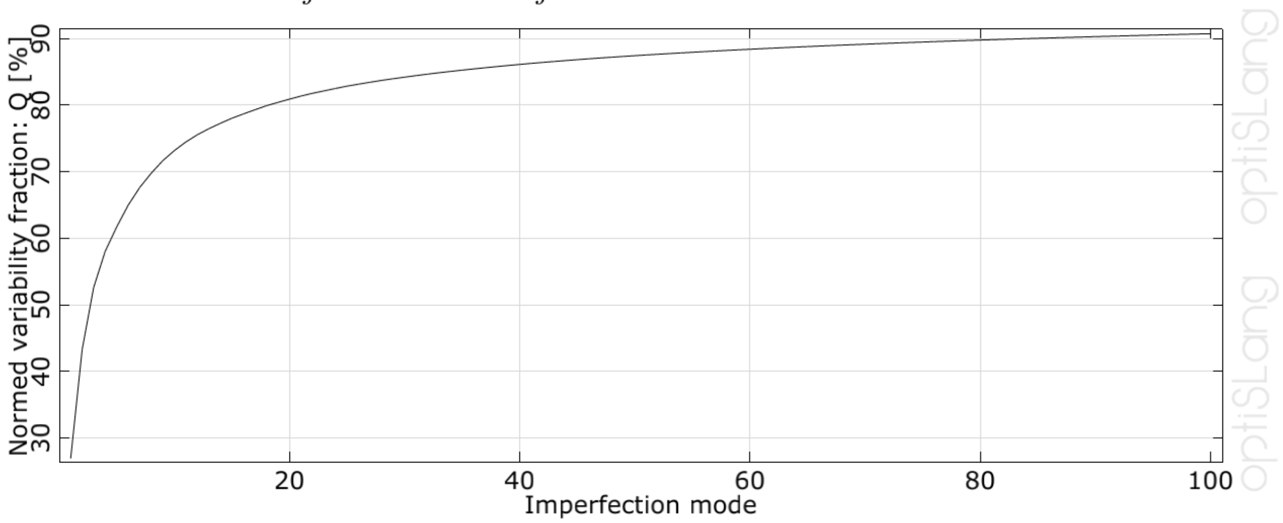
Data Reduction

- Quality of truncated series: variability fraction = 91%

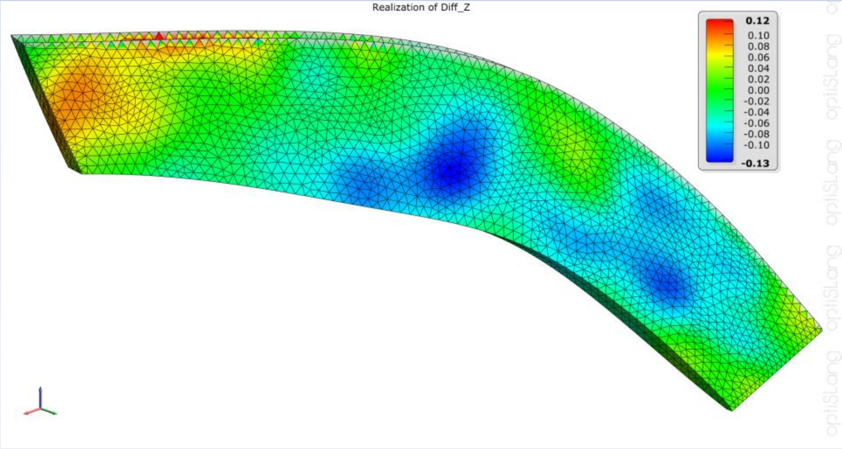
$$Q = \frac{\sum_{i=1}^{N_\lambda} \lambda_i}{\mathrm{trace}(\mathbf{C}_{XX})} \equiv \frac{\sum_{i=1}^{N_\lambda} \lambda_i \Big/ \dim(\mathbf{C}_{XX})}{\mathrm{trace}(\mathbf{C}_{XX}) \Big/ \dim(\mathbf{C}_{XX})}$$

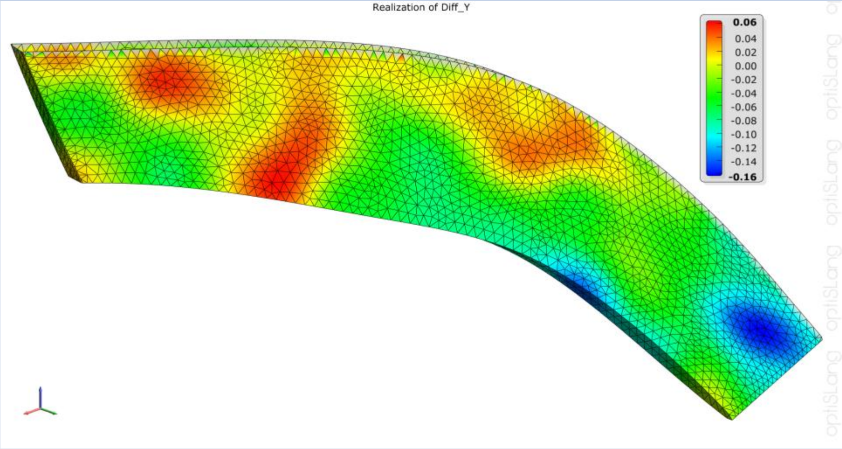
- After mesh reduction: normalize variability to number of data

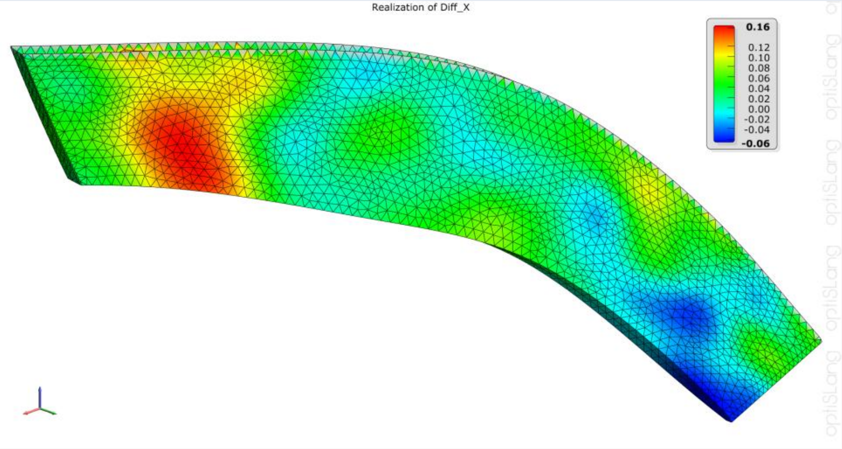
$$Q = \frac{\sum_{i=1}^{N_\lambda} \lambda_i}{\sum_{j=1}^{N_\mathrm{supports}} \sigma_j^2} \Rightarrow \frac{\sum_{i=1}^{N_\lambda} \lambda_i \Big/ \dim(\mathbf{C}_{XX})}{\sum_{j=1}^{N_\mathrm{supports}} \sigma_j^2 \Big/ N_\mathrm{supports}}$$



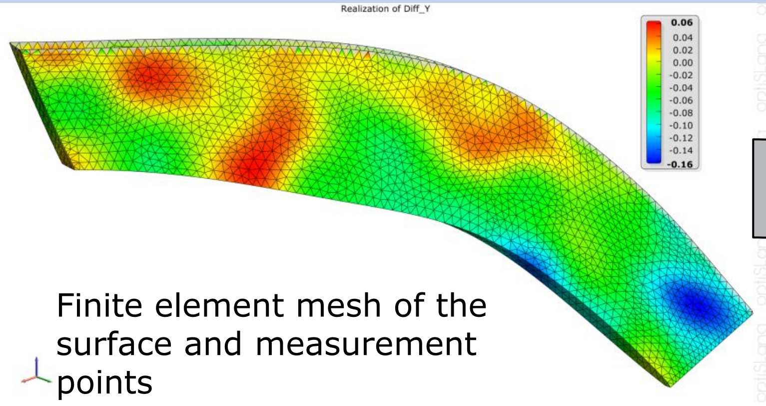
Realizations of the random field





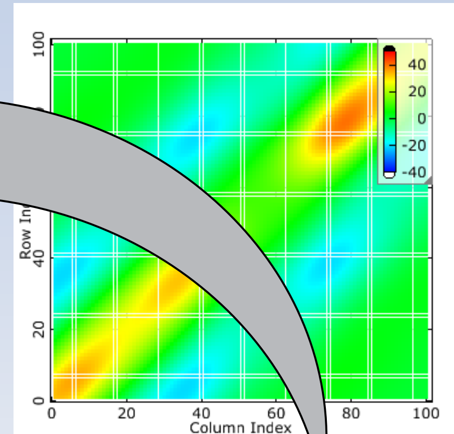


Process integration

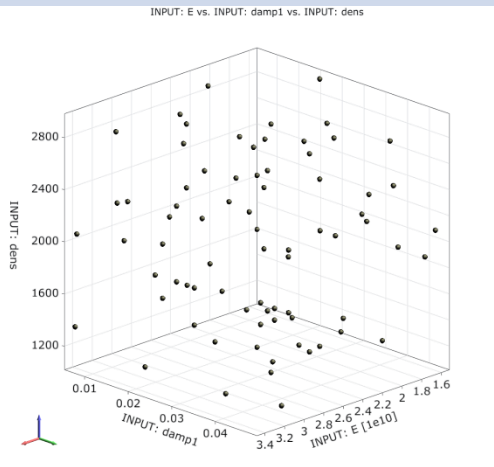


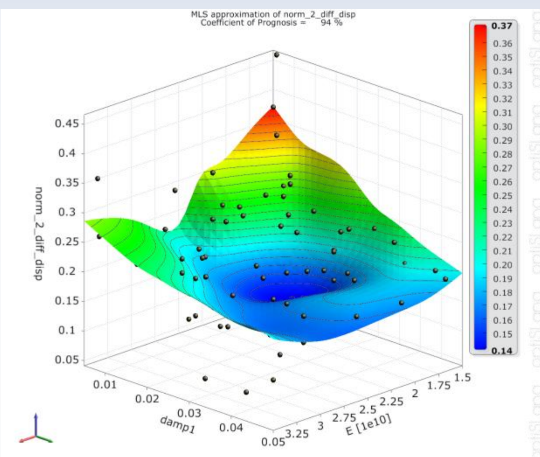
Finite element mesh of the surface and measurement points

Statistics of measurements: means, covariance matrix

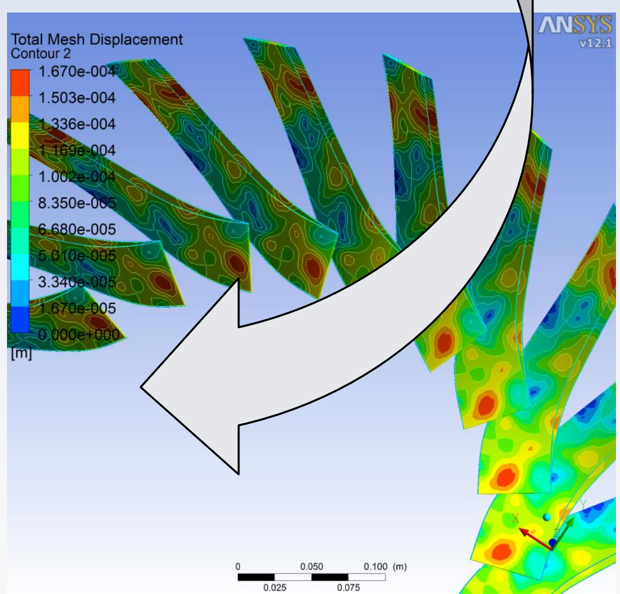


Simulate random parameters



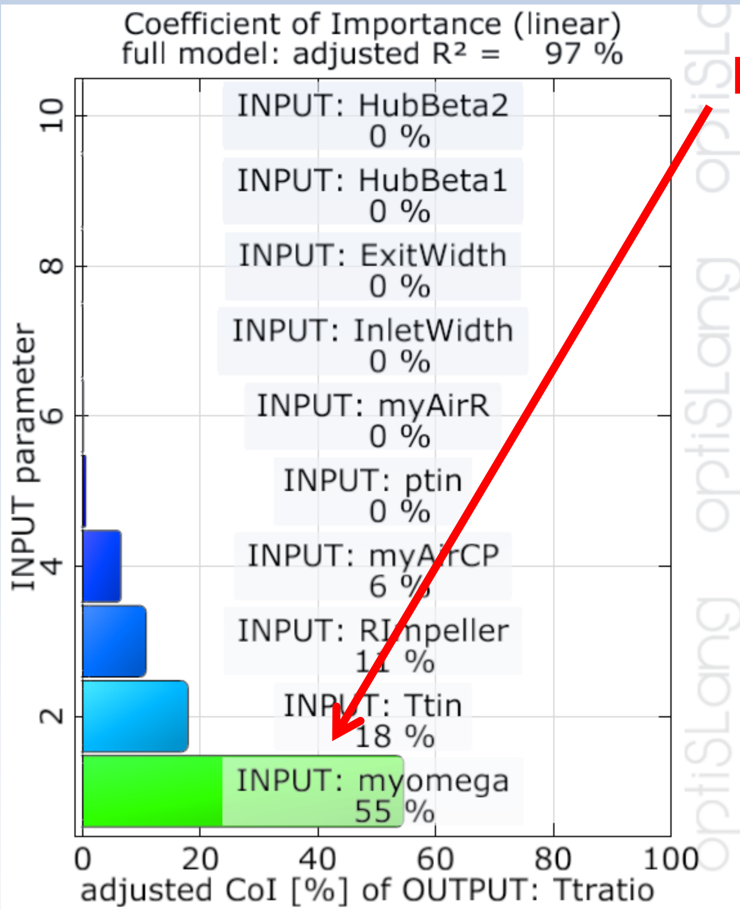


Robustness evaluation and Six Sigma Analysis



Generate set of random specimen, replace in FE assembly

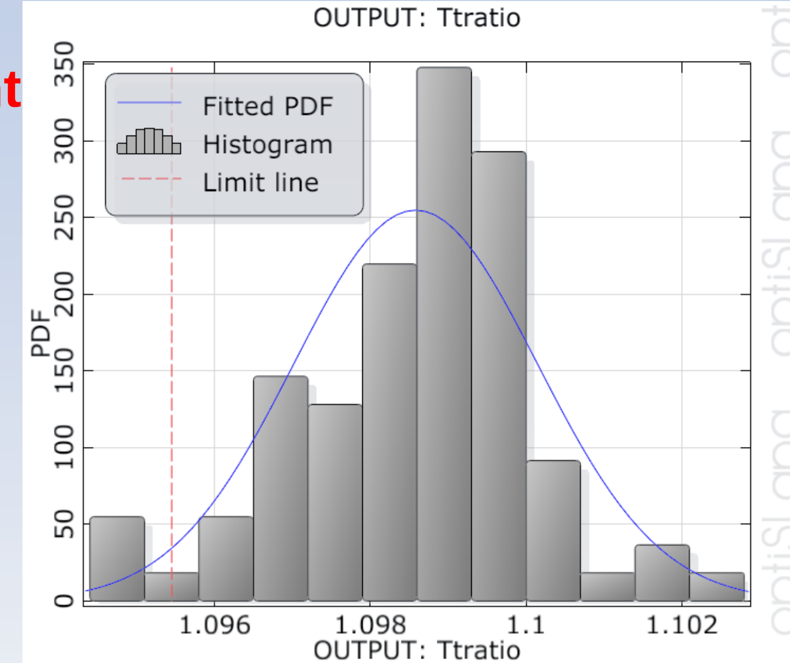
Robustness Total Temperature



Most relevant Parameter

No Tolerance limit

Objective: Θ_T = min

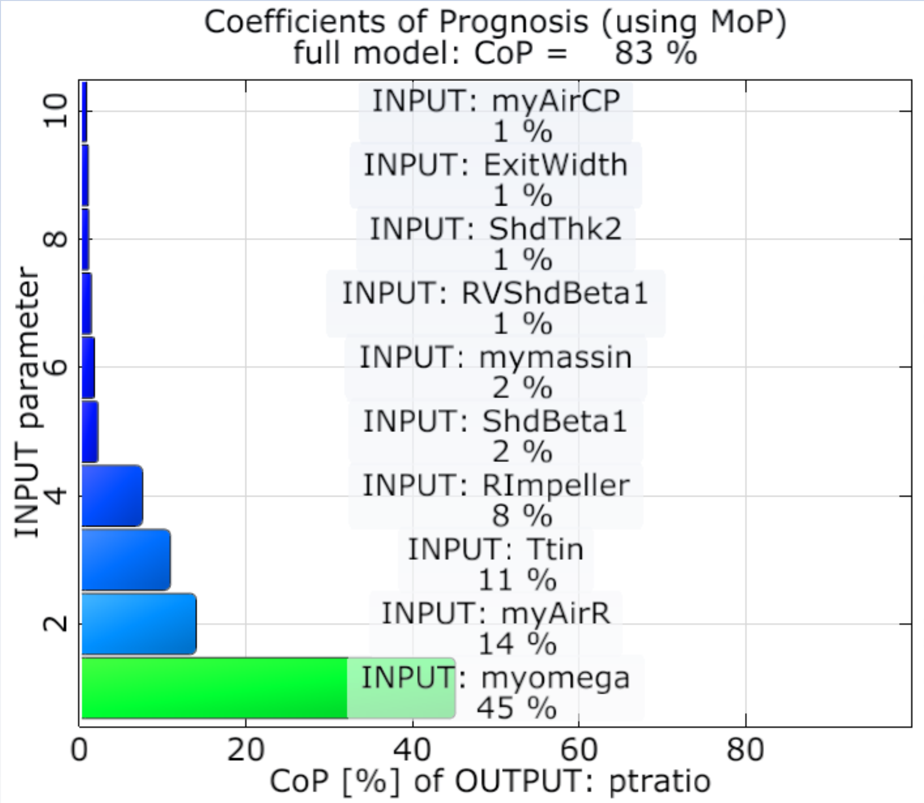


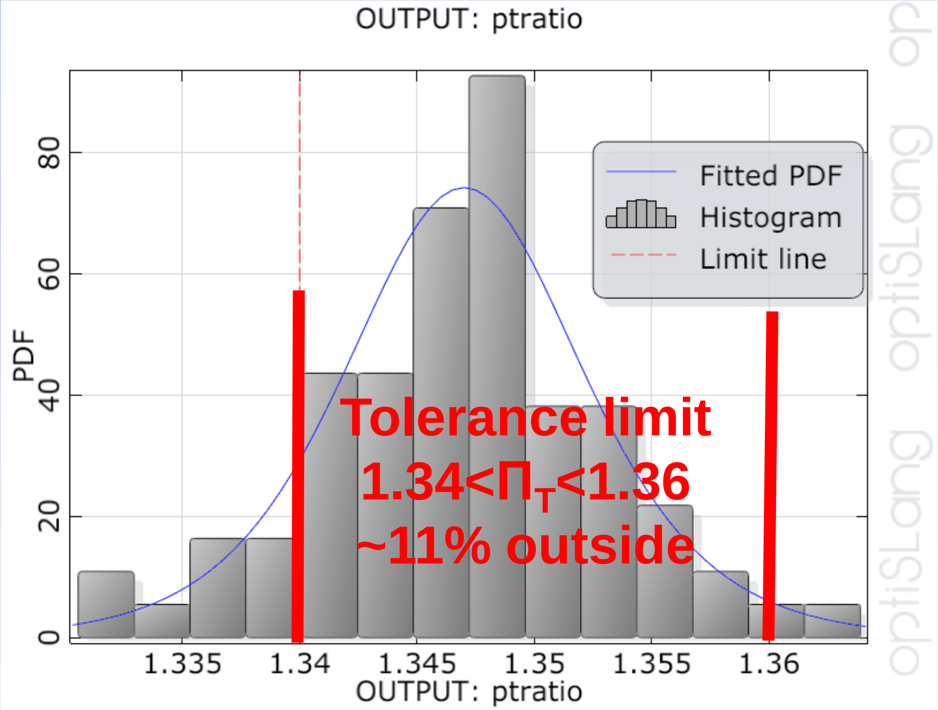
Statistic data			
Min:	1.094	Max:	1.103
Mean:	1.099	Sigma:	0.001565
CV:	0.001425		
Skewness:	-0.3148	Kurtosis:	3.606
Fitted PDF: Normal			
Mean:	1.099	Sigma:	0.001565
Limit x = 1.095			
P_rel =	0.0512821	P_fit =	0.0226627
Probability P(X<x) = 0.95			
x_rel =	1.1009	x_fit =	1.10116

No criterion of failure for Θ_T

Robustness Total Pressure

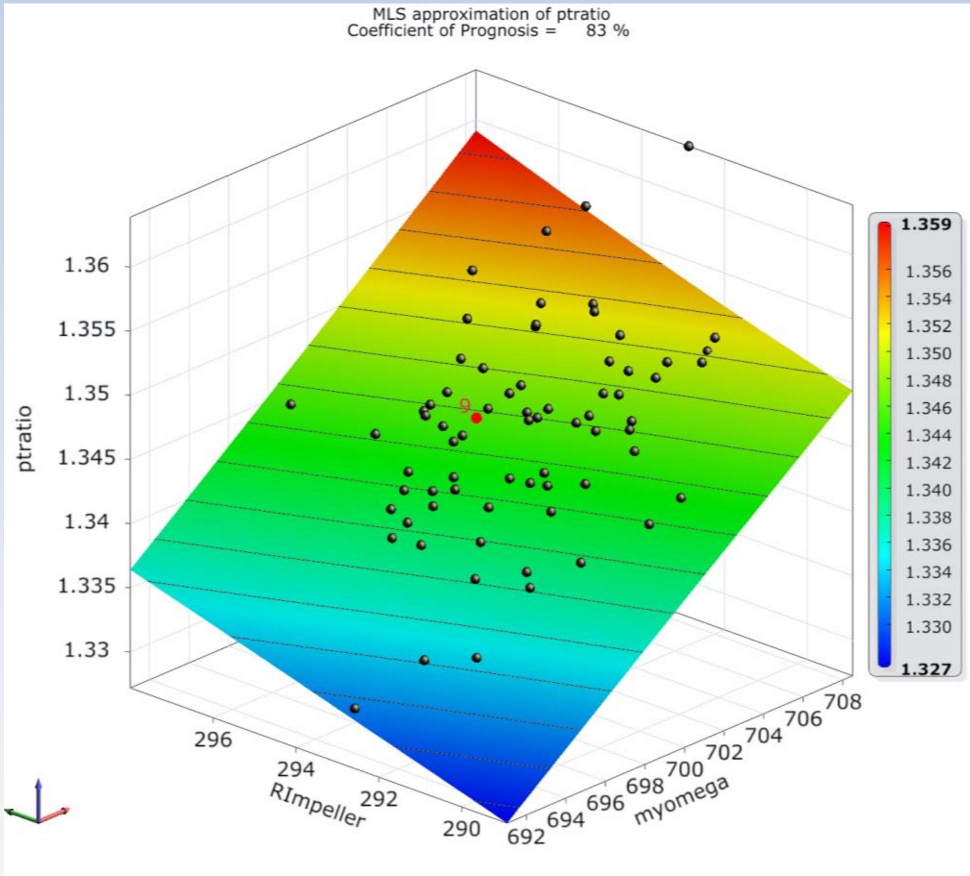
Most relevant Parameter

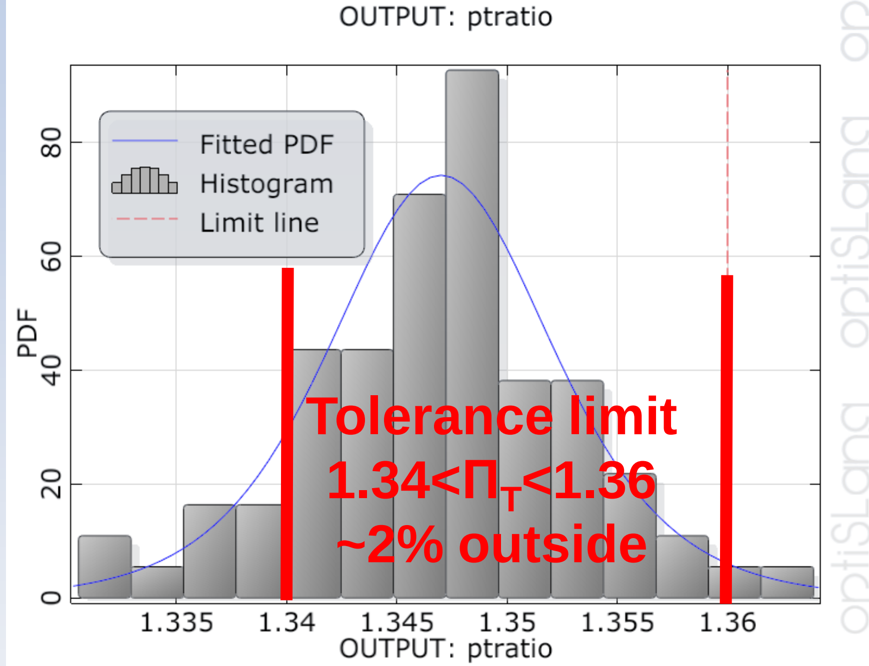




Statistic data			
Min:	1.331	Max:	1.364
Mean:	1.347	Sigma:	0.006107
CV:	0.004534		
Skewness:	-0.1683	Kurtosis:	3.589
Fitted PDF: Logistic			
Mean:	1.347	Sigma:	0.006107
Limit x = 1.34			
P_rel =	0.116883	P_fit =	0.110889

Robustness Total Pressure



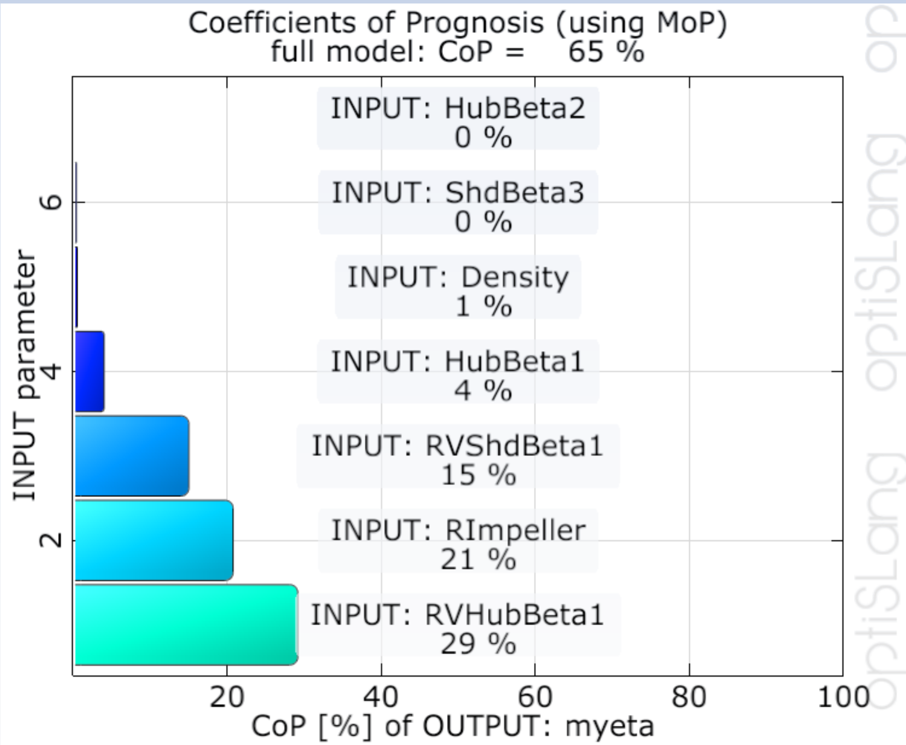


Statistic data			
Min:	1.331	Max:	1.364
Mean:	1.347	Sigma:	0.006107
CV:	0.004534		
Skewness:	-0.1683	Kurtosis:	3.589
Fitted PDF: Logistic			
Mean:	1.347	Sigma:	0.006107
Limit x = 1.36			
P_rel =	0.974026	P_fit =	0.979332

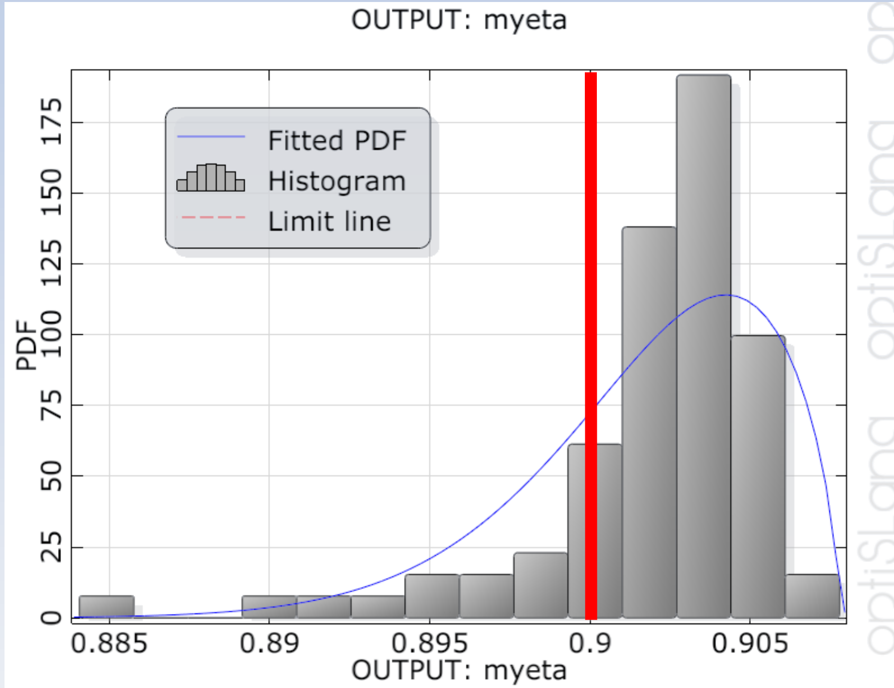
Modification of omega leads to more Robust Design

Robustness Efficiency

Most relevant Parameter

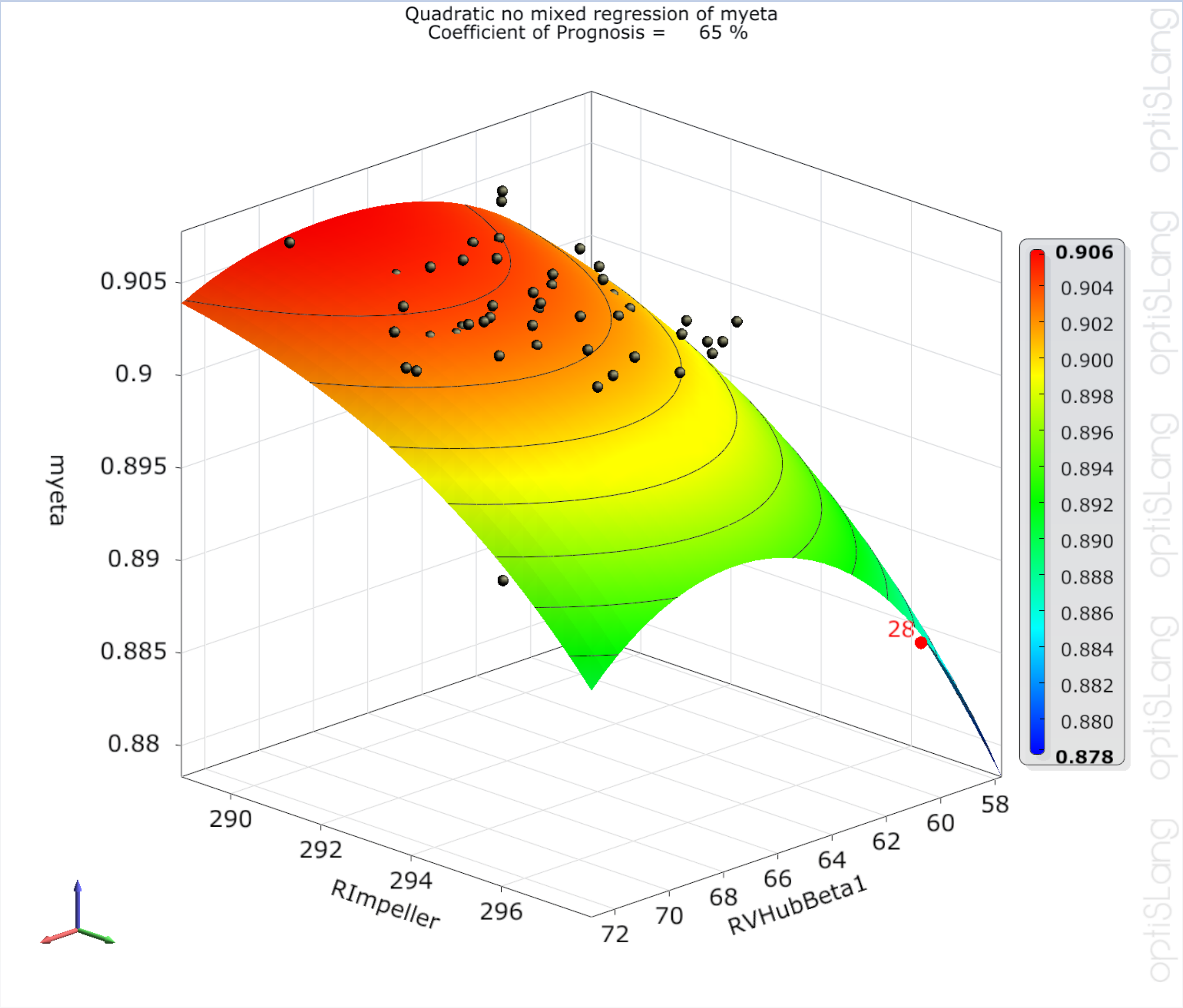


Tolerance limit η>90%
~28% outside



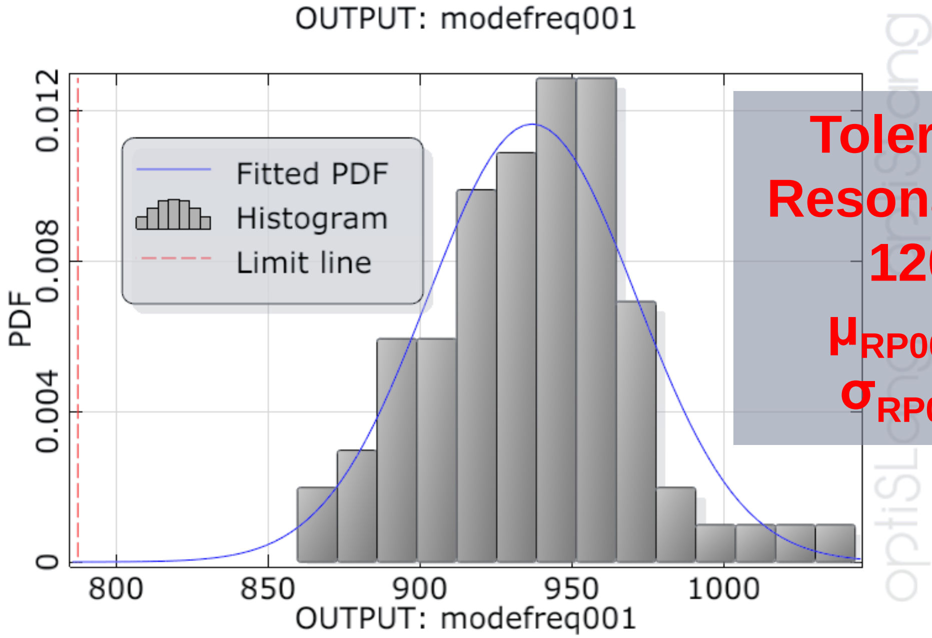
Statistic data			
Min:	0.8841	Max:	0.9078
Mean:	0.9018	Sigma:	0.003851
CV:	0.00427		
Skewness:	-2.212	Kurtosis:	9.204
Fitted PDF: Extreme Typ III (Min) Weibull			
Mean:	0.9018	Sigma:	0.003851
Upper cut:	0.9078		
Limit x = 0.9			
P_rel =	0.168831	P_fit =	0.277155

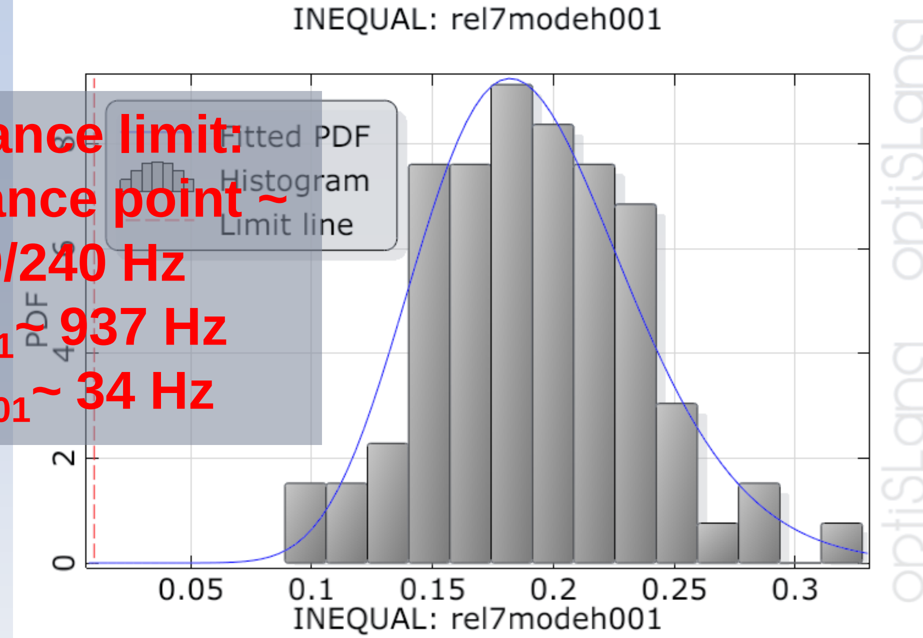
Robustness Efficiency



Modification of RImpeller and RVHubBeta1 leads to more Robust Design

Robustness Eigen Frequency Mode 1 Harmonic Index 0





Tolerance limit: Resonance point ~ 120/240 Hz

μ_{RP001} ~ 937 Hz

σ_{RP001} ~ 34 Hz

Statistic data			
Min:	859.7	Max:	1043
Mean:	937	Sigma:	34.25
CV:	0.03655		
Skewness:	0.315	Kurtosis:	3.467
Fitted PDF: Normal			
Mean:	937	Sigma:	34.25
Limit x = 787.4			
P_rel =	0	P_fit =	6.2614e-006

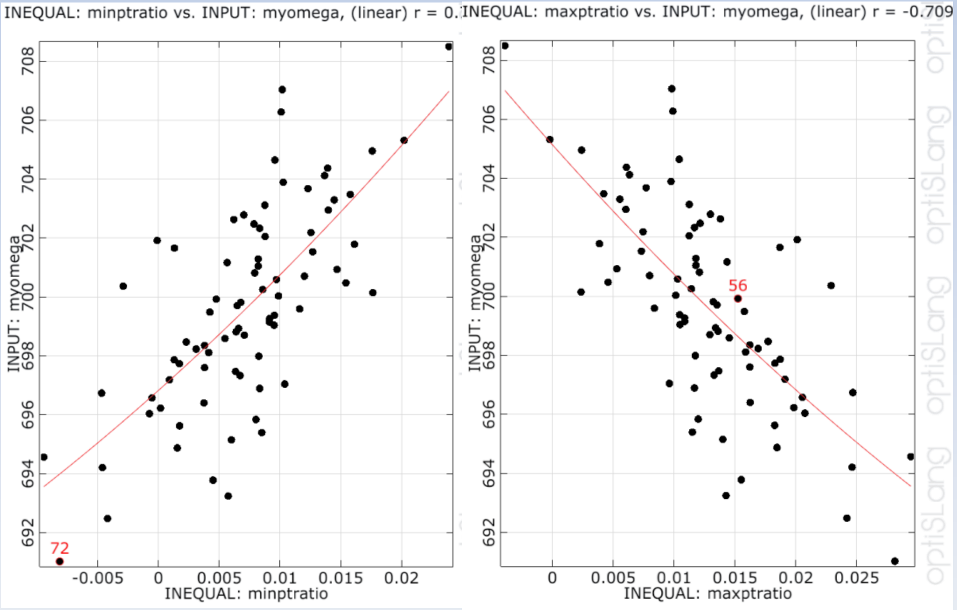
Statistic data			
Min:	0.08893	Max:	0.3278
Mean:	0.1921	Sigma:	0.0441
CV:	0.2295		
Skewness:	0.2803	Kurtosis:	3.407
Fitted PDF: Gamma			
Mean:	0.1921	Sigma:	0.0441
Limit x = 0.01			
P_rel =	0	P_fit =	2.69681e-018

$P(\mathcal{F}) \leq 3.4 \cdot 10^{-6}; (\beta = \sigma_L \geq 4.5)$

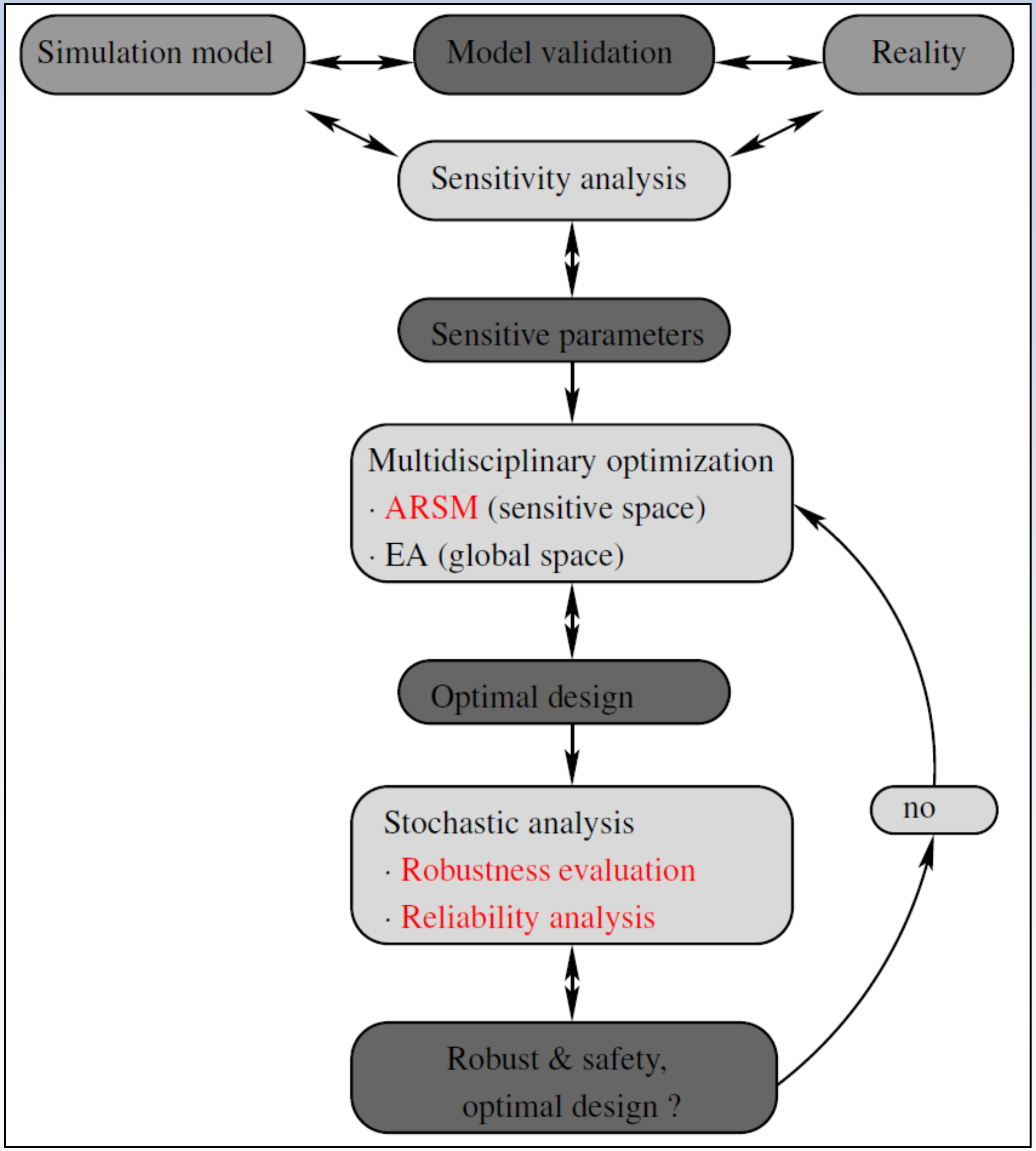
$P(\mathcal{F}) \approx \Phi(-\beta) \approx \Phi(-0.1921 + 0.01/0.0441) = \Phi(-4.13) = 1.8 \cdot 10^{-5}$

Conclusion Robustness Analysis

- Non robust behavior with respect to
  - **Efficiency**
  - **Total pressure**
- But acceptable failure probability level for structural risk
  - Estimation of a Six Sigma Design
- Efficiency: **myeta**
  - RVHubBeta1 as largest as possible
  - RVShdBeta1 as largest as possible
  - RImpeller as smallest as possible
- Total pressure: **ptratio**
  - myomega as largest as possible
  - RImpeller as largest as possible
  - ptratio mean -> 1.355



Successive Robust Design Optimization



- iterative decoupled loop approach
- in combination with identification of the most significant random and design variables using the multivariate statistic
- first step the robustness evaluation can be used to prove the predictive capability of the simulation model and to
- identify the most important parameters to solve reliability analysis, efficiently
- it is neccessary to evaluate robustness and safety of the design

Process Integration

Sensitivity Analysis

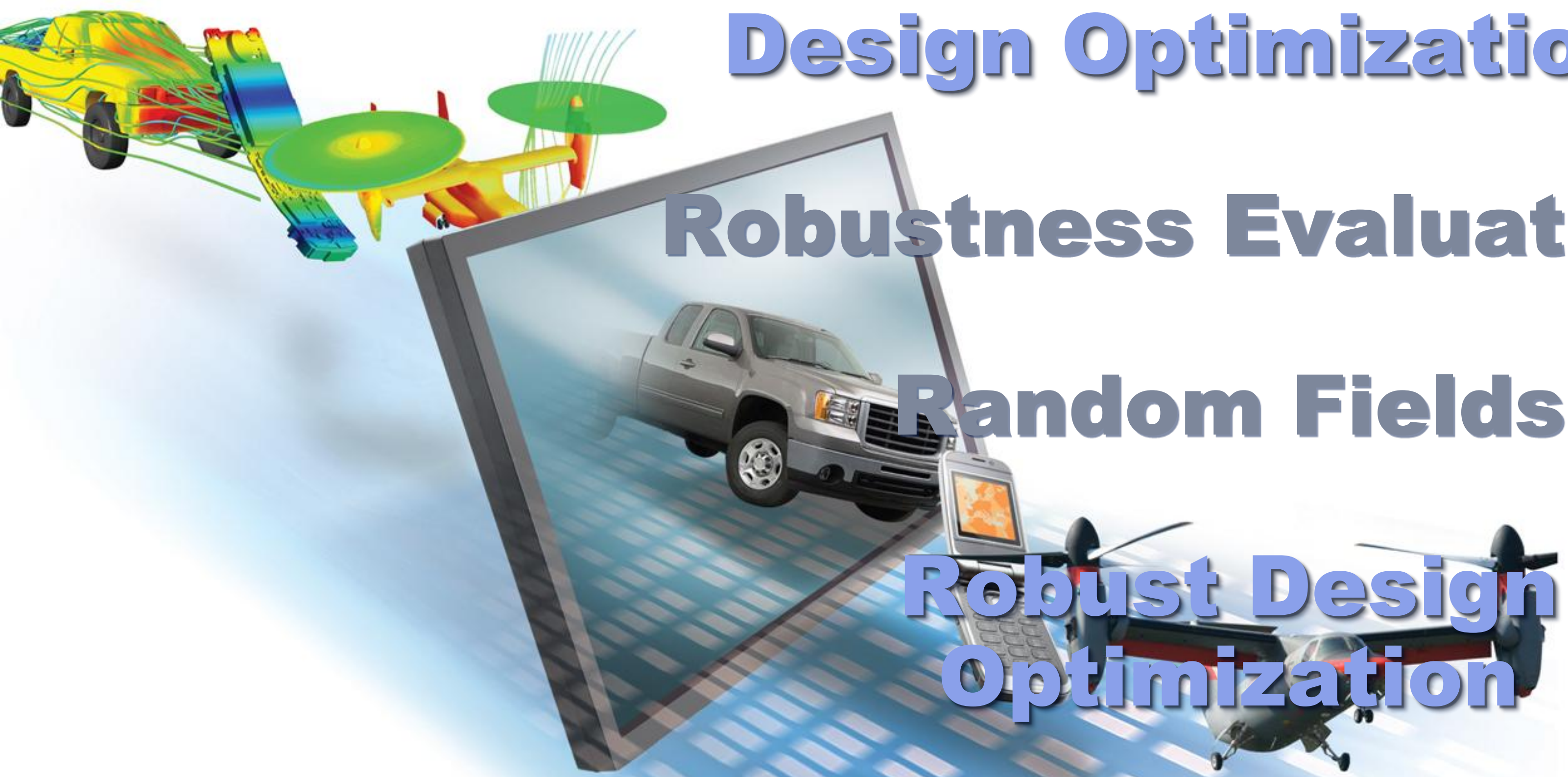
Design Optimization II

Robustness Evaluation II

Random Fields

Robust Design Optimization

Reliability Analysis



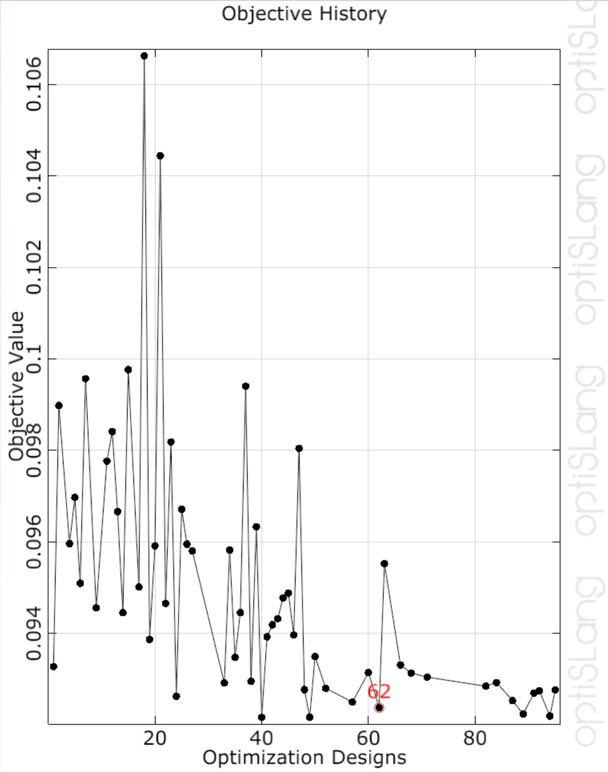
Design Optimization II

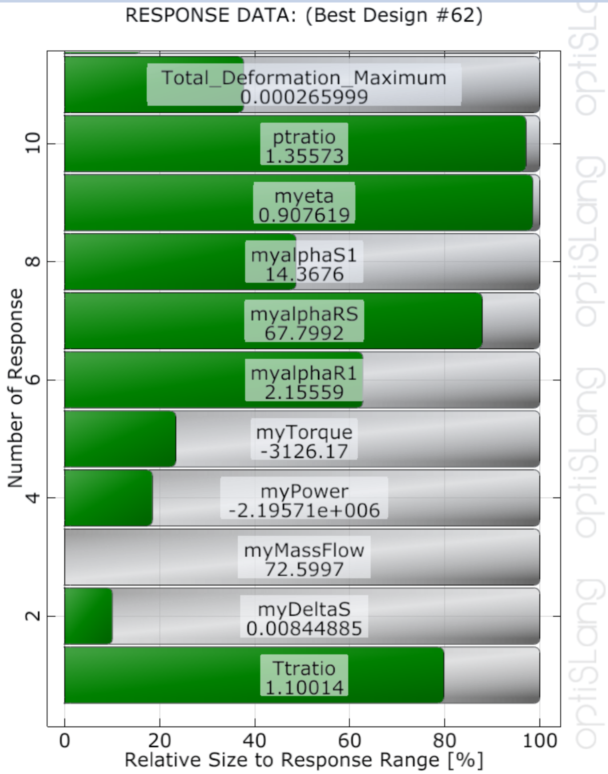
Opti | Robust | Output | Strings | Constraints | Objectives

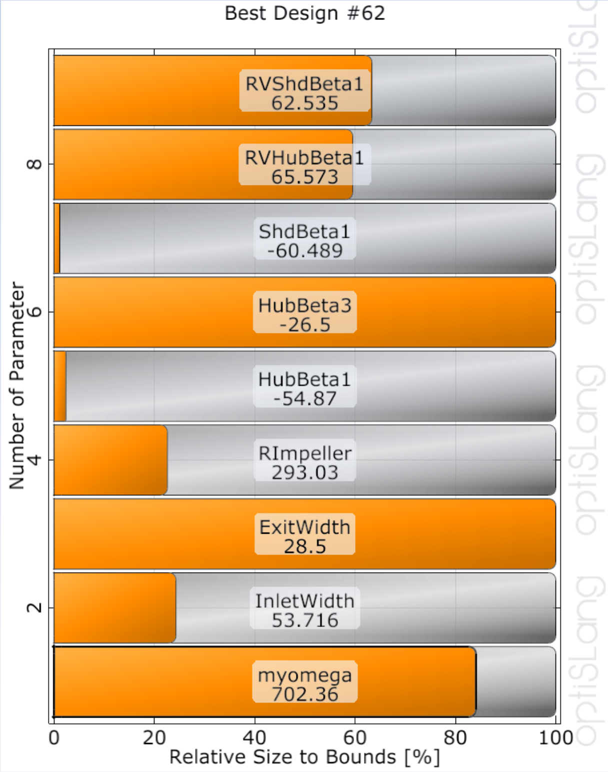
#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Type	Format	Active	Const...
1	myomega	699.76	699.76	699.0	703.0	continuous	%20.14f	✓	
2	InletWidth	53	53.6136610657...	52.5	57.5	continuous	%20.14f	✓	
3	ExitWidth	26	27.8049298398...	26.5	28.5	continuous	%20.14f	✓	
4	RImpeller	305	292.556879245...	291	300	continuous	%20.14f	✓	
5	HubBeta1	-48	-52.5	-55	-49.5	continuous	%20.14f	✓	
6	HubBeta3	-25	-27.017132519...	-28	-26.5	continuous	%20.14f	✓	
7	ShdBeta1	-55	-60.267623161...	-60.5	-59.5	continuous	%20.14f	✓	
8	RVHubThk1	45	45.0	35	66.0	continuous	%20.14f	✓	
9	RVHubBeta1	60	66.0	62.0	68	continuous	%20.14f	✓	
10	RVShdBeta1	60	62.8548646835...	60.0	64.0	continuous	%20.14f	✓	
11	RVShdThk1	45	45.0	35.0	55.0	continuous	%20.14f	✓	
12	HubBeta2	-25	-25.0	-27.5	-22.5	continuous	%20.14f		✓
13	ShdBeta2	-45	-45.0	-49.5	-40.5	continuous	%20.14f		✓
14	ShdBeta3	-30	-30.0	-33.0	-27.0	continuous	%20.14f		✓
15	HubThk1	1	1.0	0.8	1.2	continuous	%20.14f		✓
16	HubThk2	6	5.91963645103...	5.0	7.0	continuous	%20.14f		✓
17	ShdThk1	1	1.03011230706...	0.8	1.2	continuous	%20.14f		✓
18	ShdThk2	6	6.0	5.0	7.0	continuous	%20.14f		✓
19	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f		✓
20	RVBlades	24	24	21.6	28.7999999999...	continuous	%20.14f		✓

Cancel | OK

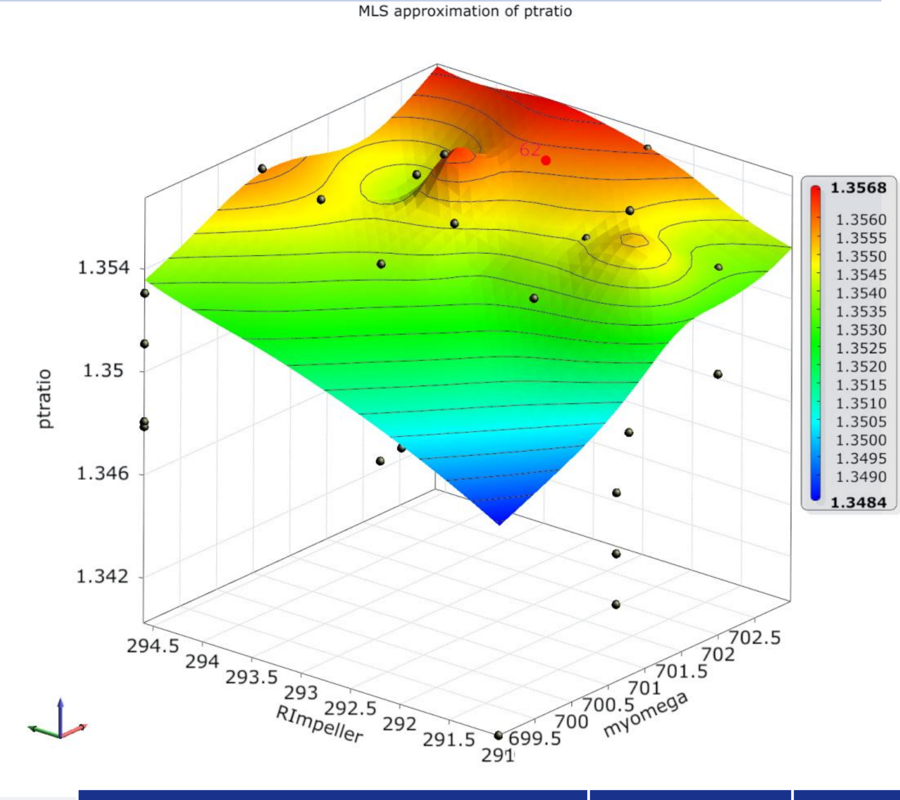
Design Optimization II: ARSM

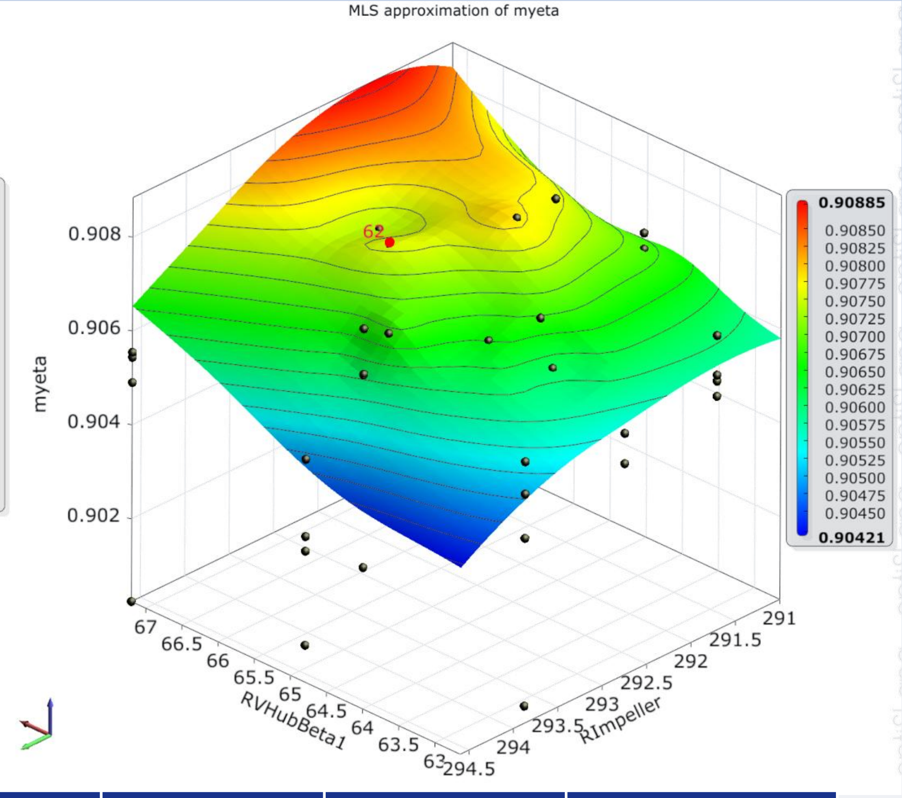




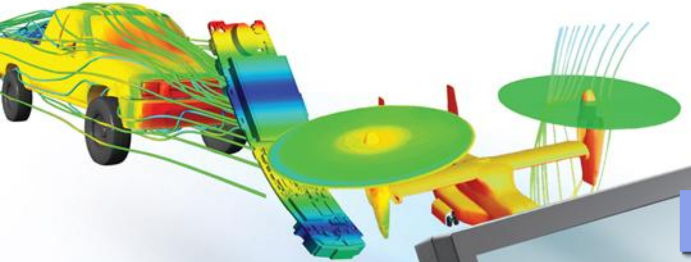


Design Optimization II: ARSM





	Initial	SA	ARSM I	EA I	ARSM II
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485	1.356
Efficiency [%]	86.72	89.15	90.62	90.67	90.76
#Designs	-	100	105	84	62



Process Integration

Sensitivity Analysis

Design Optimization II

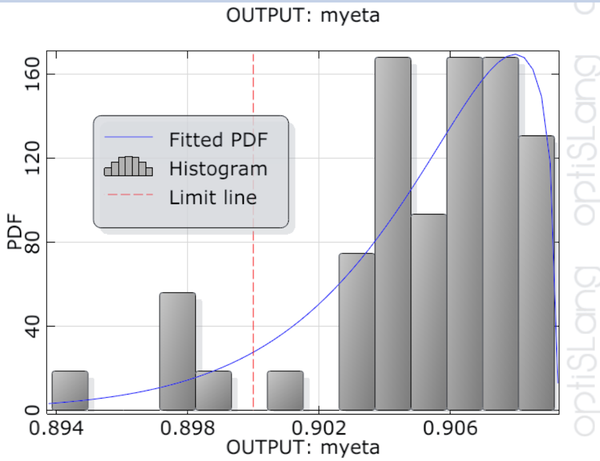
Robustness Evaluation II

Random Fields

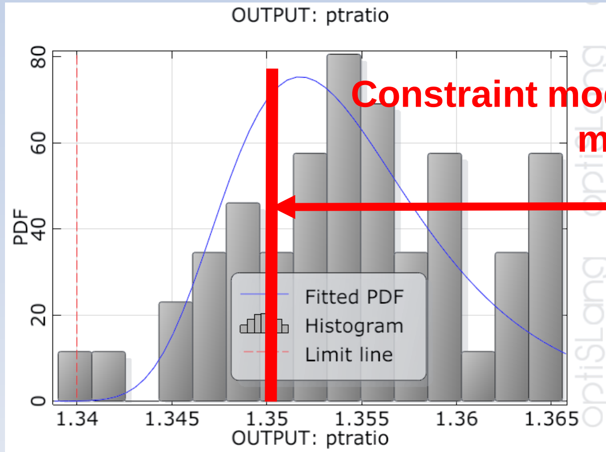
Robust Design Optimization

Reliability Analysis

Robust evaluation II: LHS

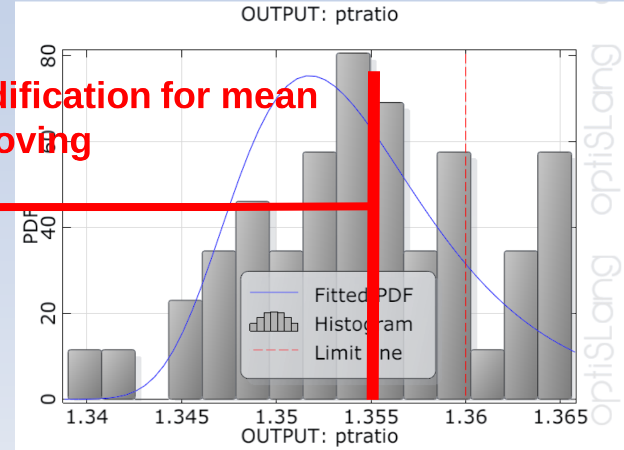


Statistic data			
Min:	0.8939	Max:	0.9092
Mean:	0.9051	Sigma:	0.00329
CV:	0.003635		
Skewness:	-1.487	Kurtosis:	4.997
Fitted PDF: Extreme Typ III (Min) Weibull			
Mean:	0.9051	Sigma:	0.00329
Upper cut:	0.9092		
Limit x = 0.9			
P_rel =	0.102041	P_fit =	0.0808577



Statistic data			
Min:	1.339	Max:	1.366
Mean:	1.355	Sigma:	0.006258
CV:	0.00462		
Skewness:	-0.1853	Kurtosis:	2.623
Fitted PDF: Fisher-Tippett			
Mean:	1.355	Sigma:	0.006258
Limit x = 1.34			
P_rel =	0.0204082	P_fit =	1.4437e-005

Constraint modification for mean moving



Statistic data			
Min:	1.339	Max:	1.366
Mean:	1.355	Sigma:	0.006258
CV:	0.00462		
Skewness:	-0.1853	Kurtosis:	2.623
Fitted PDF: Fisher-Tippett			
Mean:	1.355	Sigma:	0.006258
Limit x = 1.36			
P_rel =	0.795918	P_fit =	0.831174

Tolerance limit η<90%
~8% outside

Tolerance limit
Π_T>1.36
~17% outside

ANSYS
dynardo

Process Integration

Sensitivity Analysis

Design Optimization III

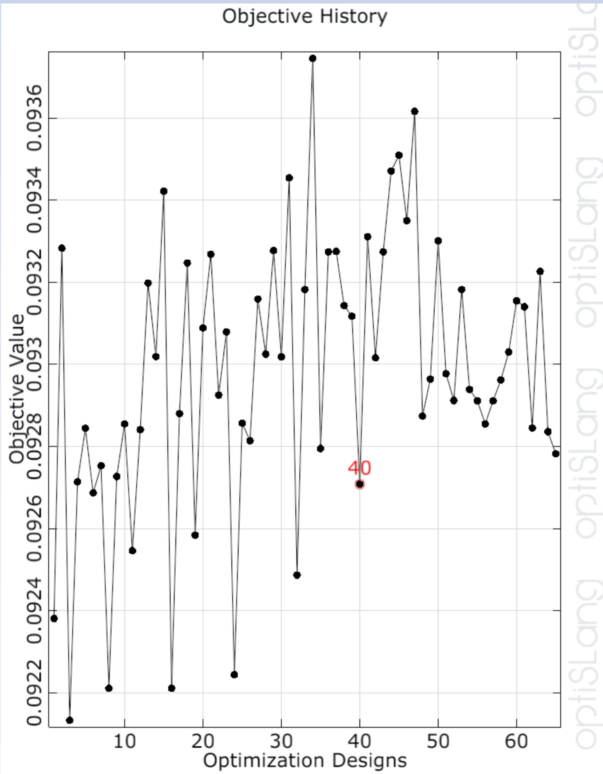
Robustness Evaluation III

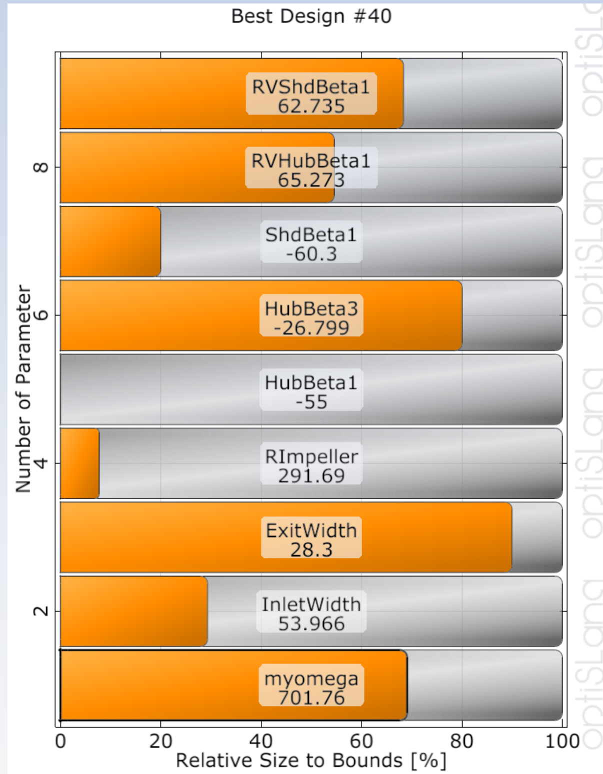
Random Fields

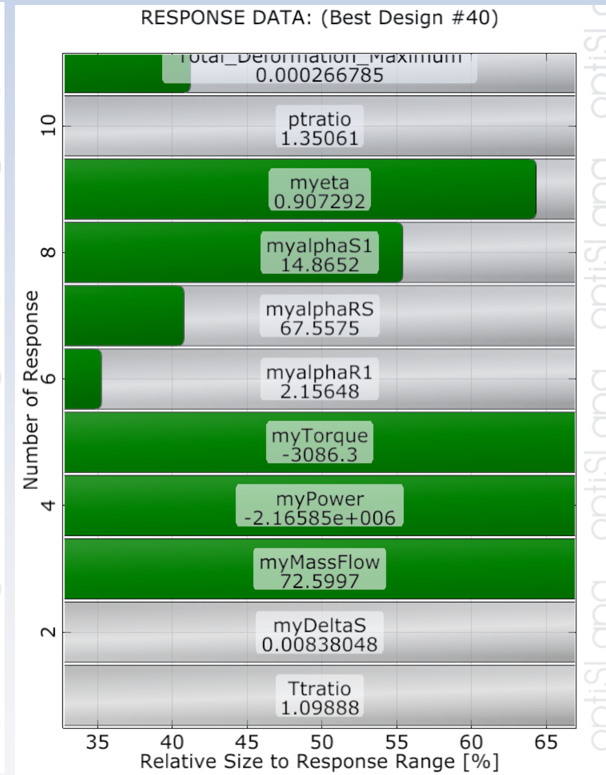
Robust Design Optimization

Reliability Analysis

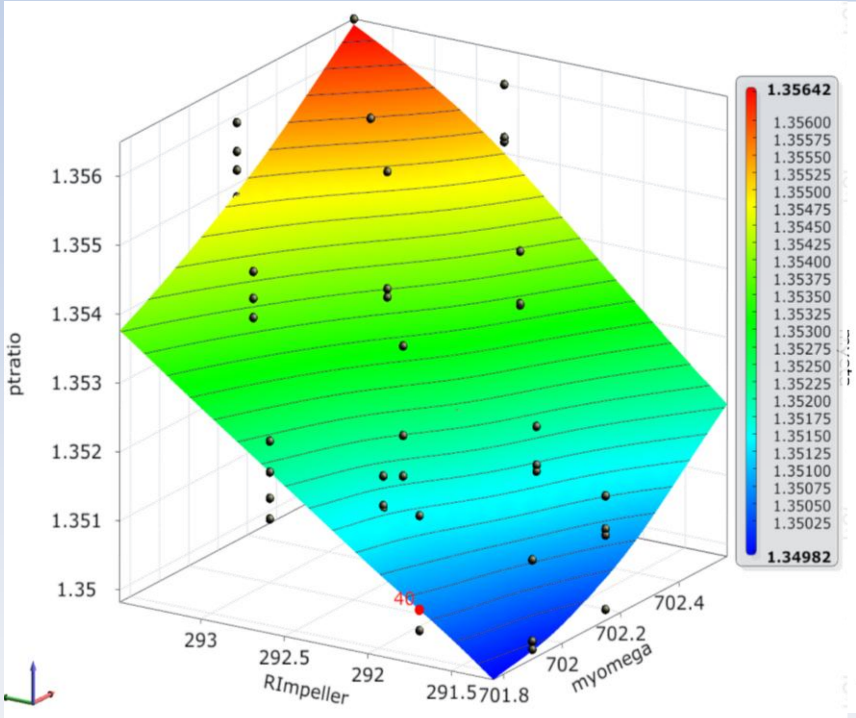
Design Optimization III: ARSM

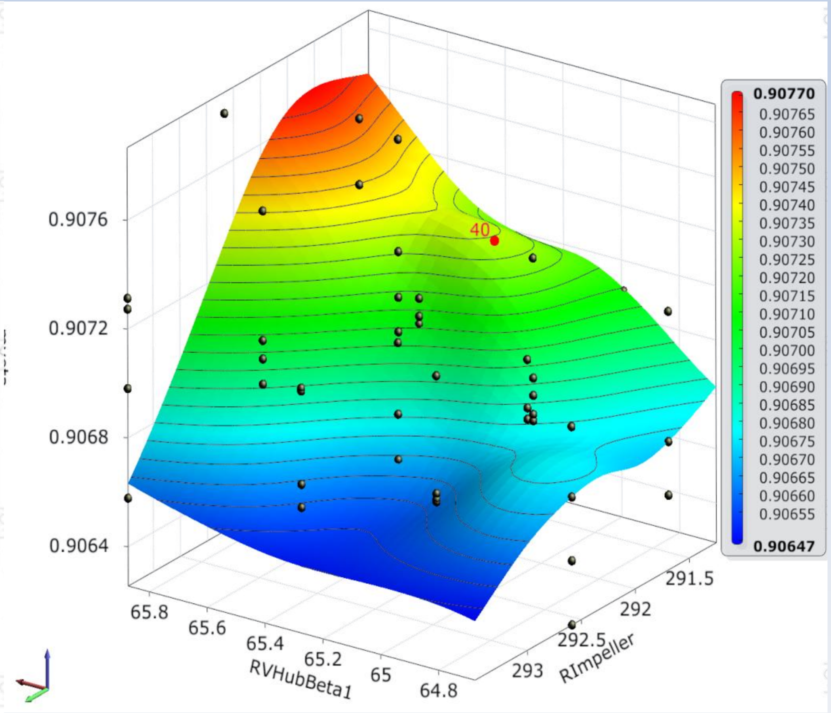






Design Optimization III: ARSM





	Initial	SA	ARSM I	EA I	ARSM II	ARSM III
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485	1.356	1.351
Efficiency [%]	86.72	89.15	90.62	90.67	90.76	90.73
#Designs	-	100	105	84	62	40

Process Integration

Sensitivity Analysis

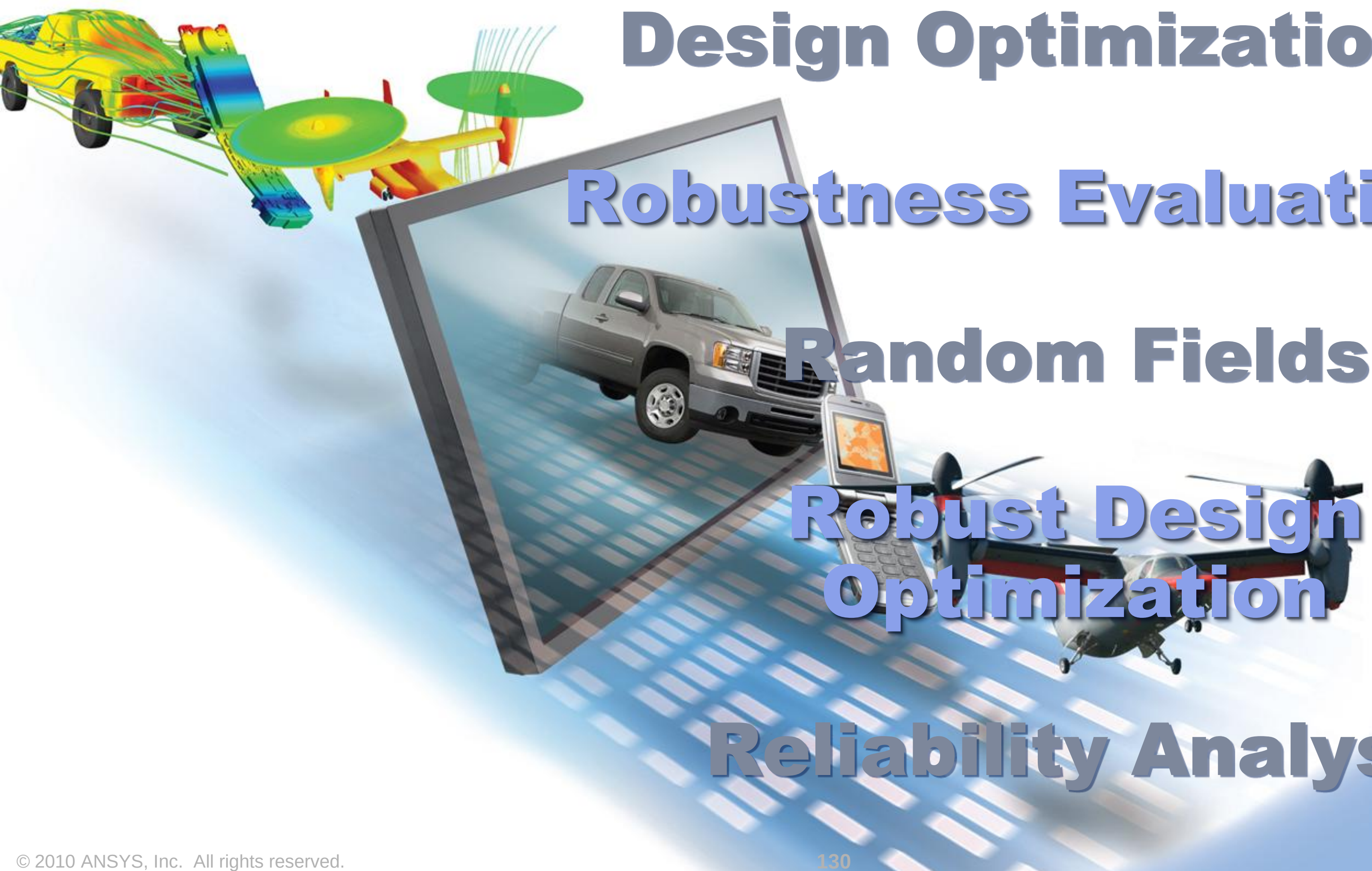
Design Optimization III

Robustness Evaluation III

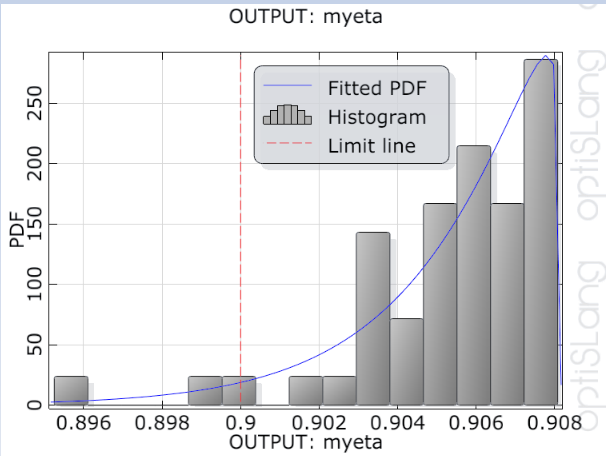
Random Fields

Robust Design Optimization

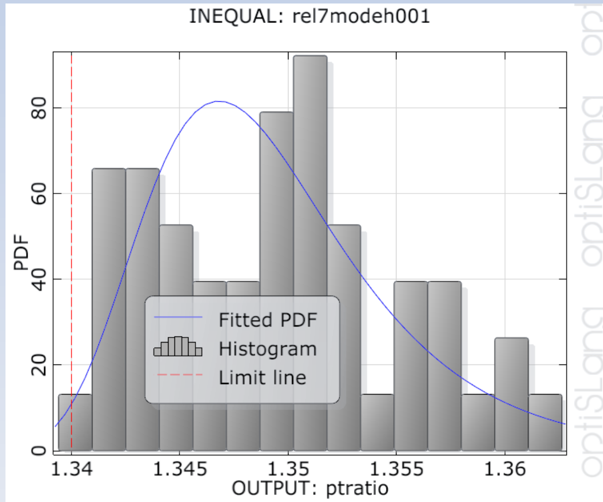
Reliability Analysis



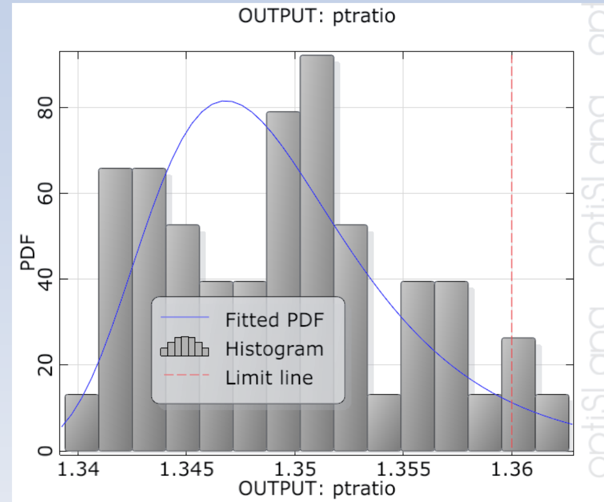
Robust evaluation III: LHS



Statistic data			
Min:	0.8953	Max:	0.9081
Mean:	0.9053	Sigma:	0.002554
CV:	0.002821		
Skewness:	-1.868	Kurtosis:	7.173
Fitted PDF: Extreme Typ III (Min) Weibull			
Mean:	0.9053	Sigma:	0.002554
Upper cut:	0.9081		
Limit x = 0.9			
P_rel =	0.0612245	P_fit =	0.0453854



Statistic data			
Min:	1.339	Max:	1.363
Mean:	1.349	Sigma:	0.005782
CV:	0.004285		
Skewness:	0.3347	Kurtosis:	2.38
Fitted PDF: Fisher-Tippett			
Mean:	1.349	Sigma:	0.005782
Limit x = 1.34			
P_rel =	0.0204082	P_fit =	0.01082



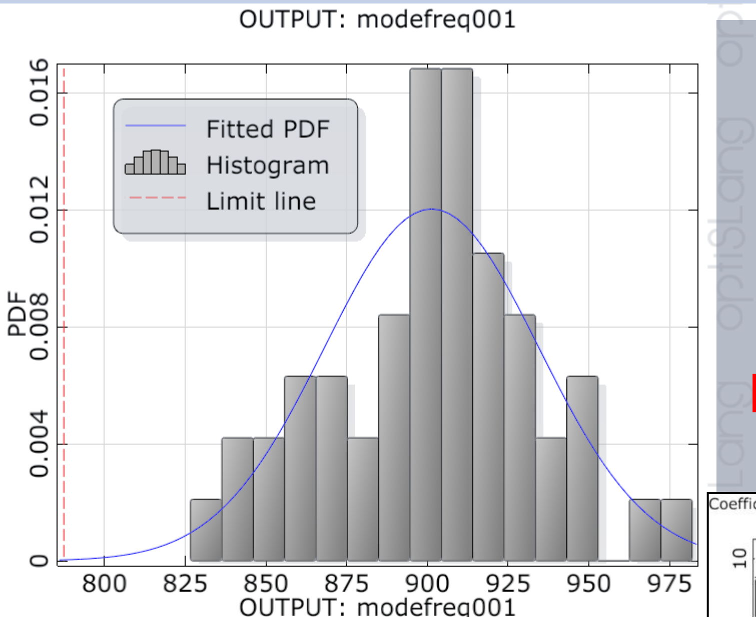
Statistic data			
Min:	1.339	Max:	1.363
Mean:	1.349	Sigma:	0.005782
CV:	0.004285		
Skewness:	0.3347	Kurtosis:	2.38
Fitted PDF: Fisher-Tippett			
Mean:	1.349	Sigma:	0.005782
Limit x = 1.36			
P_rel =	0.938776	P_fit =	0.947814

Tolerance limit η<90%
~4.5% outside

Robust Design

Tolerance limit
1.4<Π_T<1.36
~6% outside

Robust evaluation III: Eigen Frequency Mode 1 Harmonic Index 0

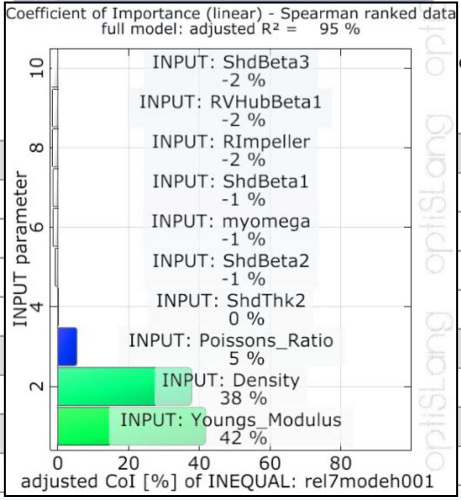


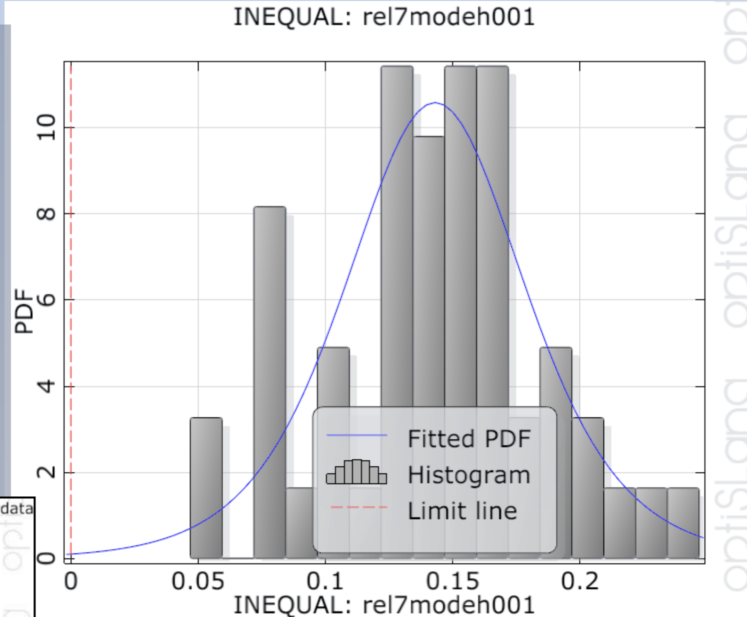
Statistic data			
Min:	826.6	Max:	982
Mean:	901.3	Sigma:	33.18
CV:	0.03682		
Skewness:	-0.04593	Kurtosis:	2.862
Fitted PDF: Normal			
Mean:	901.3	Sigma:	33.18
Limit x = 787.4			
P_rel =	0	P_fit =	0.000299089

Tolerance limit: Resonance point ~ 120/240 Hz

μ_{RP001} ~ 937 Hz

σ_{RP001} ~ 34 Hz





Statistic data			
Min:	0.04693	Max:	0.2468
Mean:	0.1431	Sigma:	0.04281
CV:	0.2991		
Skewness:	-0.1285	Kurtosis:	2.936
Fitted PDF: Logistic			
Mean:	0.1431	Sigma:	0.04281
Limit x = 0			
P_rel =	0	P_fit =	0.0023215

$$P(\mathcal{F}) \leq 3.4 \cdot 10^{-6}; (\beta = \sigma_L \geq 4.5)$$

$$P(\mathcal{F}) \approx \Phi(-\beta) \approx \Phi(-0.143 + 0.01/0.0428) = \Phi(-3.11) = 9.4 \cdot 10^{-4}$$

Safety Design?

Process Integration

Sensitivity Analysis

Design Optimization

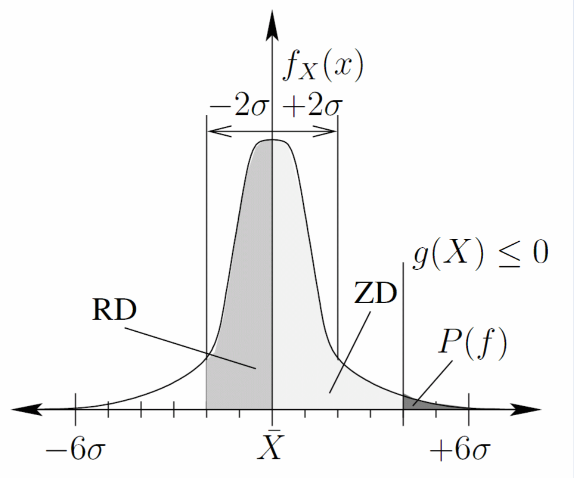
Robustness Evaluation

Random Fields

Robust Design Optimization

Reliability Analysis

Six Sigma Analysis

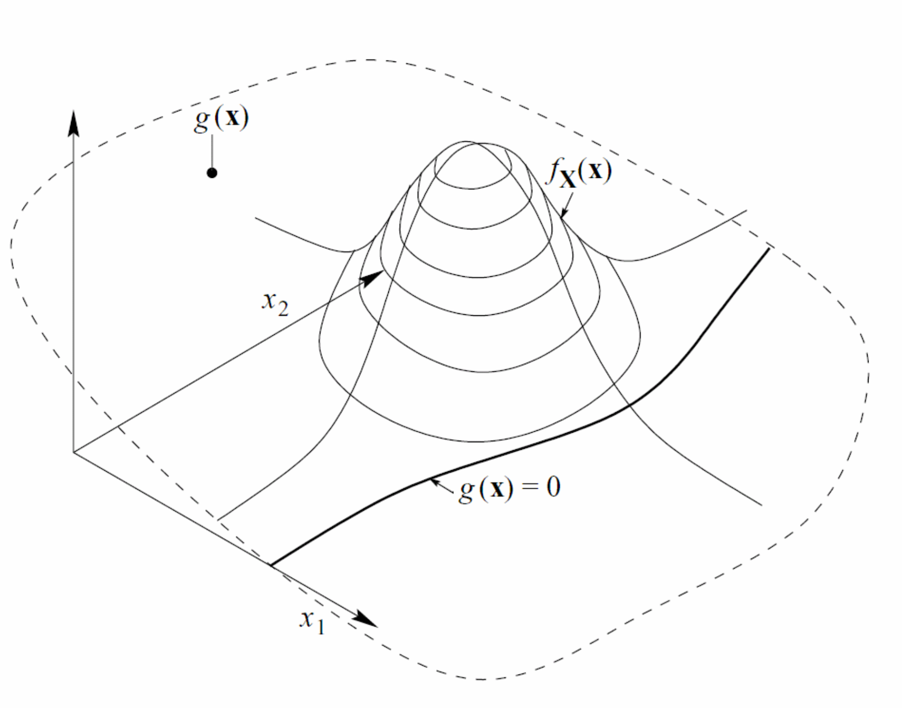


Calculation of probabilities much lower than E-2 needs detailed know how of all relevant uncertainties and reliability analysis

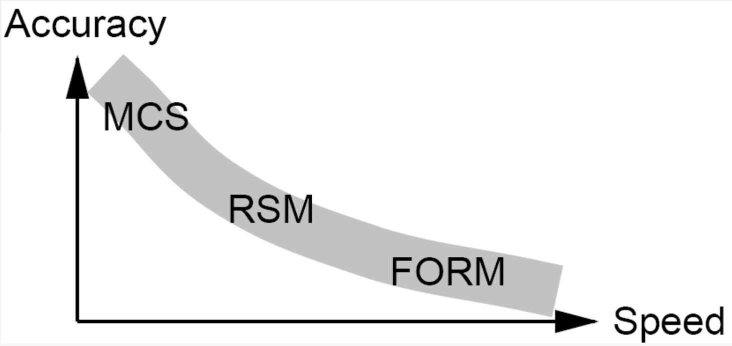
Sigma level	Varation	Probability of failure	Defects per million (short term)	Defects per million (long term – $\pm 1.5\sigma$ shift)
$\pm 1\sigma$	68.26	3.1 E-1	317,400	697,700
$\pm 2\sigma$	95.46	4.5 E-2	45,400	308,733
$\pm 3\sigma$	99.73	2.7 E-3	2,700	66,803
$\pm 4\sigma$	99.9937	6.3 E-5	63	6,200
$\pm 5\sigma$	99.999943	5.7 E-7	0.57	233
$\pm 6\sigma$	99.9999998	2.0 E-9	0.002	3.4

Methods of Reliability Analysis

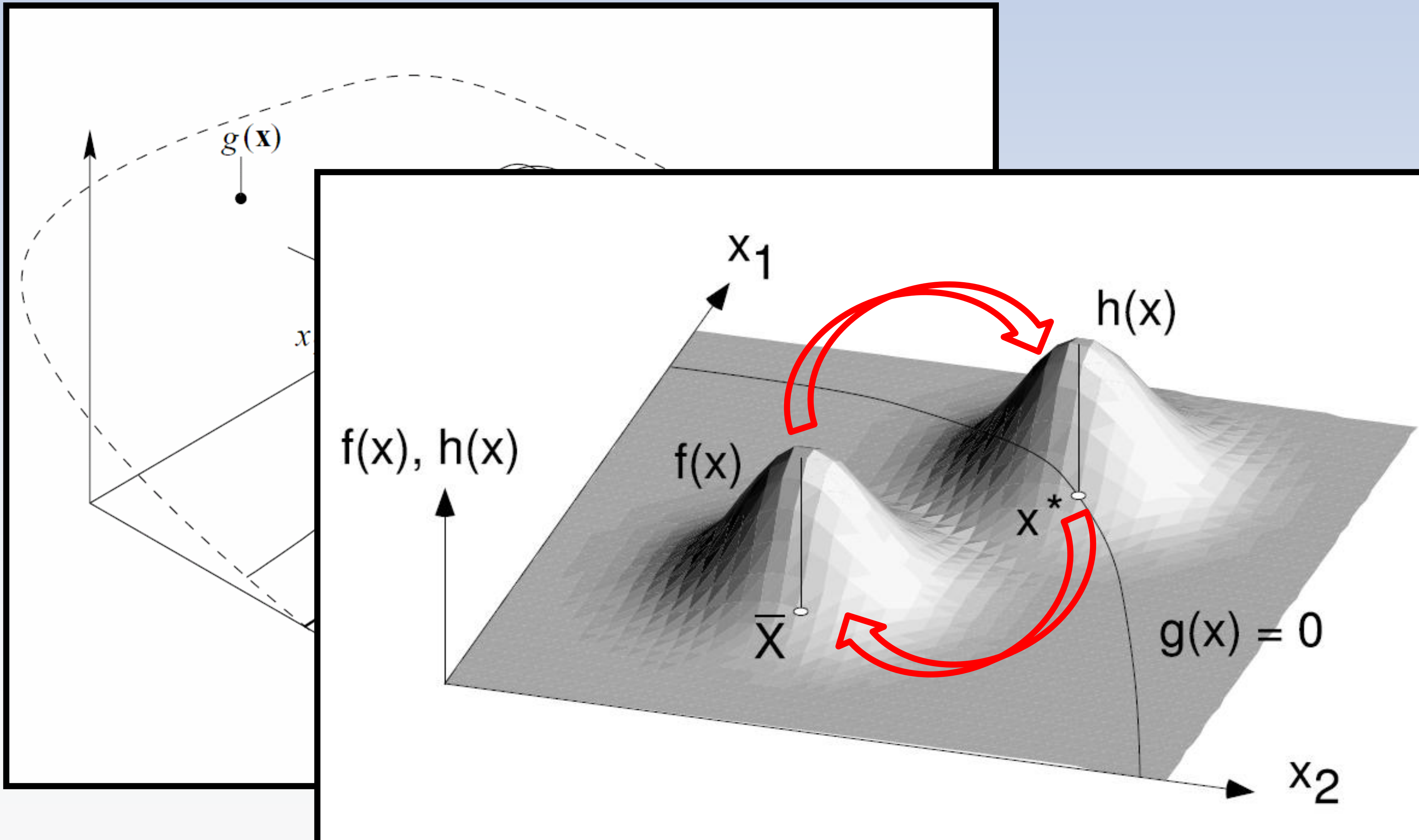
- First and second order reliability method (FORM/SORM)
  - ➲ Sigma level ≥ 2, $n \leq 50$
- Monte-Carlo-Simulation (MCS)
  - ➲ Sigma level ≤ 2, independent of n
- Latin hyper cube sampling (LHS)
  - ➲ Sigma level ≤ 3, independent of n
- Importance sampling using design point (ISPUD)
  - ➲ Sigma level ≥ 2, $n \leq 50$
- Adaptive importance sampling (AIS)
  - ➲ Sigma level ≥ 2, $n \leq 15$
- Directional sampling (DS)
  - ➲ Sigma level ≥ 2, $n \leq 20$
- Adaptive response surface method (ARSM)
  - ➲ Sigma level ≥ 2, $n \leq 15$



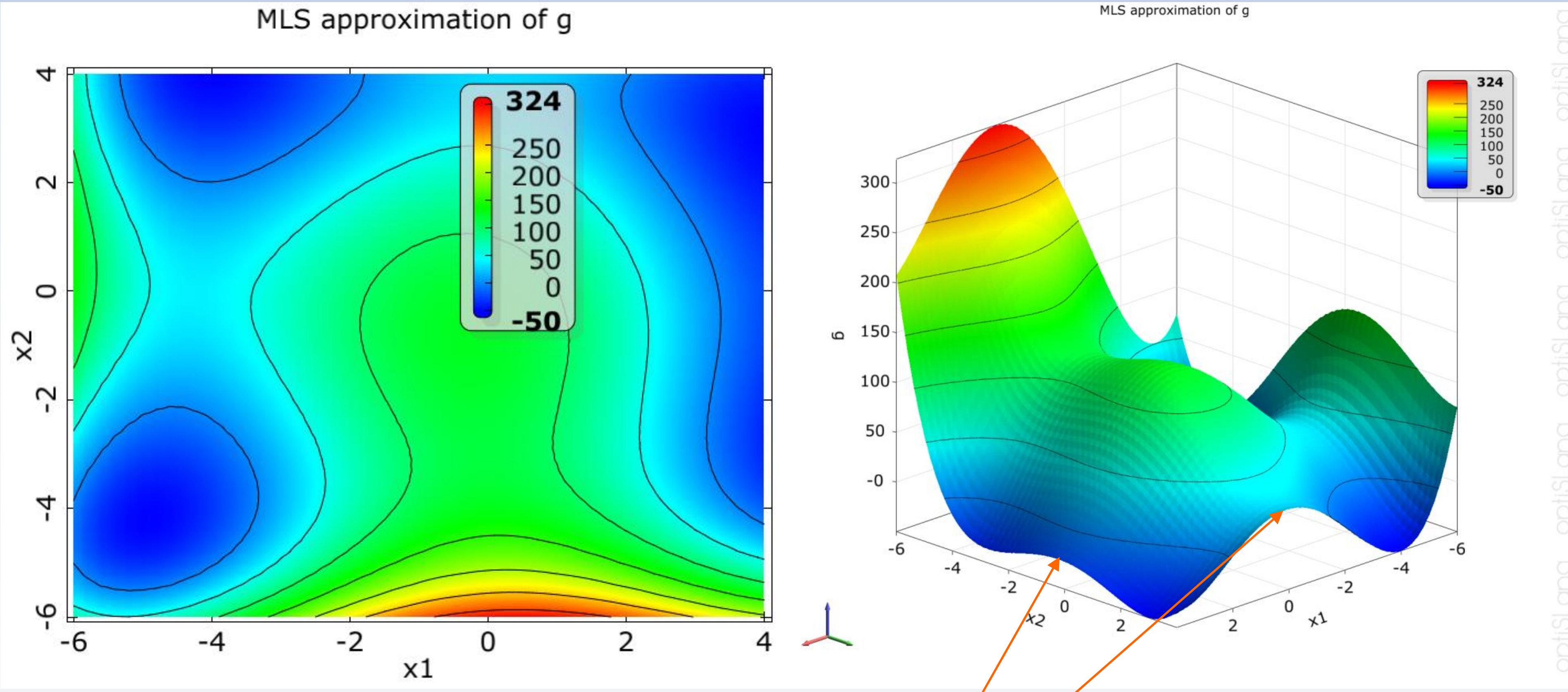
$$P(F) = P[\mathbf{X} : g(\mathbf{X}) \leq 0] = \int \overset{n}{\cdots} \int_{g(\mathbf{x})\leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$



Reliability Analysis

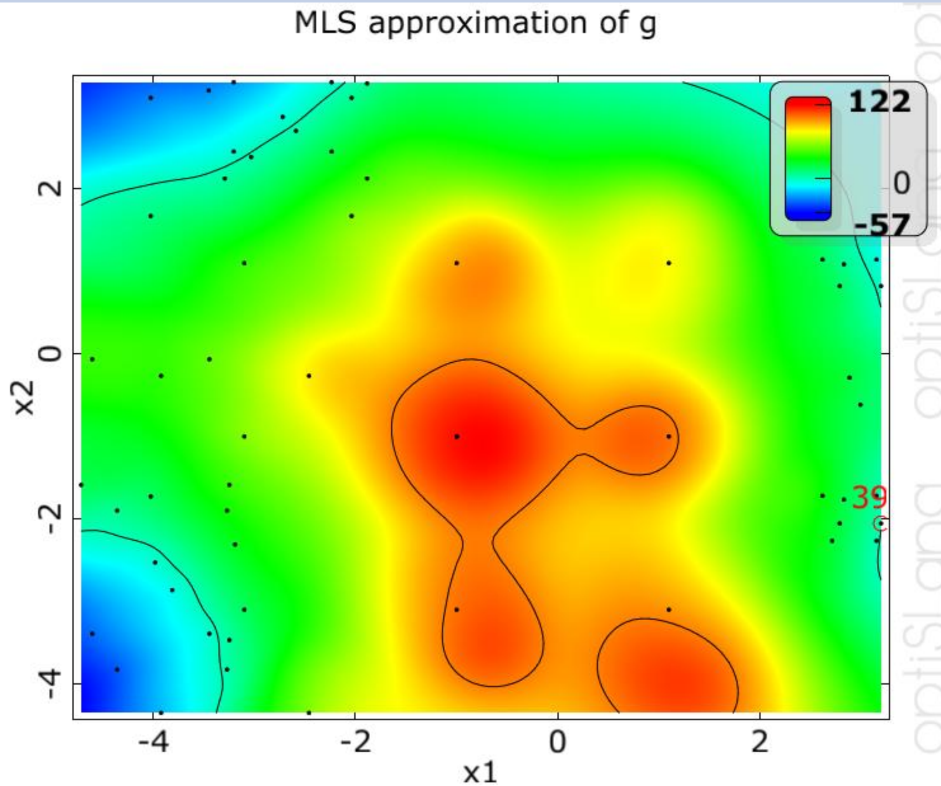


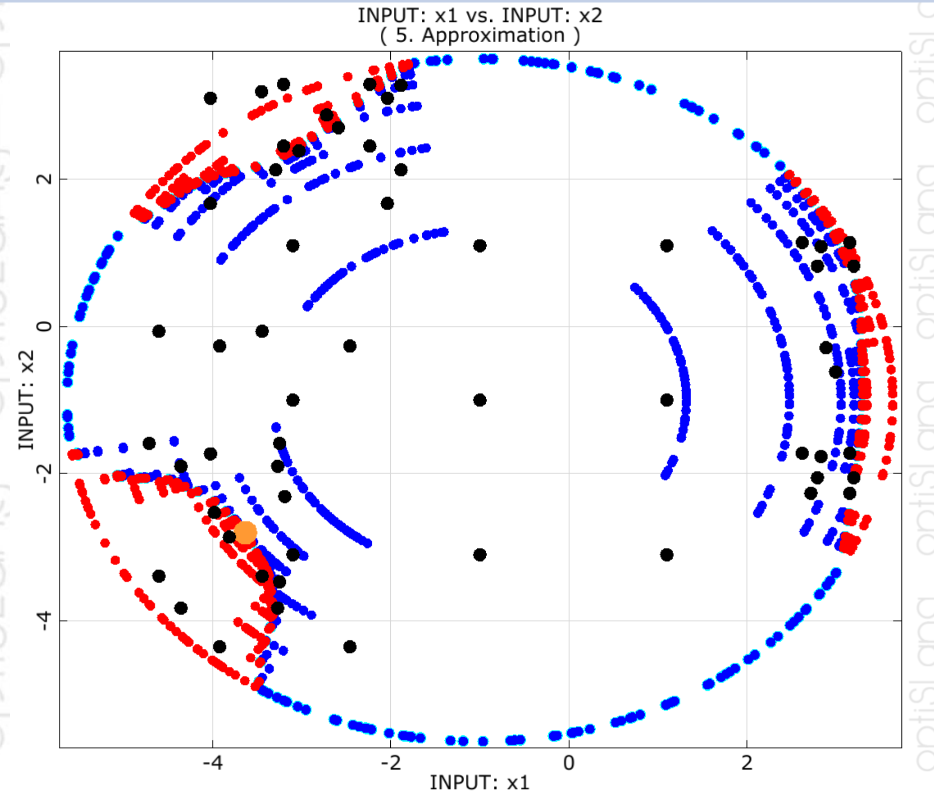
Reliability Analysis



- Himmelblau function
- Nonlinear two dimensional state function g(x1,x2)
- Nonlinear limit state function g(x1,x2)=0
- Three separated domains with high failure probability density

Reliability Analysis

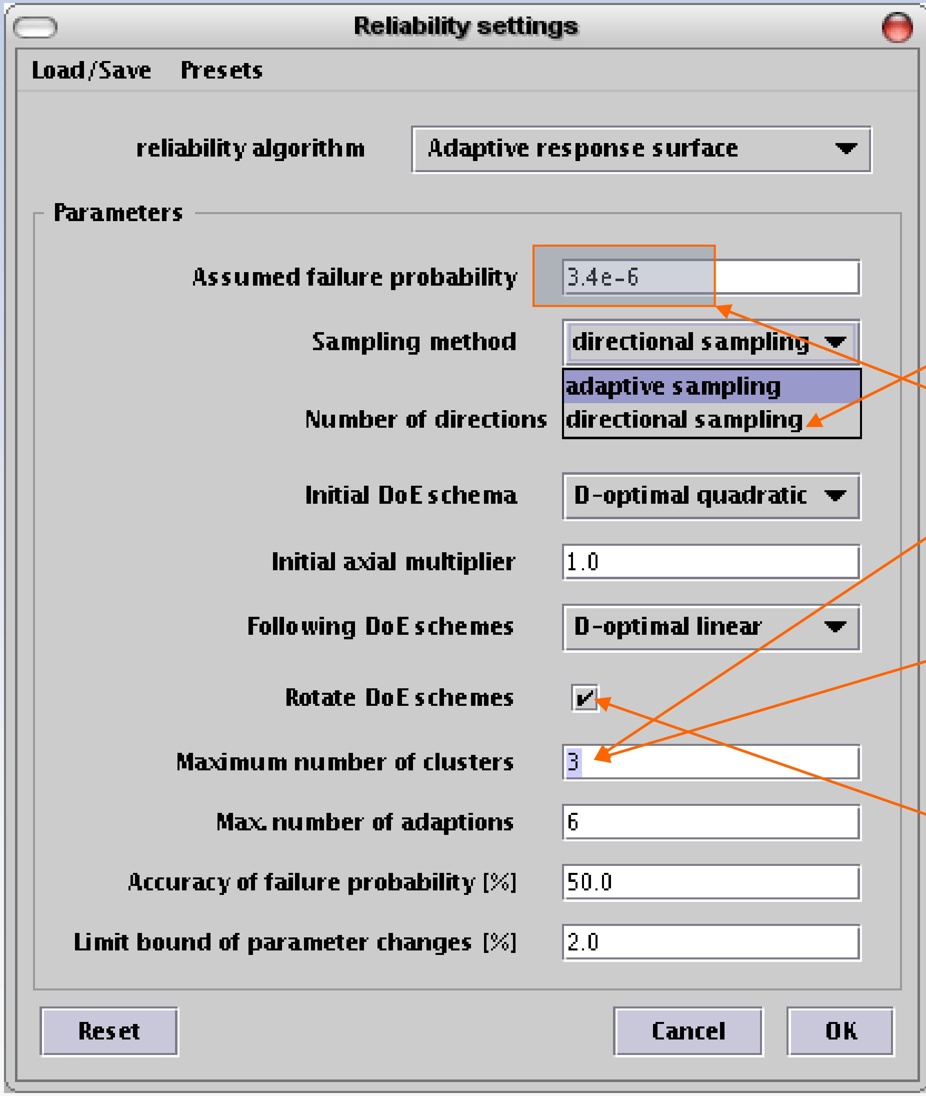




- Adaptive response surface method
- Directional sampling on MLS
- Design evaluations: 58
- PF = 1.67E-06 (1.99E-06)

- Sigma level independent
- n $\leq$ 20
- Multiple adaptive DOE
- Supports multi-domain limit states

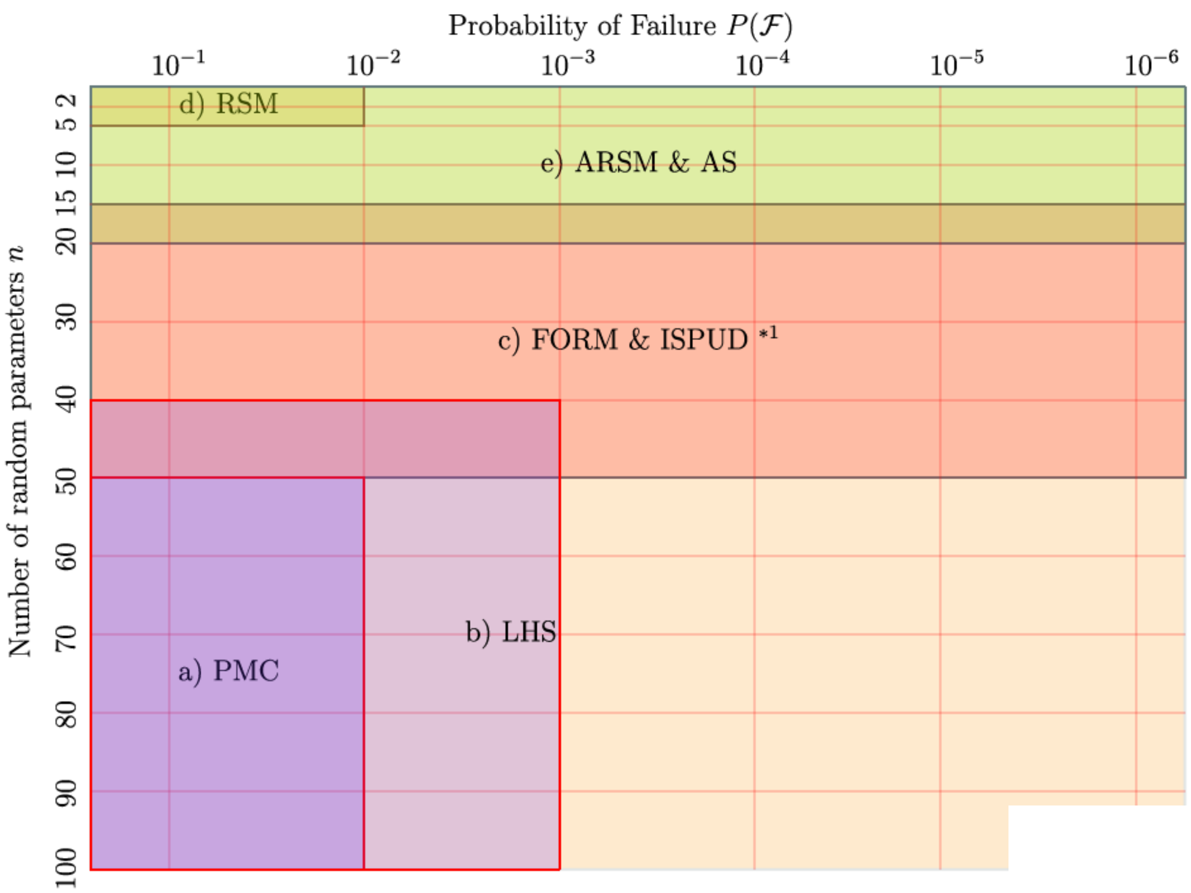
Adaptive response surface approximation



- Sampling methods on the MLS approximation:
  - Adaptive Sampling
  - Directional Sampling
    - supports more than two failure domains
    - and sigma level independent
- Cluster analysis to detect number of failure domains with high failure probability
- Rotatable adaptive designs of experiments to improve the approximation accuracy

Methods of Reliability Analysis

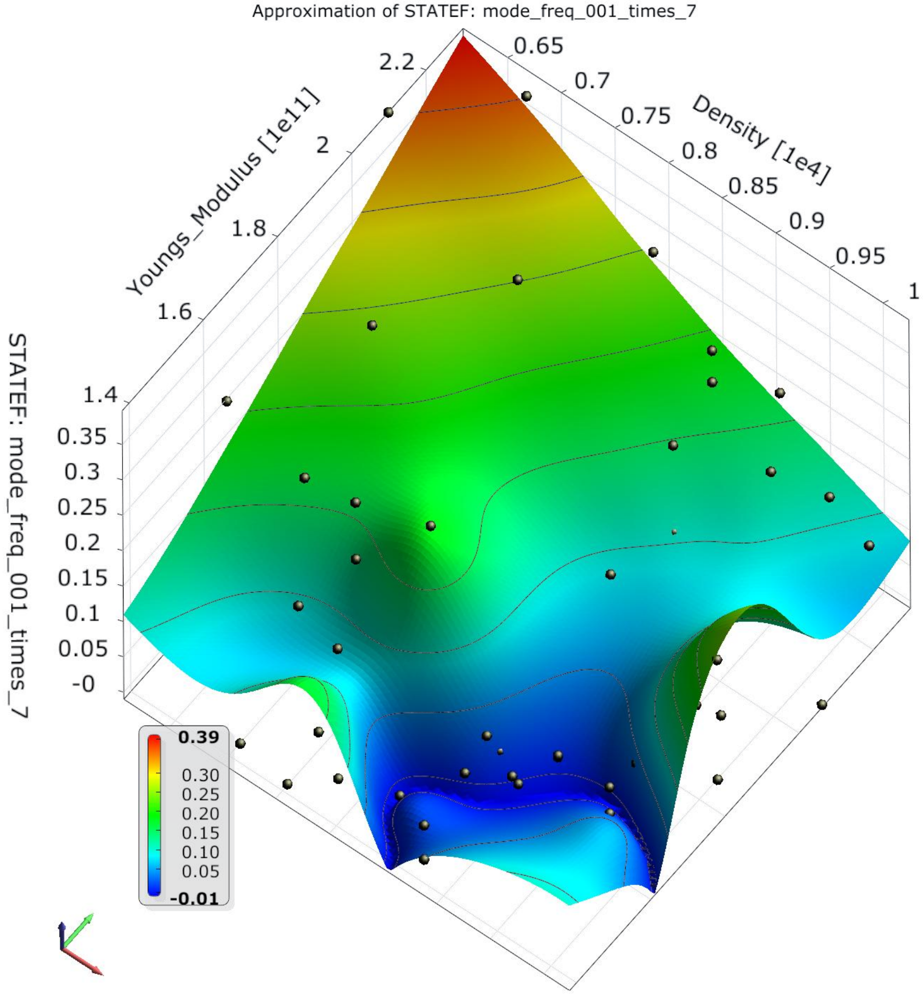
Recommended Areas of Application

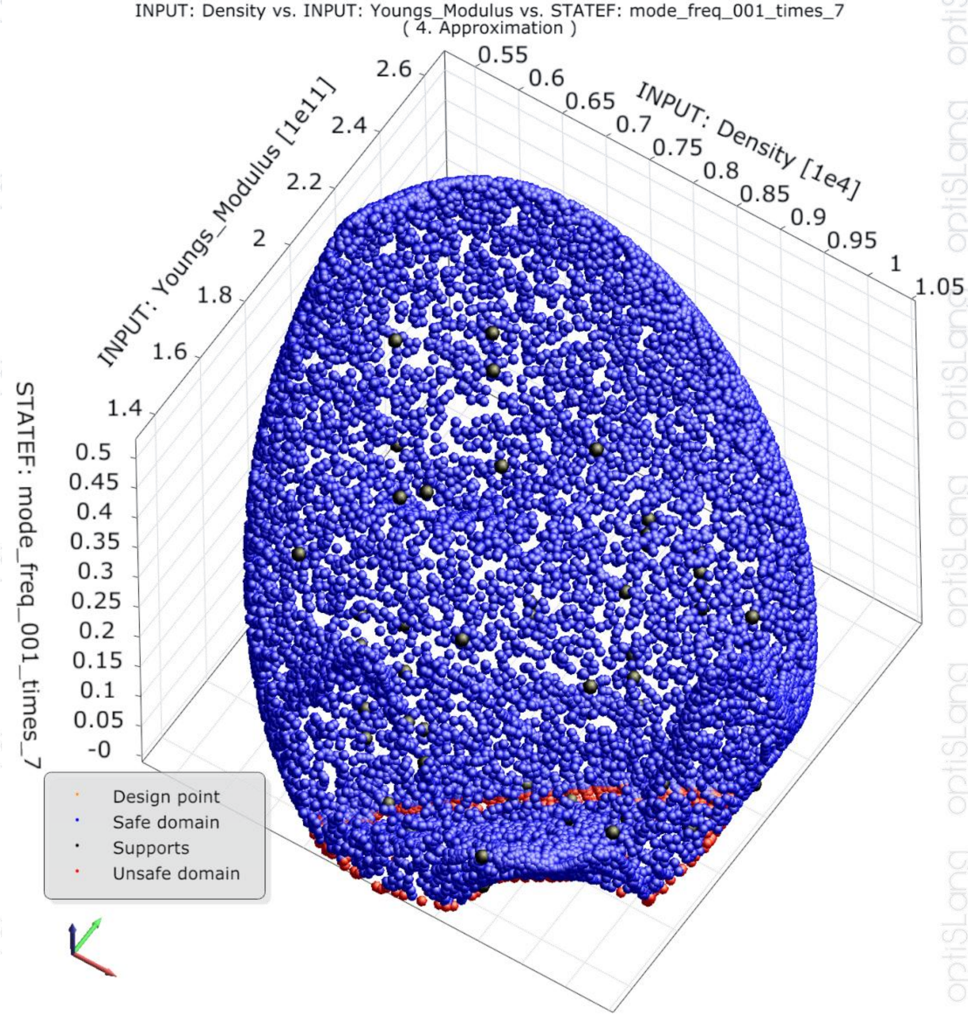


- Sigma level
- Number of random variables
- Number of β-points
- State function (linear, nonlinear, continuously differentiable, noisy)
- Limit state function (high- or slow-curved, continuously differentiable, noisy)

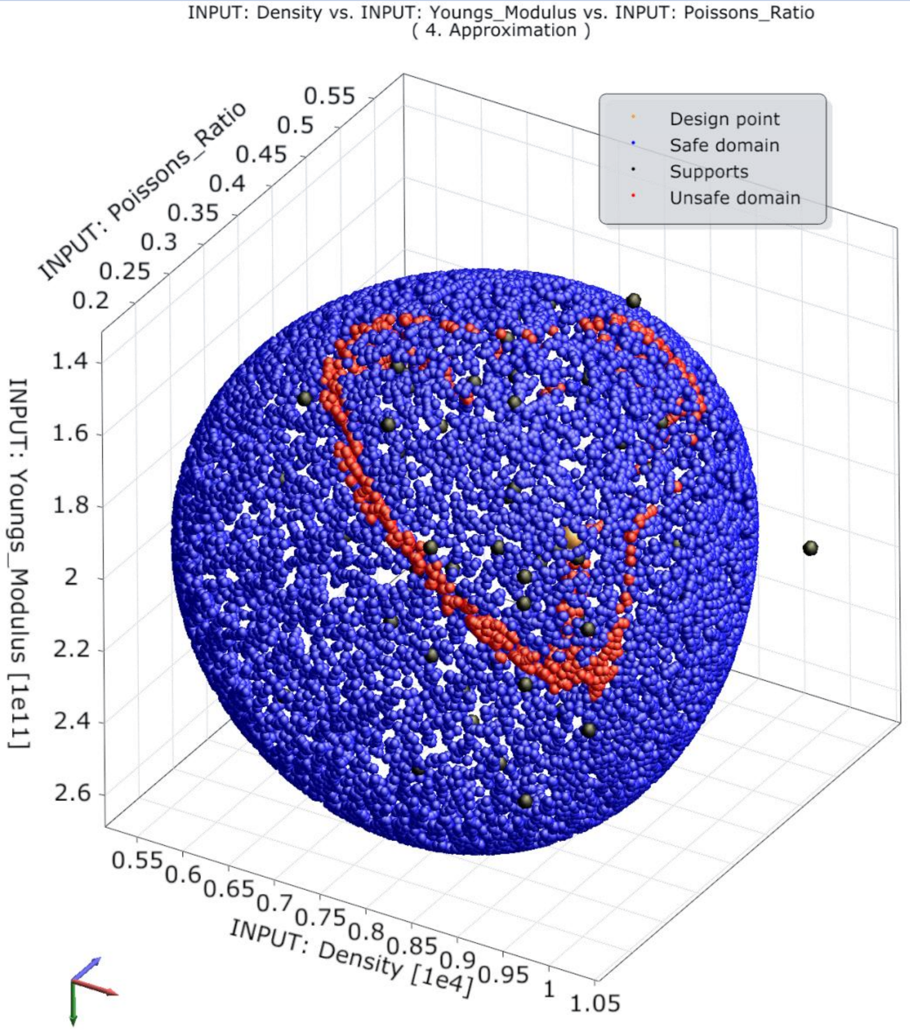
$*^1$ only applicable within continuously differentiable response functions

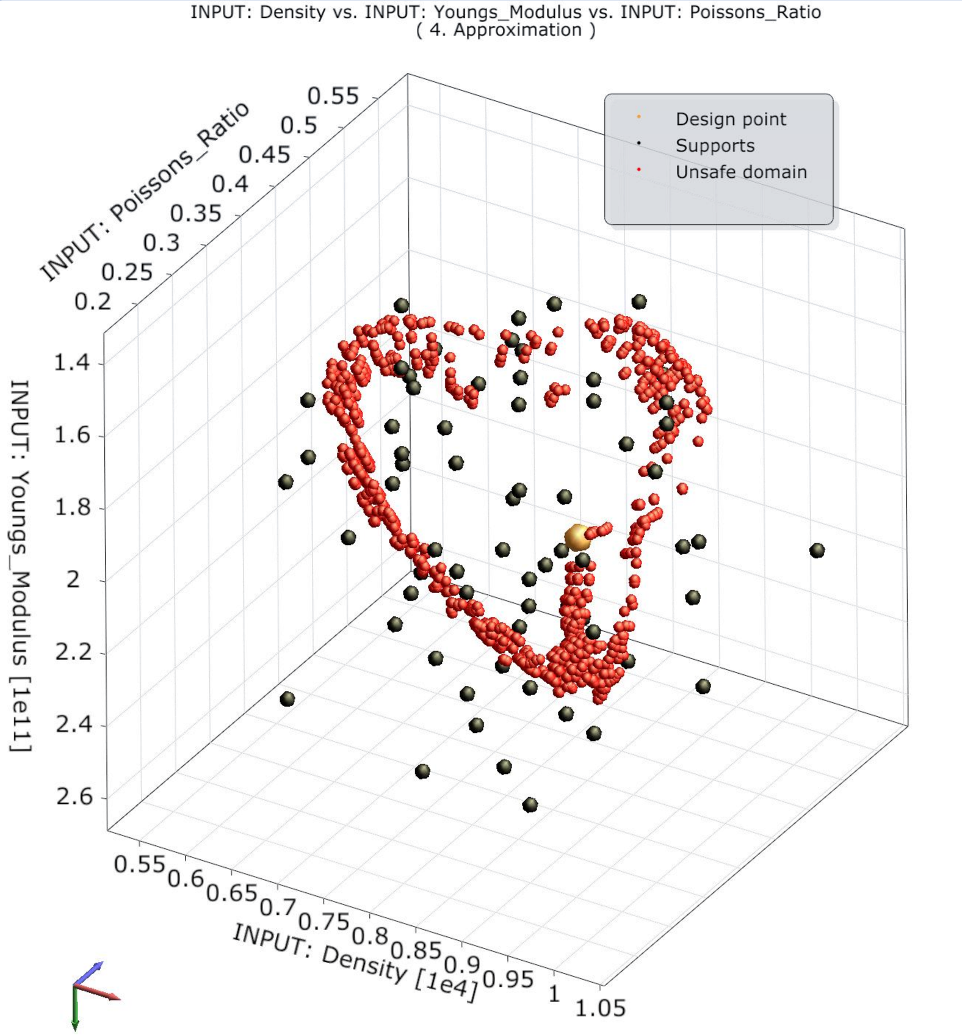
Reliability Analysis



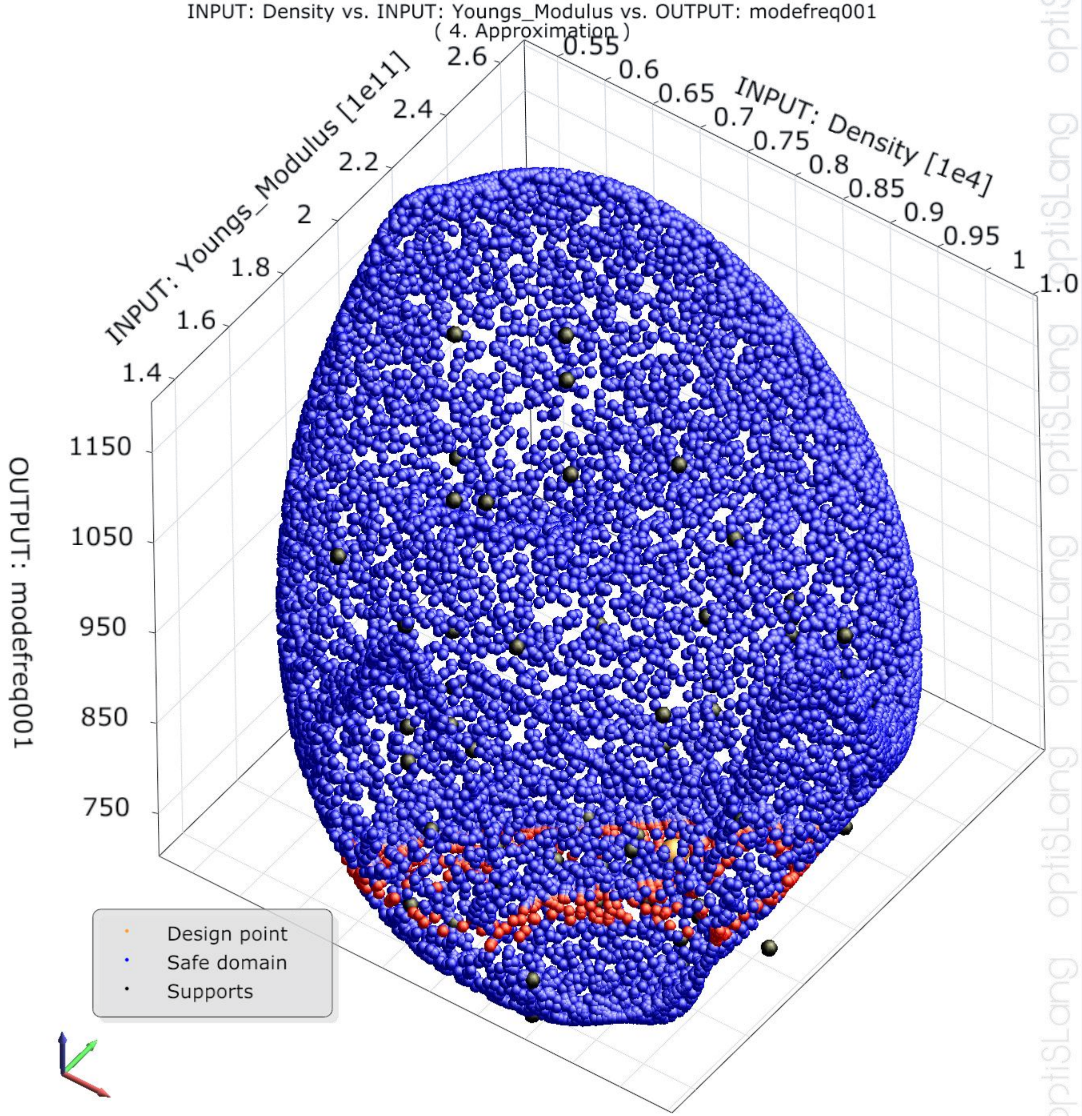


Reliability Analysis





Reliability Analysis



Method : Directional Sampling on Adaptive Response Surfaces (ARSM-DS)

Selected data : 4. Approximation

Number of designs : 68 (4 failed)

Complete directions : 10000 / 10000

Number of samples :
Total : 13640
Safe domain : 11891
Unsafe domain : 1749
Failure strings : 0
Unsuccessful : 0

Probability of failure : 2.75e-007 (2.75e-007)
Standard deviation error : 6.667e-008 (6.667e-008)

Most probable failure point:

Density :9279.86887124
Youngs_Modulus :177034428177
Poissons_Ratio :0.285958415968

Distance median - design point (beta) : 4.328

Probability of failure (FORM): 7.526e-006

- n = 3 random parameters
- N = 68 design evaluations
- $P(\mathcal{F}) \approx 3 \cdot 10^{-7} < 3.4 \cdot 10^{-6}$
- Six Sigma Design

Summary

- Parametric Workflow management
- Automatic and embedded solution
- Parallel and distributed solver runs
- Process integration within optiSLang
- Efficient Robust Design Optimization with
- Quadratic convergence rate and
- 18 design parameters and
- 26 random geometry parameters,
- including the manufactoring tolerances based random field modelling
- Optimized robust design:
  5% improvement of the efficiency (η<90%, failure rate ~4.5%)
  Tolerance limit (1.4<Π_T<1.36, failure rate ~6%)
- Optimized Six Sigma design $P(\mathcal{F}) \approx 3 \cdot 10^{-7}$
- N = 100 + 105 + 84 + 100 + 62 + 50 + 40 + 50 + 68 = 659 design evaluations
  (SA)(EA)(ARSM)(RE)(ARSM)(RE)(ARSM)(RE)(RA)
- Calculation time: 10 days (8 CPUs)